INSURANCE

A PRACTICAL EXPOSITION FOR THE STUDENT AND BUSINESS MAN

EcF
Y77i.

INSURANCE

A PRACTICAL EXPOSITION FOR THE STUDENT AND BUSINESS MAN

BY

T. E. YOUNG, B.A., F.R.A.S.,

Ex-President of the Institute of Actuaries; late President of the International Congress of Actuaries; ex-Chairman of the Life Offices' Association; member of the Actuarial Societies of Belgium, France, and America; late lecturer on the Principles of Insurance at the London School of Economics and Political Science.

SECOND EDITION, REVISED AND ENLARGED

8 2588
6/6/07

"Alternate progress and impediment,
"And yet a growing prospect in the main."
The Recluse.—WORDSWORTH.

LONDON:
SIR ISAAC PITMAN & SONS, LTD., 1 AMEN CORNER, E.C.
BATH AND NEW YORK.

UNIVERSITY OF TORONTO LIBRARY

Printed by Sir Isaac Pitman & Sons, Ltd., Bath.
G
(1555)

PREFACE

THIS book is designed to afford a clear and succinct exposition, in an elementary form, of the principles, theoretical and practical, upon which insurance administration is based and conducted.

Art in every department of executive work naturally precedes science or organized knowledge: actual experience must provide the materials out of which the intellect must educe into generalisations and formulae the scientific principles which the practice implicitly involves; and the rational prosecution of Art into its most manifold and widest forms is conditional upon the clearer and exacter manner in which its intellectual elements are unfolded and systematized.

The art, or practical conduct, of insurance will thus prove to be more readily intelligible to the apprehension of the man of business if the principles which it embodies are explicitly presented; and the scope of this book accordingly comprises, with this object in view, the concurrent description of administration and of the rational methods, derived from experience, on which the actual processes are founded.

In pursuance of this course, an attempt has been made to explain the mode in which the interpretation of the Board of Trade Returns of Companies may be rendered practically serviceable in discerning the financial position of Assurance Offices and the prospects of profits which they are competent to present.

An exposition has also been introduced of the various modes in which the universal and flexible principles of insurance have been applied to provide for popular and commercial needs which the complex development of social and industrial activities demands.

The book, it will be observed, is not intended for the use of experienced experts; but for the student who aspires to the character of an expert, and for the busy practical man, every effort has been employed throughout the volume, though in a rudimentary form, to produce a clear, sound, and intelligible guide to this department of business in its multiform shapes.

In the Examples furnished in the book, different Tables of Mortality have been utilized according to convenience: and this course introduces no difficulty of perception or result since each illustration is of homogeneous character in itself, and the express purpose, moreover, in every part, has been simply the exhibition of *comparative* effects.

It need hardly be necessary to state that no particular Company or method of administration is recommended or disapproved; where any existing or suggested method, however, required definite comment for its complete understanding, a reasonable criticism has naturally been submitted.

The author, while drawing mainly upon his own experience and thought as a practical manager and actuary, gratefully acknowledges his deep indebtedness to the numerous valuable papers and discussions which the *Journal of the Institute of Actuaries* contains; nor should he omit a special reference to Part II of the *Text Book* which has been issued by that Institute—an Association, it may justly be added, which has signally conferred upon the scheme of life assurance, and incidentally upon the general principles underlying every phase of insurance, a distinctive character as a body of scientific truths.

The author is also greatly indebted to Mr. C. J. Bunyon's admirable works upon the *Law of Life* and *Fire Insurance*, and to Mr. William Gow's excellent *Handbook upon Marine Insurance;* he has also helpfully drawn upon the resources of other writers.

A certain amount of repetition has, unhappily, been unavoidable, since some subjects possess aspects which relate to questions discussed in more than one chapter. This fault, he trusts, will be readily pardoned, and he has endeavoured to reduce it to the briefest extent.

November, 1903.

PREFACE TO THE SECOND EDITION

THE present Edition has been most carefully revised. The Author has attempted to remove obscurities ; fresh explanations have been furnished ; and many important additions have been incorporated, so that the work may fulfil more adequately its purpose as an efficient and practical Elementary Treatise.

The Author cordially thanks his many professional friends for the valuable assistance which they have rendered in the improvement and extension of the different sections of the book—a help generously afforded, and here most gratefully acknowledged.

It is deeply gratifying to him to learn that the Authorities of the honoured Yale University have included the work in their list of Text-books.

It may be convenient to state that, in Chapter II. of this Edition, the Author has now introduced his own views upon the principles which should regulate the Limitation of Risks ; he has also, in Chapter V., endeavoured to furnish a simple explanation of the force of Mortality, with an illustration of the conception of the " infinitely small " to certain Assurance functions ; Chapter XIV. on Marine Insurance has been thoroughly revised to the present date ; Chapter XV. upon Accident Insurance has been entirely rewritten and expanded ; and throughout the Volume additional and interesting Tables have been inserted.

August, 1906.

ANALYTICAL TABLE OF CONTENTS

		PAGE
	Preface	iii
	Preface to the Second Edition	v
	Preliminary Remarks	xix

CHAPTER I.

THE NUMERICAL BASIS OF ASSURANCE CALCULATIONS, WITH EXPLANATIONS AND DISCUSSIONS OF COGNATE SUBJECTS.

i.	An exposition of Bernouilli's Theorem of Large Numbers as the basis of the General Doctrine of Probabilities and of the system of Life Assurance	24
ii.	Illustrations of this principle, and the analogies in Life Assurance observations: Defect of Tables showing the progress of a Life fund	25
iii.	The Probable Error, with numerical illustrations	28
iv.	The meaning of Error	29
v.	The detection, by observation, of data more precise than the observation itself	29
vi.	Detailed illustration of the Probable Error	29
vii.	The scientific distinction between an Average and a Mean ..	30
viii.	Explanation of an Average	31
ix.	Explanation of a Mean	32
x.	The Weights of observations, and their use	32
xi.	An illustration of the formation of a Mean	34
xii.	Recapitulation of the distinction between an Average and a Mean, and their respective values	35
xiii.	Illustration of erroneous modes of expression in Life Assurance language	37
xiv.	Criticism of the expression—the Expectation of Life ..	37
xv.	The meaning of the value of a Life Annuity and a Reversion	39
xvi.	The fallacy of Percentages	40
xvii.	The defects of many statistical deductions	40
xviii.	An enumeration of the accidental errors which occur in all observations, and the consequent need of the adjustment or graduation of the results..	40
xix.	The object of graduation	40
xx.	A brief description of some methods of Graduation.. ..	41
xxi.	An explanation of " portions " and " proportions " of diminution in the power to resist death	43
xxii.	The conditions of Validity of a method of Graduation ..	44
xxiii.	The assignment of weights to observations in any system of Graduation	45
xxiv.	The necessity of Graduation in forming a basis for Assurance Premiums	45
xxv.	The deletion of actual physical phenomena occasionally produced by some systems of Graduation, with an illustration..	45
xxvi.	A suggestion for the concurrent presentation of original and adjusted data	46
xxvii.	The Method of Least Squares, with illustrations of its application	46

CHAPTER II.

THE RISK AND ITS LIMITATION; WITH EXPLANATIONS AND DISCUSSIONS OF COGNATE SUBJECTS.

		PAGE
i.	The Nature of Risk generally in the doctrine of Probabilities	50
ii.	The risk in Life Assurance	50
iii.	The diminution of the amount at risk in consequence of the Reserves created	51
iv.	The mode in which the determination of the Limit of Risk in Assurance Companies has been suggested in the past	51
v.	As the area of distribution of risk is widened, the chance of inordinate loss and the chance of the absence of loss concurrently diminish	52
vi.	Illustrations and examples	52
vii.	The practical method of settling the amount of the Limit on a single contingency formerly suggested	53
viii.	The views upon the subject of the present Author	55
ix.	The successive increase of the limit of risk	60
x.	The adoption of the maximum limit in the acceptance of "rated-up" lives	60
xi.	The assurance of special risks, and the conditions on which this course depends, with explanations and illustrations	61
xii.	The Expectation of Loss	64
xiii.	The most probable number of claims and losses	65
xiv.	The number of observations on lives required to provide for fluctuations of occurrence within specified limits; and various illustrations of the mode of calculation	66
xv.	Remarks upon the necessity of an adequate number of risks in the Valuation of various Societies	69
xvi.	The mode of constructing the "loading" added to the pure premium for expenses and profits, with illustrations in each instance	70
xvii.	Will a percentage addition to the pure premium compensate the same percentage addition to the rate of mortality?	74
xviii.	A consideration of the uniform or level, and Assessment, scales of Premium	75
xix.	Tontine schemes	76
xx.	The mode of acceptance of lives for Assurance	77
xxi.	The question of "rated-up" lives: the incidence of additional mortality; the removal of extra premiums; the schemes proposed for avoiding the imposition of an extra premium in the ordinary form; and the lives of teetotallers	78
xxii.	The mode of assessing a general extra premium, with examples of Female lives and Publicans	85

CHAPTER III.

THE MEANING AND EFFECT OF SELECTION.

i.	The meaning of the term Selection in Life Assurance	87
ii.	The two-fold process of selection concurrently operating	87
iii.	The selection exercised by the Company: its object and effect	87
iv.	The mode of "rating-up" inferior lives	88
v.	The selection exercised by the Policy holder, and its effects	89
vi.	A numerical example of the probabilities of death, deduced from a Mixed Table of Mortality and a Table of Select Mortality	89
vii.	Explanation of these examples	90
viii.	The operation of Mortality in Successive years of Assurance	90
ix.	The rate of Mortality among Assured lives is a function of two variables	90

		PAGE
x.	Explanation of the terms employed in connection with Selection	90
xi.	The duration of select effects	91
xii.	The consequent construction of the H^m (5) and O^m (5) Tables, and their employment	91
xiii.	Table of the Mortality per cent. for certain periods of Assurance	91
xiv.	Notes on the Table..	92
xv.	Explanation of the annual rate of Mortality	92
xvi.	Mixed and Select Tables: the mode of construction of a Mixed Table	92
xvii.	Composite character of the rate of Mortality among Assured lives	92
xviii.	The mode of construction of a Select Table of Mortality, and its character	93
xix.	The advantage of the employment of Select Tables, especially in the formation of Premiums	94
xx.	The scientific character of Select Tables..	94
xxi.	An explanation of the effect of employing Mixed Tables upon the Premiums and Annuity-Values for young and old lives..	95
xxii.	Examples of the probabilities of life, premiums, annuities, and reserves by a Mixed and a Select Table	97
xxiii.	The appropriateness of Select Tables in the settlement of Surrender-Values	97
xxiv.	A description of the symbols used for the functions derived from Select Tables	98
xxv.	An explanation of the differences between the values of premiums, annuities, and reserves produced by Mixed and Select Tables	98
xxvi.	An explanation of the comparative effect of Mixed and Select Tables at old ages	99
xxvii.	Examples of reserves according to Mixed and Select Tables..	100
xxviii.	An explanation of the differences in the reserves	101
xxix.	The addition to be made to the Annuity-Value in obtaining the Reserve in a single instance and in a classified mass of Policies	101

CHAPTER IV.

Surrender Values.

i.	Popular misapprehension upon the subject	103
ii.	The absorption of part of the contract premium in the payment of claims and of the other portion in expenditure ..	103
iii.	An illustration of Reserves in relation to the Premiums received	104
iv.	In an Assurance Company the interests of all members must be equally considered	104
v.	Lapses and surrenders increase the rate of mortality among the remaining members, since on the whole, the withdrawals occur upon sound lives	105
vi.	The surrender-value, therefore, will be governed by equitable treatment of all the members	105
vii.	A discussion of the effects produced by withdrawals ..	105
viii.	A numerical illustration of the proposition that the withdrawal of members exercises the same effect upon the general rate of mortality of the continuing members as the introduction of a number of inferior lives	107
ix.	A description of an attempt to measure numerically the result of withdrawals, and the defect of that attempt: the true method mentioned	110

		PAGE
x.	The adverse effect of withdrawals on the rate of expenditure in a twofold aspect	111
xi.	The conditions on which surrender-values may be granted in excess of the ordinary scale	113
xii.	The preceding considerations apply to forfeited Policies	114
xiii.	Companies endeavour to prevent lapses and surrenders by means of advances	114
xiv.	The Policy may be maintained in force while the surrender-value suffices to provide the premiums	114
xv.	The amount of surrender-value to be granted in the case of "rated-up" lives	115
xvi.	A history of the published methods suggested for the computation of surrender-values	117
xvii.	A summary of the principles underlying the calculation of surrender-values	122
xviii.	The adoption of a uniform scale of surrender-values	123
xix.	Extra premiums possess no surrender-value	123
xx.	The amount paid in surrender-values in a year by Assurance Companies	123
xxi.	Popular misconception of the amount of surrender-values corrected by a numerical illustration of the progress of a life fund	124
xxii.	The coincidence thereby exhibited of the retrospective and prospective methods of valuation	126
xxiii.	The percentage of the surrender-value upon the reserve should be an increasing one	126
xxiv.	The value to be allowed for the surrender of a Bonus	126

CHAPTER V.

The Meaning and Process of the Periodical Valuations of the Liabilities and Assets.

i.	A valuation ascertains the relation of actual experience to that anticipated with respect to mortality, interest, and expenses, and by means of a distribution of profits effects an equalization of these sets of elements	127
ii.	The three-fold meaning of a valuation explained	128
iii.	The skill, knowledge, and prevision which a valuation involves	128
iv.	An illustration of the distinction between the vitality of males and females, with an explanatory analysis, and the consequent difference of judgment in selecting the appropriate rates of mortality and interest	129
v.	The process of a valuation described	131
vi.	The identity of results obtained by a valuation of Policies individually and in classes	132
vii.	The mode of classifying Policies for valuation at a common age	133
viii.	The classification for valuation purposes of "rated-up" lives	134
ix.	A complete example at a particular age of the method of valuation, extracted from the Board of Trade Returns (with proportionately modified figures to avoid identification)	135
x.	Check Valuations	137
xi.	The production of the valuation balance-sheet	137
xii.	The surplus, equilibrium, or deficiency resulting; and the mode of rectification in the latter event	138
xiii.	The consideration of the proper proportion of the resulting surplus to be distributed	139
xiv.	The conservation of the interests of existing members, and the regard also of the advantages to fresh entrants	139

		PAGE
xv.	The question of a considerably reduced rate of interest for valuation purposes	140
xvi.	Definition of the pure premium	141
xvii.	Definition of a pure premium valuation..	141
xviii.	The homogeneity of a pure premium valuation	141
xix.	A statement of a heterogeneous though sound mode of pure premium valuation in respect of the rate of interest employed..	142
xx.	An illustration of an instance where different rates of mortality are involved	142
xxi.	The mode of valuing Assurances upon " rated-up " lives ..	143
xxii.	The method of forming the adjustment of the customary valuation factors..	145
xxiii.	The adjustment of the values of the reversions employed ..	145
xxiv.	The adjustment for the unequal incidence of the premium revenue	146
xxv.	The adjustment for provision of loading on Assurances subject to single premiums and to a limited number of premiums	148
xxvi.	The reserve for Policies subject to discounted-bonus premiums	148
xxvii.	The reserve for temporary Assurances	149
xxviii.	The reserve for extra premiums consequent upon foreign residence or hazardous occupation	149
xxix.	The question of negative values	150
xxx.	The treatment of the loading in a valuation, with illustrations	152
xxxi.	The necessity, for an analysis of resources, of obtaining the value of the loading, and the mode of comparing the reserved ratio of loading to premiums with the rate of expenditure actually incurred	152
xxxii.	Remarks upon a valuation at $2\frac{1}{2}$ % interest in relation to the loading	153
xxxiii.	A qualification of the criticism upon the absorption of a portion of the loading involved in a valuation at $2\frac{1}{2}$ % ; with a numerical illustration of the results obtained in two Companies, one valuing at $2\frac{1}{2}$ % and the other at 3 % ..	155
xxxiv.	A 3 per cent. Valuation appropriate under present conditions ; with considerations on the amount and purpose of the sum to be retained as a precautionary Reserve	156
xxxv.	The effect of employing a mixed table of mortality for the valuation of Assurances upon lives recently selected ..	156
xxxvi.	The effect upon valuation reserves of an alteration in the rates of interest and mortality	157
xxxvii.	The effect upon pure premiums and reserves of the absence of interest altogether	163
xxxviii.	A description of the retrospective and prospective methods of valuation, with a numerical illustration	165
xxxix.	The re-insurance or hypothetical method of valuation ..	168
xl.	A statement of the conditions under which the valuation-reserves of the hypothetical and pure premium methods coincide	169
xli.	A discussion of the pure premium method of valuation under varying circumstances	170
xlii.	Consideration of the suggestion of a special modification of the pure premium method where the new business of a Company is procured at inordinate expense	171
xliii.	The mode of tabulating the materials for a Valuation ..	174
xliv.	The Force of Mortality (in one of its forms) mentioned ; with illustrations relating to the increase of the rates of premium	

		PAGE
	at the older ages compared with that at younger ages; the abrogation of Reserves under certain conditions; and other questions	174
xlv.	The actual rate of interest in relation to income-tax and depreciations of value	176
xlvi.	The mode of treatment of the investments and securities at a valuation	177
xlvii.	The considerations which should govern the assessment of the values of the investments	177
xlviii.	Illustrations of the effect of variations in prices	178
xlix.	The general practice of Companies in this respect	179
l.	The "quarter-average" plan..	179
li.	Suggestion of a basis founded upon the relation between solar energy, physical conditions, and human enterprise ..	179
lii.	The course recommended until some such scientific mode is found to be feasible	180
liii.	The Force of Mortality defined and explained, with examples; illustrated by a fundamental principle in Kinetics and in all measurements of Change; its relation to q_x; its approximate values; its relation to the Central death-rate; a rough concrete illustration..	181
liv.	Appendix I.: On the proper value and amount of a sum in respect of fractions of a year	189
lv.	Appendix II: An explanation of the expressions "infinitely small and great" and "limits" in connection with the Force of Mortality; with an application to certain Assurance functions	190

CHAPTER VI.

The Sources of Profit and its Distribution.

i.	Confusion of terms: the definitions of surplus and profit ..	194
ii.	Considerations to be kept in view in deciding upon the amount of surplus to be distributed as profit	195
iii.	The sources of surplus: excess interest, moderate expenditure, and favourable mortality	195
iv.	The question of surplus from lapses and surrenders.. ..	196
v.	The surplus from reversions purchased, and its treatment ..	196
vi.	Interest as the chief source of surplus	197
vii.	Capacity in Finance the supreme requirement of the Actuary	197
viii.	Surplus from loading	198
ix.	Economy of management a principal duty and test of the Actuary or Administrator	199
x.	The considerations which should regulate the amount of new business	199
xi.	A tabular statement of the proportion of the new premiums in a year which is absorbed in expenditure	202
xii.	Consideration of the suggestion that the analysis of the total expenditure between the new and renewal business affords a test of comparative economy	202
xiii.	The conditions which should control the extent of new business	203
xiv.	The case of a Company which has fallen into an enfeebled condition	203
xv.	The suggestion that the benefit of selection should be utilized as a counterpoise to increased expenditure	204
xvi.	Should the first year's premium with the same object be excluded from participation in profits?	204

		PAGE
xvii.	Life Companies exist for family provision and profit, and not for general social ends	205
xviii.	The progress of Life Assurance expenditure	205
xix.	The demands which new premiums are required to satisfy, and the effect on the bonuses of existing Policy holders ..	205
xx.	An illustration from the Board of Trade Returns of the relation between rates of expenditure and profits	206
xxi.	Every aspect of Life Assurance administration possesses actuarial implications	207
xxii.	Practical suggestions in assessing the true burden of expenditure upon the premium revenue	208
xxiii.	Mortality as a source of surplus	209
xxiv.	The case where a Company would genuinely obtain a profit from mortality	209
xxv.	The case where Companies adopt a general Table of mortality as the basis of premiums and reserves correspondent with the class of lives admitted : does the excess of the expected number of deaths (or claims) above the actual number constitute surplus in the sense of profit ?	210
xxvi.	A greater number of deaths and a larger demand in claims than those anticipated may be more favourable than a fewer number and a reduced amount, and *vice versâ* ..	211
xxvii.	Description of a mode of ascertaining the surplus or deficiency due to the mortality experienced	212
xxviii.	Numerical illustration of the method, showing the futility of the usual statements made respecting the favourable or adverse result of the mortality experience	214
xxix.	Description of a method for deducing the beneficial or unfavourable results of the mortality experienced during the course of a quinquennium	216
xxx.	The surplus on mortality a function of the individual amount assured, or of the rate of premium per unit assured ..	217
xxxi.	Analysis of a Company's actual experience of the proportions of surplus from interest, mortality, and loading	218
xxxii.	The principles upon which the method of distribution of profits should be founded ; with incidental remarks dealing with the creation of separate series of participating Policy holders ; the attempted analysis of different scales of contribution to profits ; the probably main future factor in modes of distribution, and other questions	218
xxxiii.	A statement of some curious modes of division in the past, and of one even now, though very rarely, employed ..	222
xxxiv.	Other methods described and criticised :—	
	(1) In proportion to the accumulated premiums ..	224
	(2) In proportion to the reserves	225
	(3) The deferment of participation until the accumulated premiums equal the sum assured	226
	(4) The compound reversionary bonus scheme	226
	(5) Consideration of the conditions on which the compound system is feasible, with a numerical illustration ..	226
	(6) The mode of obtaining the uniform annual reversionary addition	228
	(7) A method based upon the specific appropriation of the surplus-interest earned	229
xxxv.	The difficulty introduced into the distribution of profits by the present preponderant increase of Endowment Assurances	231
xxxvi.	The extension of Endowment Assurances largely aided by the agitation relating to Old Age Pensions	231

PAGE

xxxvii. The mode in which a change in the rate of mortality affects the two component elements of an Endowment Assurance 231
xxxviii. The prospect of profit from Endowment Assurances .. 231
xxxix. A comparison of the rates of mortality under whole-life Policies and Endowment Assurances.. 233
xl. The unsatisfactory effect on the genuine scheme of Life Assurance of the increase of Endowment Assurances .. 233
xli. The experience of a Company in respect of the rate of mortality in Bonus options 235
xlii. The mode of converting a cash bonus into its equivalent reversionary amount 236
xliii. The plan to be pursued when a change in the method of distribution has become imperative 236
xliv. The expenses of Industrial Companies 238
xlv. The relation between the general rate of expenditure and the average amount per *new* Policy exhibited by an analysis of the Board of Trade returns and other statistical sources.. 239

CHAPTER VII.

The Investments of a Life Office.

i. Life Assurance Administration a commercial trust with wide powers 240
ii. The paramount value of capacity in finance and economy of management in successful Assurance work 240
iii. Advice to students upon these points 240
iv. The principles of Life Assurance finance described, with illustrations 242
v. A statement of securities which as a rule should be avoided 245
vi. The question of deposits in discount and banking institutions 246
vii. Mortgages upon real estate in the Colonies and Foreign countries 246
viii. The purchase by a Company of its own shares discussed .. 247
ix. Examples, with observations, of the mode in which mortgages upon leasehold properties, and upon reversions, and advances on personal security, should be scrutinized and conducted .. 248
x. The fallacy of the dictum that high interest signifies imperfect security 258
xi. The mode of accounting when reversionary interests are purchased 259
xii. The retention of a portion of the surplus from the early realization of reversions for the purpose of compensating the deficiency of return in reversions whose falling-in is postponed 260
xiii. The elements of computation of the value of a Life Interest and an Absolute Reversion.. 261
xiv. The analysis of the conditions in reversions in order to provide a formula, with an explanatory example 264
xv. The mode of expressing a professional opinion upon the value of reversionary estates 266
xvi. The duties of the Administration of a Life Office in obtaining the highest reasonable rate of interest.. 266

CHAPTER VIII.

The Interpretation of the Board of Trade Returns.

i. The value of the Life Assurance Companies' Act of 1870 .. 268
ii. The relation of State control over the basis of valuation to Governmental responsibility for solvency 269

		PAGE
iii.	A brief analysis of the requirements of the Act	270
iv.	The rates of mortality on participating and non-participating Policies	270
v.	The two primary elements to be considered are the ratio of loading reserved compared with the rate of expenditure, and the realized return from interest in relation to the valuation rate	271
vi.	The particulars of a Company selected, and an analysis exhibited of its returns in the preceding respects.. ..	271
vii.	Explanation of the formula for obtaining the effective rate of interest	274
viii.	A note upon a completer demonstration of the formula for determining the effective rate of interest	276

CHAPTER IX.

The Winding-up, Transfer, and Amalgamation of Assurance Companies.

I. Winding-up.

i.	Causes of insolvency in Life Assurance enumerated.. ..	277
ii.	The provisions of the Act of 1870	277
iii.	An explanation of the fact that in a reconstruction the Policies of longer duration sustain a heavier reduction than more recent Assurances	278
iv.	Methods of valuation as a basis for reconstruction	280

II. Transfer.

i.	Definition of the term	281
ii.	Basis of valuation precedent to a Transfer	282
iii.	The limits of the scale of valuation	282

III. Amalgamation.

i.	Definition of the term	283
ii.	Adjustment of the interests of each Company for equivalence of future conditions	283
iii.	A description of methods proposed for this purpose.. ..	284
iv.	Powers of Transfer and Amalgamation contained in some deeds of settlement	288
v.	Novation	288
vi.	Observations upon Amalgamations generally	288

CHAPTER X.

The Uses of Life Assurance to the Business Man.

i.	A Pure Endowment, or the provision of a fund at a deferred age	290
ii.	An Endowment Assurance, or the provision of a fund at a deferred age or at earlier death	290
iii.	An Endowment Assurance, with an income receivable after the deferred age	290
iv.	Policies where the sum assured at death is paid by instalments	291
v.	Policies remaining at death in the Company's guardianship, with the payment of a settled rate of interest	291
vi.	Joint-Life Policies on husband and wife	291
vii.	Policies to secure a life-estate which will be forfeited to the family by death	291

		PAGE
viii.	The provision of a fund for children on the attainment of adult age or marriage	291
ix.	The provision of a Whole-Life or Endowment Assurance Policy for a child at a low rate of premium, provided that death occur after adult age..	292
x.	A Policy for a term of years to protect temporary business interests	292
xi.	Partnership Assurances to compensate the abstraction of capital from the firm on the death of a partner	292
xii.	Policies to render absolute and marketable a contingent reversionary interest	293
xiii.	Policies to protect the expectancy to property which would be defeated by the birth of issue	293
xiv.	Policies to protect the value of estates which are held on condition of the retention of a special "Name and Arms"	293
xv.	Policies to protect a pecuniary interest in the event of a present lunatic holder recovering sanity, and otherwise disposing of it	294
xvi.	Policies to provide against the loss of capital invested in the purchase of leasehold properties, and the cancelment of the premium on Bonds redeemable at par	294
xvii.	Policies to protect debts	294
xviii.	Policies effected by the wife or the husband for the benefit of the family under the Married Women's Property Acts ..	294
xix.	The question of Trusteeship under such Policies.. ..	294
xx.	Policies which include exemption from the payment of premiums during physical or mental incapacity produced by accident	295
xxi.	Policies for the provision of estate duty under the Finance Act of 1894	295
xxii.	Policies effected at minimum rates of premium in two modes	295
xxiii.	Policies without medical examination	296
xxiv.	Annuities of various kinds	296

CHAPTER XI.

A Brief Summary of some chief legal principles affecting Life Assurance.

i.	The nature of the contract of Life Assurance	297
ii.	The principle of *Caveat emptor* inapplicable	297
iii.	All material facts to be disclosed, and the interpretation of materiality of statement	297
iv.	The nature of the answer to be returned to any question if positive evidence be not possessed	298
v.	In the employment of an agent in the submission of a proposal, the obligation of complete disclosure exists ..	298
vi.	The distinction between warranties and representations ..	298
vii.	The responsibilities of referees	299
viii.	A curious experience of the author cited respecting a referee's ambiguous replies	299
ix.	The proposer is not responsible for the fraud of a referee if he himself be innocent	300
x.	The nature of the requisite insurable interest, and a detailed statement of its constitution under various conditions and contingencies	300
xi.	The form of Policy: its construction and conditions; with the mode of rectification of any errors; the payment of the premium, and renewal notices	303

		PAGE
xii.	The Assignment of Policies, and Notices of Assignment; the priorities of encumbrances treated in detail; the Customs and Inland Revenue Act of 1888 described and discussed; the effect of Bankruptcy; a statutory form of Assignment	305
xiii.	Death, and the presumption of death; a striking experience of the author	310
xiv.	Payment of the sum assured into Court	311

CHAPTER XII.

Fire Insurance.

i.	The question of the reduction of fire insurance statistics to a scientific form	313
ii.	The difficulties of an attempt of this nature involved in partial loss, moral hazard, and other elements of risk	313
iii.	Description of two attempts to introduce a quasi-scientific method into the tabulation of Fire Insurance statistics	315
iv.	The Contract: its nature and conditions	318
v.	The Proposal and its statements; the Cover note; special incidents that may occur; warranties and representations; the survey	318
vi.	The Insurable Interest: the terms "legal" and "equitable" defined: Trustees: Mortgagees: protection of purchasers	320
vii.	The nature of the Policy: Specific: the average clause: Floating Policies: Valued Policies	321
viii.	The period of protection under Fire Policies	324
ix.	Alterations in the premises	325
x.	The causes leading to the forfeiture of a Policy	326
xi.	Assignments and Notices	326
xii.	The Loss, with its incidents and conditions, including a loss of profits; the payment of expenses; partially damaged goods; with illustrations of the measure of loss	327
xiii.	Reinstatement	330
xiv.	Arbitration	331
xv.	The adjustment of the loss	331
xvi.	Successive losses	331
xvii.	Double Insurance	332
xviii.	The Insurance of rent	332
xix.	Insurance by a tenant	332
xx.	The Fire Offices' Committee	332
xxi.	The great advantages to the public conferred by this Committee; the tabulation of combined statistics of experience as the foundation for rates	333
xxii.	The London Salvage Corps	334
xxiii.	Non-Tariff Companies	334
xxiv.	The preservation of Property from Fire a National concern	334
xxv.	Fire inquests	335
xxvi.	Municipal Insurance criticised	335
xxvii.	The change of style in the Calendar: an incident	337
xxviii.	The Reserve for unexpired risk	337
xxix.	The comparatively small profits of Fire Insurance shown over a series of years	338

CHAPTER XIII.

Marine Insurance.

i.	A brief history of Lloyd's	340
ii.	Other historical notes, including the public services rendered by Lloyd's	343

CHAPTER XIV.

Marine Insurance (*continued*).

		PAGE
i.	A Contract of Indemnity	348
ii.	Consideration of the anticipated profits from the sale of the goods insured	348
iii.	The conduct of the business	348
iv.	Shipowners' Mutual Clubs and Protecting Associations; their origin; modes of business; and uses	351
v.	Insurable interest fully described under various conditions and contingencies; the principle of Subrogation	351
vi.	The Policy; Voyage and Time Policies; Valued Policies; Floating Policies; explanation of the terms employed in a Policy; the course of the voyage; the insurance of goods; the expression "f.o.b."; the definition of voyage; deviations and permissible deviations; the definition of freight; charterer and charter party; the construction of clauses	353
vii.	Assignments	358
viii.	Perils of the sea: Jettison: Barratry	359
ix.	Loss of the vessel and cargo: Abandonment and the course of business then pursued: Constructive total loss	361
x.	Average: its meaning and effects: General Average: Particular Average, with detailed explanations and illustrations	362
xi.	Free of Particular Average, and of All Average	364
xii.	Wear and tear	364
xiii.	The stranding of a vessel	365
xiv.	Burning	365
xv.	All other perils, losses, and misfortunes	366
xvi.	Time Insurance, and the presumption of loss; remarks on the importance of Time Insurances	366
xvii.	Essential elements of a valid Insurance, and the warranty of seaworthiness; the warranties imposed by three authorities	367
xviii.	The adjustment of losses	367
xix.	Bottomry; not now greatly used, and why	368
xx.	Demurrage explained; the York-Antwerp rules; the three-fourths' indemnity; the shipowners' liability for different kinds of losses	368

CHAPTER XV.

Accident Insurance; and other Descriptions of Insurance against Casualties.

i.	The basis of the assessment of premiums	370
ii.	A brief history of Railway Accident Insurance; the definition of a railway accident by the Act of 1864; and the compensations there prescribed	370
iii.	The extension of Insurance to accidents generally, and the compensation provided	371
iv.	The extension of insurance to cover practically all diseases	372
v.	A detailed statement of the different descriptions of accidents now insured; and of other contingencies (in social and financial life) protected by guarantees	372
vi.	The compensation for accidents at Common Law stated	377
vii.	The Employers' Liability Act described	378
viii.	The Workmen's Compensation Act of 1897 and its provisions in respect of death from accident and disablement	378
ix.	The extension in 1890 of the benefits of this Act to agricultural labourers	379

		PAGE
x.	The rates of premium assessed as a percentage upon the wages; the need for this course in place of individual insurances; the loading on the premium in relation to the costs of possible litigation	379
xi.	The marked characteristics of Workmen's Insurance ..	380
xii.	At the outset of the business the official statistics of Germany and Austria were utilized; Tables of these statistics ..	380
xiii.	The experience of a British traders' Insurance Association in respect of the number of accidents, claims, and their increasing average cost; deductions from the preceding experience	381
xiv.	The element of Deferred Liability discussed	382
xv.	The dangers to be guarded against in Workmen's Compensation Insurance; and how they are tentatively met in practice..	383
xvi.	The indirect consequences of these Compensation Acts; the difficulty to older men in retaining and finding employment; the extent of litigation which has ensued largely demanded for the interpretation of novel Acts	383
xvii.	The over-sanguine estimates formed at the commencement of this class of Insurance in the United Kingdom, and its consequence in heavy losses	384
xviii.	The acceptance by workmen of a capital sum in place of an annuity; the possible unhappy results of such an act ..	384
xix.	The necessity of a Central Organization for the analysis and combination of experience as a basis of rates	385
xx.	The need of placing financial strength and profit before mere extent of business; commercial training and insight required in the Manager of an Accident Insurance Company ..	385
xxi.	Remarks upon the indirect consequences of these Acts ..	385
xxii.	The dictum of Bastiat in relation to social legislation ..	386

PRELIMINARY REMARKS

A FEW suggestions may be offered to the student who may peruse this volume, both in relation to his mode of study and his future career.

The primary conditions of success, equally in the earlier and the later stages of his professional labour, consist of accuracy and exhaustiveness of work ; and in no sphere of intellectual and practical activity is this element more imperative than in the actuarial profession. Important financial interests are committed to the actuary's trust—interests not limited in their consequences to the present time or to the immediate future, but ranging in influence and effect over a prolonged series of years. And not infrequently a result of investigation, on which judgment requires to be based, may appear to be valid which a more exhaustive analysis (accompanied by a numerical measurement) will prove to be fallacious.

The student, again, should never be satisfied with simply the symbolical or analytical process of reasoning by which a result may be attained. Analytical language is merely the shorthand expression of the natural modes of induction and deduction which alone retain throughout the process a steady and clear perception and apprehension of the *facts* investigated ; and the student, though he may employ the mathematical method as an abbreviation of mental labour, should always, prior to his decision, translate the symbolic language into ordinary terms, and justify or modify the analytical course and its deductions by the usual mode of reasoning which deals directly, without the intervention of algebraical mechanism, with the realities which the symbols represent. Numerical computation and comparison of results should invariably accompany the mathematical processes. One of the most famous mathematicians has pointed out, from personal experience, the extreme facility of error where analytical reasoning is unattended by rigorous calculation. Speaking of the consideration of a certain problem, he refers to a plausible mistake which would have occurred, *nisi nos calculus aliud docuisset.* The aim of accuracy not alone suggests this procedure : mental discipline and power of

grasp are more adequately evoked into exercise and expression when the method of deduction is conducted with the actual facts of the inquiry ever present to the mind without their complete relegation to conventional and representative symbols.

As the student advances to a higher post of responsibility, he will discover that a primary and essential distinction in competency of administration lies in the circumstance that one actuary will burden himself with the details of his work and thus effectually impede the capacity and opportunity of serene judgment, while another will attain a more substantial result by delegating the minor portions of service to carefully selected subordinates, and thus possess, while necessarily controlling and supervising their labours generally, the time and mental freedom requisite for attention to that wider and more fruitful origination and direction on which achievement depends. This devolution of charge in less important concerns is conditional for its success upon skill and insight in the choice of men; and we thus arrive at the practical conclusion that the student, from the outset, should cultivate, by actual personal intercourse with his fellows, the faculty of analyzing and judging human character and its gifts and deficiencies.

At every stage of his career the student should sedulously develope the power of original thought. However limited may be the degree in which it is inherently possessed, it still exists to some extent, and by judicious and assiduous toil and concentration can be expanded. The universe, whether in the practical or the more intellectual sphere, would prove a weary and monotonous abode were the vast majority of voices to consist simply of iterated echoes. This force of originality can be evolved by avoiding servile acquiescence, and pursuing a serious personal investigation, with independency of judgment, into every subject on which the student is required to decide; and, in addition to the value conferred upon the character by the impress of original mental and persistent effort, the student will gradually become a master, and not a mere servant, of his methods and principles, using and adapting them intelligently as instruments instead of simply performing, as a servitor, the work they suggest. The obligation is again enforced upon the student, for the acquisition and expansion of this power, of ceasing to be an occupant chiefly of the study or a dweller in the speculative domain, and of entering into commercial intercourse with men of

business, with a vigilant observation and a tenacious retention of the modes in which he perceives affairs to be actually conducted, and keen, though kindly, attention to the diverse characters of men.

The author may here record a result of personal experience. It is too frequently the fashion, in correspondence, to address all men in virtually an identical form of expression or exhortation or suggestion : by personally mingling with men of different dispositions, diversely responsive to the same appeal, he gained the valuable practical lesson that, for the purpose of securing the most effective support and the most earnest and vigorous efforts of men, he must address them by letter in a mode of special overture, individually adapted to their several peculiarities and idiosyncrasies of character.

The end of all practical life, and the potent instrument of successful administration, consists of judgment, or the delicate and exhaustive weighing of evidence in favour of alternative lines of action, and a steady forecast of each probable result. This power is naturally an acquisition of gradual growth and widening experience ; of serene demeanour in success and teachable humility in defeat ; incompetent of cultivation by mere study or speculation, it simply arrives as the slow and finished product of intimate communion with men and affairs, and the patient observance and analyzed retention in memory of the varied results which ensue upon different courses and under diverse conditions. The requisite training involves the remembered perception and experience of the causal connexions between actions and consequences, motives and effects, so that, when a novel problem is presented for decision, the memory may promptly revive and supply the analogous example from its storehouse of past instances which is adapted to the circumstances of the case. In every question, the views of capable and experienced men should be sought, but they should be utilized as data only, since, in each critical epoch of life, the individual judgment must be responsibly supreme.

The student should, again, remember that our mathematical conclusions in Assurance, like the premises from which they are derived, are necessarily more or less approximate. The entire range of science enforces this practical lesson ; even in the apparently finished analytical sphere of astronomy—frequently cited as the exemplar of strictest mathematical reasoning—approximation is superposed upon approximation. The decimal results of his

work, consequently, and the comparatively tentative nature of his methods should always be borne in mind, and utilised with judgment based upon the circumstances of the problem ; so that an affected and artificial appearance of rigorous accuracy—for example, in the exhibition of Reserves which formerly used to be expressed in decimals—may be avoided. Discrimination and experience will gradually teach the practical value of the degree of minuteness to which results should be conducted—retaining in view the approximate character of all Assurance work—data and processes alike.

INSURANCE

CHAPTER I

The Numerical Basis of Assurance Calculations, with Explanations and Discussions of Cognate Subjects.

The student is presumed to be acquainted with the elementary principles of Probabilities and the modes of their expression in numerical calculation. It is worth while, perhaps, to mention that if, out of $(m + n)$ trials or experiences the result A has happened m times and the result B, n times, the probability that the succeeding trial will present A is strictly $\frac{m+1}{(m+1)+(n+1)} = \frac{m+1}{m+n+2}$, and not $\frac{m}{m+n}$ This proposition is based upon the assumption that all values of the required probability are *a priori* (or prior to trial) equally likely.

If this assumption were true in mortality-measurements, that is, in respect of the probabilities of death, (which it is not), the chance of a person, aged 35, (taking one with another), dying before the attainment of the succeeding age, would be $\frac{30}{2002}$ (and not $\frac{29}{2000}$) if the Table of Mortality showed that, out of 2,000 persons alive at the former age, 29 died before reaching the age of 36.

In many applications of the Theory of Probabilities, the disturbing factor of human feeling or expectation enters; and although such emotions, associated with any specified future event, vary widely in intensity in individuals, they yet tend, in aggregates or societies, to present a comparative uniformity of expression. The student, however, is happily exempt from the complicated task of assigning a numerical measure to these subjective states, for the processes of Life Assurance, obviously, and of other statistical applications of the Doctrine of Probabilities, are totally independent of the mental phenomena of expectation (moral expectation, as it is sometimes termed). These processes proceed upon the assumption, experienced universally in the Continuity of Nature, that the future will present

an essential resemblance to the past in each department of operation ; that, under similar circumstances, the same event will tend to recur with a definite numerical frequency without the intrusion of any calculation of the vigour or deficiency of human hopes and fears.

It is accordingly intended in this chapter to describe briefly the basis on which the Doctrine of Probabilities in general and the system of Life Assurance, as a practical application, are founded, with a view especially to directing attention to current errors of speech and the paramount necessity (particularly in the complex problems which occur in Life Contingencies) of accuracy of expression. For obscurity of statement is not merely the product of indeterminateness of thought, but the origin again of wider mental confusion, with the consequent negation of valid reasoning.

This general basis is presented in James Bernouilli's *Theorem of Large Numbers*, which may be enunciated in two propositions : (1) The first proposition affirms that in a very extensive number of trials or observations there exists a demonstrably greater probability that the events under examination will occur in numbers proportioned to *their respective chances in a single trial* than in any other *specified* proportion. Thus, if an accurately constructed coin be spun in the air without any bias in either the mode or the extent of the spin, the chance of a head appearing is $\frac{1}{2}$, or an even chance, with a similar probability of the obverse being presented. The *Theorem* accordingly shows that, in an adequately prolonged number of tossings, the probability of the total appearances of a head and of the obverse will be identical with the respective results of the individual trial. (2) The second proposition asserts that a number of trials or observations may always be assigned of so large an extent as to make the probability of the events happening in numbers within any specified limits of deviation from the proportion just mentioned, however narrowly those limits may be fixed, approach to certainty as closely as we please by multiplication of the experiments.

The latter proposition is especially valuable in estimating the probable results of a given number of trials. This celebrated *Theorem* has also been thus expressed : it is always possible to increase the number of trials or experiments until it becomes a certainty that the proportion of occurrences of the event will differ from p, the probability on a single trial, by a quantity less than any assignable.

The following illustrations will elucidate the significance of these propositions and explain their relationship to life contingencies. Turning to proposition (1), the births of 1,000 children are accurately registered, and the ages at which the children successively die are recorded : if, then, 900 remain alive at the end of the first year, the probability of an infant (taking one with another), on actual experience, surviving for one year, under conditions corresponding with those affecting the original observations, is expressed by the fraction $\frac{900}{1000}$ or $\frac{9}{10}$; and assuming that the number of registered children were infinite, this result would cease to be approximate, and the proportion of the numbers alive on the termination of the year to the entire number observed at birth would be precisely and absolutely equal to the probability of an infant under similar circumstances surviving one year.

And, obviously, with every increase of the number of observations within this limit, the proportion exhibited would approach with closer and closer approximation to equality with the ratio ascertained at the outset. Let us adapt the proposition to the illustration usually furnished in treatises on probability. If we know the odds in favour of an event to be 3 : 2, as, for example, in that of drawing a white ball from an urn containing three white and two black balls, we should judge that if five trials were made we should be more likely to draw white three times and black twice than any other combination. We should still feel, however, that this result was uncertain in so few a number of experiments : instead of three white balls we might draw white 0, 1, 2, 4, or 5 times. But if 1,000 trials were attempted we should feel assured that, although the numbers of white and black balls might not occur in the ratio of 3 : 2, they would be ascertained to be very nearly in that proportion. And the more the experiments were multiplied, the more precisely would this proportion be discovered to exist.

Laplace justly stated that the theory of probabilities consisted simply of good sense reduced to a system of calculation. In the ordinary affairs of life, whether in private, social, or commercial transactions, every mind, basing its expectations upon experienced results without transforming those records into a calculus, actually employs the relations, whether concurrent or sequent, observed between past occurrences as the foundation of judgments respecting

future events of corresponding nature and happening under congruent conditions. And the doctrine of probabilities simply erects this inherent tendency and necessity of all minds into a general and organized scheme of numerical expression.

We thus observe that probabilities are transmuted into certainties when the series of experiments or trials is infinite, and approach to practical certainties when the observations are exceedingly numerous; since in the long run, every event that *can* occur *will* occur, and the action of fortuitous causes in operation will disappear as the amplitude of survey is enlarged.*

Experiments have frequently been made upon the results of the tossing of a coin. In a well-known instance—one, indeed, which may be termed historical—a coin was spun, and, in 4,040 trials, heads appeared 2,048 times and tails 1,992; while, in a second attempt, the corresponding events were 2,048 heads and 2,044 tails out of 4,092 instances, or practically each result occurred an equal number of times. The chance of head appearing at the first trial is $\frac{1}{2}$; and the extended observations just described express in these particular cases the doctrine of proposition (1), that if the experiments be multiplied the events will occur in the proportion indicated at the single trial. In a third set of experiences, it was ascertained that, out of 12,000 tossings of a coin, the occurrence of tails happened in 50·16 per cent. of the total observations, while, when the experiment was extended to 24,000 trials, tails showed the closer equality to heads of 50·05 per cent.

The more extensive, therefore, be the mass of observations upon the duration of human life, the more profoundly is the expectation confirmed that a persistent regularity of results will be displayed.

In the trials already mentioned, moreover, sequences of the following order were observed. In the first set of the preceding experiments, heads appeared at the first throw in 1,061 cases; in the second set, in 1,048; no head occurred until the second throw in 494 instances in the one set, and in 507 cases in the second; and

* These probabilities, which become more allied to certitude as the observations are extended, are frequently termed *moral* certainties: probabilities in practical life, that is to say, which assume so sure a form that, though failing in final demonstration, they are adequate as a basis for "*mores*" or "*manners*," and afford consequently a clear and sufficient guide for the practical conduct of the affairs of life.

omitting intermediate events of the same kind, no head appeared until the seventh toss in twenty-five cases in the one series, and in seventeen instances in the second. That is to say, selecting one specimen only, the seventh, in the first set there occurred a run in the appearance of tails in twenty-five instances before a head turned up. It will thus be observed that, since the contrary events in the totality coincide in extent, a consecutive appearance of one event during any duration of experiment will be followed by a diminished sequence of occurrence of that event at another stage, and vice versâ. This phenomenon is paralleled in life assurance by the occurrence at one or more periods of numerous deaths followed by a briefer sequence, or a short succession preceding a more extended one, but in such a mode as, in the mass, to render the total result accordant with the observations from which, for example, the premiums are deduced. It follows that the basis of life assurance consists in the condition that those who survive longest and contribute most largely to the common fund,—in excess of the payments which will be received by their representatives,—form a necessary constituent of the scheme by balancing the experiences in which death occurs at an early date and where consequently the representatives of the assured obtain a benefit considerably superior to the premiums discharged.

We revert now to the second proposition of Bernouilli's *Theorem*, which asserts that, with a multiplied series of observations or trials, the chance of the events occurring in numbers comprised within specified limits of departure from the result shown in a single experiment, however brief may be the interval between those limits, may be made to approach as closely to certainty as is desired, according to the extent of the series of experiments employed : that is to say, the probability of occurrence within those boundaries can be rendered as near as we please, by enlarged trials, to certainty or unity.

(Although the *certainty* of an event is not customarily conceived as a *probability*, or the numerical measure of the expectation or belief that the event will occur, yet the continuity of mathematical language requires the identification, in the doctrine of probabilities, of the chance which possesses unity as its measure with certainty, just as the impossibility of the happening of an event is registered on the scale of probabilities as zero, or o,—the terms o

and 1 forming really, in mathematical symbolism, the *limits* of the probability of occurrence.)

This proposition has been illustrated by anticipation in the preceding examples. The subject will also be specially considered and exemplified in Chapter II, where the question of the stability of results, and the range of deviation from an expected experience, are discussed.

Interesting tables, it may be anticipated, are frequently furnished, both for Life Assurance and Friendly Societies, where the progress of the Assurance Fund is exhibited year by year, with increments by premiums and interest and diminution by claims, so that, on the extinction of the lives involved, the final balance is exactly sufficient to discharge the concluding claim. This is, of course, theoretically accurate; but the student will observe that it is practically impossible. When the numbers at risk become so reduced that uniform and regular results (whose occurrence implies an adequate mass of cases) fail to be practicable, the Company would then require, if it ceased to supplement its diminishing area by fresh entrants, to transfer its liabilities and assets to another Office which possessed the requisite basis of numbers.

The meaning of the term "probable" (or, as it is frequently named, the Median) error of an observation or average of observations should here be introduced. This function expresses the range of deviation from the average within which, taken positively and negatively, that is, on one side or the other, it is an even chance (or $\frac{1}{2}$) that the truth or observation will lie. If there exist an even chance (A being the exact value) that the result of an observation shall lie between $A-a$ and $A+a$, a is termed the probable error of the observation. In other words, the extent of divergence from the precise (or true) result within which, on one side or the other, in excess or deficiency, it is an even chance (or it is as likely as not) that the truth will be discovered, is named the probable error of the observation to which the calculation refers. Thus, if the chances be 1 to 1 (or equal) that the error shall be found between 0 and 10, and the same chance exists that it shall exceed 10, 10 is named the probable error. Or again, if 5·45 be the mean of all the determinations of the density of the earth, and ·2 be approximately the probable error, the meaning implied is that the probability of the actual density of the earth falling between 5·25

(5·45 — ·2) and 5·65 (5·45 + ·2) is one-half. Or we may more graphically define the probable error as that error which we are as likely to exceed as to fall short of, so that if we arranged all the errors committed in the order of their magnitude the probable error would correspond to that one of them which precisely bisected the row.

The term " error " signifies a discordance in observations of an accidental character—that is, one of which the cause is unknown, and which in the long run occurs as frequently in the one direction as in the opposite direction ; if the cause could be assigned, the error would disappear, or cease to be an error, in consequence of its dependence upon a recognized cause, and thus proving predictable. An error is thus, in mathematical reasonings, the difference between the real value in nature of an observed quantity and the result of its actual measurement. If Z represent the true value, and M be the product of measurement, the difference $Z - M$ is designated the Error.

It may appear impracticable by means of *observation* to discover data more precise than the observation itself. Numerical error being necessarily incidental to every observation, the latitude of this error (or digression from the truth) is obtainable by repetition of measurements under varied conditions : it is, therefore, the number of careful observations and the ascertainment of their mean value which—with the probability that in this proceeding the errors of opposite kinds will at length be compensatory—bridges over the interval, and conducts with minuter nearness to the truth.

The student must proceed further in his mathematical education before he can appreciate the doctrine of the probable error, and as an illustration, he must accept at present on trust the following example of the mode of calculation :—

1. Find the mean of all the observed results.

2. Ascertain the excess or defect of each observation from this mean—that is to say, the *error* of each result from the mean quantity.

3. Square each of these reputed errors.

4. Sum the squares thus obtained (which will be, of course, all positive).

5. Divide this sum by one less than the number of observations which furnishes the *square of the mean error.**

6. Extract the square root of this final result, which exhibits the *mean error of a single observation.*

7. Divide by the square root of the number of observations, and thus obtain the *mean error of the mean result.*

8. Multiply by the natural constant ·6745 (or, say, $\frac{2}{3}$) (a constant entering into formulae for determining the probable error), and we thus ascertain the *probable error of the mean result.*

In illustration of this process, let five measurements be taken of the height of a hill which produce severally the numbers of feet of 293, 301, 306, 307, and 313; and assume that the goodness or validity of the observations is equal: we desire to ascertain the probable error of the mean or 304, that is to say, its departure from the true value of the altitude.

The differences between this mean and the preceding numbers, without regard to their direction in excess or deficiency, are 11, 3, 2, 3, and 9; their squares are 121, 9, 4, 9, and 81; and the sum of these squares of errors is consequently 224. The number of observations being 5, divide the preceding sum by 5−1 or 4, and the quotient is 56. This number constitutes the square of the mean error; and extracting its square root, we obtain 7·48 (or $7\frac{1}{2}$), which expresses the mean error of a single observation. Dividing by 2·236, the square root of 5 (the number of observations) we find 3·35, or the mean error of the mean result: then multiplying by ·6745, we arrive at the probable error of the mean result, or 2·259, or about $2\frac{1}{4}$. This final number indicates that the probability is $\frac{1}{2}$, or the odds are even, that the exact height of the hill lies between $301\frac{3}{4}$ (i.e. 304−$2\frac{1}{4}$) and $306\frac{1}{4}$ (i.e. 304 + $2\frac{1}{4}$) feet.

We deal more fully with this question in Chapter II in relation to the necessary extent of assurance risks for securing the occurrence of results within stated limits of deviation.

An important distinction has been pointed out by physicists and mathematicians between the significance of an average and a

* The mean error signifies that of all the positive or all the negative, errors; and is obtained by applying the same principle of the arithmetical average to the errors themselves that is applied to the original magnitudes. (It will be noticed, from the examples and illustrations adduced in the text, that the distinction explained in succeeding paragraphs between "average" and "mean" has not been steadily and universally observed by scientific writers.)

mean; and this accuracy of scientific expression possesses a definite bearing upon our current modes of speech. In ordinary language, and even in the statements of scientists themselves, accustomed though they be to verbal precision, the average is frequently confounded with the mean. So prevalent has proved this confusion that it has been proposed, for the purpose of a definite separation of the terms, that the ordinary arithmetical average should be designated the "fictitious mean." If there be 6 quantities—2, 3, 4, 6, 13, and 14—the arithmetical average is obtained by dividing the sum of the quantities by their number, when a quantity is produced intermediate at some place in the set, so that the sum total of the excesses of certain quantities over the average is equal to the sum total of the deficiencies of the other quantities which fall below the average. In the preceding example the average is $\frac{42}{6} = 7$: the excesses are 6 and 7 or 13, and the deficiencies are 5, 4, 3, and 1 or 13. Now it has been justly pointed out that an average value involves no conception of a normal and recognizable central magnitude, all divergencies from which in the observed results should be regarded as deviations from a natural standard. The mere average of the altitudes of several trees of the same species probably fails to furnish the height even of any individual specimen, and exhibits no indication of the type of altitude expressed in Nature as that to which the variously divergent heights constantly tend.

An average merely furnishes a general notion of the objects compared in respect of the particular feature selected for examination: it represents no actually existent quantity, and simply forms a convenient mode of conception and expression, enabling us to apprehend and convey in a single result a multitude of details, and to compare, with mental ease, the results of one set of observations with those of another.

It would be quite impracticable, for example, to form a definite conception of the duration of human life, except in the concentrated mode of expressing the length of life (or expectation of life) which each individual would possess if the years attained by each were equally distributed among the whole body of persons observed. But though this compact form of statement presents a convenient and helpful aid in comparison, it obviously involves

no indication of an actually existing fact pertaining to the order of human mortality in the shape of a uniformity of nature.

The average thus affords, in an individual result which can be readily pictured to the mind and retained in memory with facility, a *general* notion of a number of quantities, and provides us with the means of conveying the same notion to other minds without the necessity of presenting a lengthy list of the isolated quantities themselves from which, in their multiplicity, no lucid and definite conception could be constructed with ease. If, for example, we formed the average in the ordinary manner of the heights of 100 houses in a street, the result would not represent any real entity, any individual house, but would merely furnish a fictitious height expressing as nearly as possible the several altitudes of the individual buildings. An average accordingly fails to serve as a guide to the future based upon the experience of the past.

A mean, on the other hand, it is rightly contended, does indicate an approximation to a definite existing quantity; and, applying language of a metaphorical character, points to some model or standard inherent in Nature, some real existence expressive of law or uniformity or intention, to which the various observations obtained by human measurement or induction refer, by the symmetric nature of their deviations, as the type to which they severally tend.

A well-known illustration may be considered, but for its appreciation and a complete conception of this scientific and natural significance of a mean a brief explanation of a certain mode of measurement, to which the name of "Weights" is applied, requires attention,—"weights" consisting of numbers expressing the relative practical worth or value of observations.

An observer of some event, repeatedly examined, may consider that some of his measurements possess a greater trustworthiness and accuracy than others: he then, by the impression produced upon his mind, at the time of observation, of their respective merits of value, affixes to each trial its relative weight. Suppose that A_1, A_2 . . . be the results of n observations of a stated object or event, and that the numbers c_1, c_2 . . . express their presumed proportional validity or scientific goodness (i.e., c_1, c_2 . . . are the "weights" measuring the respective values of the observations): then, instead of employing $\frac{\Sigma A}{n}$ (where Σ is the

symbol of summation) as the Average, the observer substitutes the mean of $\frac{\Sigma cA}{\Sigma c}$, where ΣcA consists of $A_1 c_1$, $A_2 c_2$, ... , and Σc is the sum of the "weights" of c_1, c_2, ...

Expressing the proposition in a numerical form, assume that the observations effected cannot all be accepted with equal confidence: the observer must then, from inquiry and judgment, make the different observations be reckoned as different *numbers* of observations, allowing any one more acceptable result to count as more than one if it appear to be superior to the remainder in accuracy and care of measurement—taken, for example, under more favourable physical conditions, or with the mind and senses more alert, or with an instrument of observation more exact. Three observations, we will suppose, furnish the results of 26, 28, and 29; a close examination suggests that 28 is to be preferred to the rest, and 29 to 26, so that, for example, 28 ranks in order of substantial correctness as though it possessed the value of a mean of 8 observations, 29 of 6, and 26 of 4. It must then be considered that 8 + 6 + 4 or 18 observations have been effected, of which 8 have exhibited the result of 28, 6 that of 29, and 4 that of 26. These numbers—8, 6, and 4—are termed the "weights" attached to the several observations 28, 29 and 26 as the measures of their values; and in obtaining the mean result of the total observations, each is multiplied by its specialized weight; the sum is then divided by the sum of the weights. Thus we obtain, $8 \times 28 + 6 \times 29 + 4 \times 26$ or 502; and dividing by 8 + 6 + 4, or 18, the number 27·89 is produced as exhibiting the most probable result, or that which most closely represents the actual truth in Nature. The mere arithmetical mean or average would be $\frac{26 + 28 + 29}{3}$ or 27·6.

Without affixing weights of value dependent upon the observer's personal judgment, and therefore liable to error through defective recollection, obscure impression, or imperfect surmise, the weights themselves may be obtained direct from Nature. Thus, the quantity of each observed phenomenon, amid a multitude of divergent observations, should be multiplied by the number of times which that individual fact is perceived in Nature. An actual example which is frequently adduced in books on

probability will exhibit this genuine and trustworthy method of ascertaining the mean in the clearest light.

First recurring, however, to our previous illustration, if, instead of taking the average measurement of the heights of 100 different houses, we made 100 equally sound measurements of the height of the *same* house, the mean of these results would very approximately express the actual altitude of the house, or the value of the real existent fact. Or—a happier illustration still—we revert to the former reference to the height of the trees of a selected species. If we construct a schedule of heights of various trees, and multiply each height by the number of trees which as exactly as possible present it, we obtain a mean value (as distinguished from a mere Average resulting from a purely arithmetical process), which genuinely furnishes an index to that altitude in Nature which constitutes, so to speak, the ideal or typical altitude, from which the isolated heights are deviations, and towards which, in the order of Nature, and persistently, they tend.

We now proceed to present the example, to which a reference has just been made, which exhibits the mode of formation of a Mean quantity, and which, from the manner in which it is obtained, ascends beyond the mechanical conception of an Average, and justly and distinctly expresses a phenomenon or type in Nature.

The dimensions of the chests of 5,738 soldiers of different regiments belonging to the same country were measured, and the results, represented in inches, were expressed in tabular form, proceeding by differences of one inch, and treating 2·5 to 3 inches inclusive as 3 inches, while 2·4 inches would be regarded as 2. The appended table contains the facts—

Inches	33	34	35	36	37	38	39	40	41
Number of cases observed	3	18	81	185	420	749	1073	1079	934

Inches	42	43	44	45	46	47	48
Number of cases observed	658	370	92	50	21	4	1

Multiplying the number of inches in each column by the number of the instances where that dimension was discovered, we obtain,

for example, in the case of 36 inches, the product of 36 × 185, or 6,660; summing these several products, the total of 228,555 results, which, divided by 5,738, the number of observations, furnishes the mean quantity of 39·83 inches. We perceive a mode of progression in the groups, increasing to a maximum, and then diminishing; and the ascertained mean result of 39·83 inches points us to the groups of 39 and 40 as exhibiting the most approximate tendency to this apparent centre or type of magnitude existing as a definite model in Nature.

The mean, thus scientifically obtained, it is contended, may be accepted as furnishing a confident index to the future, while the average possesses no such inherent property of assurance; and the validity of all statistical records depends upon the apprehension of this distinction between these two modes of measurement. The ordinary course of averaging without the introduction of the weights would have afforded a result of 40·5, or an error of about 1·7 per cent.

In the application of this doctrine of weights to the subject of mortality, it is obvious that greater influence should be attached in a Table of Mortality to a considerable number of observations at one age or set of ages than to the smaller numbers appearing at other ages, if the natural phenomena of the rate of human mortality are to be educed. It is evident that the more extensive be the numbers of the living or of the deaths observed, the greater are the "weights" assigned by Nature for detection of the inherent uniformity of progression which the phenomena manifest. We refer to this question at a later stage.

The student should carefully ponder the deep divergence of significance and value in statistical inquiries between the result termed an average, which is the product of a mere arithmetical operation, and the Mean, which indicates some permanent fact in Nature and in the numbers exhibited in natural processes. His modes of expression, consequently, in describing the results of investigation, will then assume a determinate and scientific form.

A brief recapitulation accordingly will be valuable.

1. The object of research, in whatever sphere of science it be pursued, is the discovery of the facts and uniformities of co-existence or succession displayed in Nature itself, as an integral

portion of its constitution; in Physics, the elucidation of laws, as they are termed, which shall serve, by rendered obedience, as guides to the mastery of natural processes; in the region of observations upon human beings, relevant to the structure of assurance, the regularity exhibited, under corresponding conditions, of the force of mortality.

2. An average presents no feature of intrinsic value in this aspect: it furnishes merely a convenient measure of comparison of a purely arithmetical and conventional character.

3. The mean genuinely possesses a natural capacity of indicating the actual types and standards towards which aberrant observations, when duly and precisely weighted in respect of accuracy and existence, apparently and normally tend.

This precision of measurement and the resulting approximate closeness to the natural phenomena themselves are obtained either (*a*) by applying "weights" expressive of the degrees of intimate accuracy, furnished by the observer's own inspection and judgment; or (*b*) by adopting the "weights" provided directly by Nature, as exemplified in the ascertainment of the standard circumference of soldiers' chests which has been cited. The relevant "weights" presented in Mortality statistics for the discovery of any uniformity in the intensity and incidence of the force of death would consist of the adequate extensiveness at the several ages of the areas of observations of the "existing" and the deaths, which are utilized in our search.

The Mean implies an assurance that the future will be similar to the past (which an average cannot indicate), and the recognition of a mean, among a series of results distributed in order, depends upon the observance of a conformity between the law of progression in the magnitude of the distributed groups and the reality of *some* central truth.

4. It will be noticed that in the formation of an arithmetical average the mere multiplication of the different quantities by the numbers of each is not of the nature of the process of multiplication by natural "weights." The former numbers furnish simply the respective areas in which these quantities occur, but without the genuine implication of supplying, so to speak, natural indications of the gradation by which the various quantities may

be assumed to approximate to an existent standard inherent in the constitution of things.

5. The average is concerned with different quantities of the same kind, and provides a general, superficial notion of magnitude and comparison only; the mean deals with the one object and educes by its process a true phenomenon embodied in Nature. In brief—and adopting an ancient philosophic formula—the mean seeks to decipher, with exactitude and clearness, the "one in the many," while the average presents merely a superficial and general resemblance between the "many."

Certain current improprieties of speech in life assurance may be exposed on the teaching of these remarks.

It is constantly stated that a Table of Mortality must be employed which furnishes "average results"; that "on the average" a certain consequence may be expected; that the doctrine of "averages" applies. It is clear from the preceding exposition that the term "average" is improperly and irrelevantly introduced into these expressions, and that what is intended to be conveyed, and should be definitely affirmed, is—taking the several statements in their order—that a Table of Mortality must be employed which is "sufficiently extensive in range"; that "in the long run," or "in the mass," or "on the whole," or "taking all events together," a certain result may be expected; and that the doctrine of "large numbers of observations" applies. It is unfortunate that we do not possess abbreviated forms of expression which would convey the significance we rightly intend; but this explanation will be useful if it teach the student to measure his terms by the exactness of his meaning.

If the new assurances of a Company comprise one for £200, another for £300, and a third for £400, we properly employ the term "average" in stating that the average amount is £300: where a serious divergency, however, exists between the several sums involved, the average possesses little value.

The phrase of the "expectation of life" may be specially mentioned. The term "expectation" is here erroneously employed: "expectation" involves a *belief* in the occurrence or the order of occurrence of events in the future; but no such state of mind is implied in the preceding phrase, and the adoption of the term has frequently conduced to the fallacious notion that the

function expresses the period which a person may be "expected" to survive. The quantity denoting the expectation of life is simply the numerical result of distributing the total durations of life severally enjoyed by persons successively surviving after a given age among the whole number of persons existing at that age, so that the term "average" applies, since no *mean* result is displayed. Examine the process of calculation by the reverse mode. In the O^m Table it is observed that the expectation of life at age ninety-five is 1·188 years. Now the following facts appear in that table :—

Out of the number of persons alive at the age of ninety-five—

1	died	at	the	age	of	102	and	had consequently lived	7 years
2	,,	,,	,,	101	,,	,,	,,	6	,,
4	,,	,,	,,	100	,,	,,	,,	5	,,
8	,,	,,	,,	99	,,	,,	,,	4	,,
15	,,	,,	,,	98	,,	,,	,,	3	,,
28	,,	,,	,,	97	,,	,,	,,	2	,,
49	,,	,,	,,	96	,,	,,	,,	1	,,

Multiply the several years of life enjoyed by the respective numbers of survivors, and we obtain 1×7, 6×2, and so on, or a total of 221. Dividing the sum of these products by the number existing at age 95, or 186—among whom these several periods are distributed—we arrive at the expectation of 1·188. This result is a purely arithmetical one, in no way dependent upon any uniformity disclosed in Nature, and hence is properly designated, as we have said, a mere average, and not a mean or naturally significant quantity.

The use of the function of the expectation lies, in conformity with previous remarks, in the facility of comparing the results of one Table of Mortality with those of another, in a compact form. It would be impossible to acquire a clear and definite conception of the significance of a Mortality Table if we were compelled to retain in memory the multitude of the successive probabilities of survivance or death ; but this power of perception and comparison is furnished by the compression, so to speak, of the entire area of the several results into a single expression : we thus gain our knowledge at a ready glance in place of the wearisome inspection of a prolonged series of isolated facts.

It may be added incidentally that the suggestion has sometimes been made that the basis of comparison between two Tables of

Mortality would be adequately presented by a conspectus of the annuity-values derived. But, besides the difficulty we have pointed out—namely, that we should here be reduced to a series of separate inspections instead of a single and unified view—the proposition imports an extraneous element, contrary to scientific usage, in the form of the rate of interest, which is not relevant, from this aspect, to the probabilities of life.

The distinctly recognized element of multiplicity instead of unity in his conceptions will save the student from many errors of statement, and through reflex action (to employ a relevant phrase), from much obscurity of thought.

Two writers of the highest scientific eminence have erroneously mentioned respectively, in standard treatises, the probability of a *specified individual* surviving for a year, and the probability of an *individual* person attaining a deferred age. Probabilities, it is obvious, are merely expressive when involving numbers and not units.

The value of an annuity at age x (independent of the accompanying life assurance in calculating the market-value) is clearly not the value in relation to an individual life ; and should be expressed in the phrase that the value at any age is the average value of a large number of annuities completed at that age, taken one with another. We have just referred to the introduction of a protecting life policy in ascertaining the market value of an annuity, $\left(\frac{1}{d+p}-1=a\right)$; but although the isolated security is thus rendered complete in itself, and we may accordingly describe the value in this aspect as appertaining to a *single* life, the element of mass or number of transactions is still inherent, for the effected policy involves the notion of large numbers in the data employed by the Company by which the assurance is granted.

The purchase of an absolute reversion as an isolated transaction will range between the limits of a considerable profit and a considerable loss (in comparison with the calculated rate of return) dependent on the epoch of the advent of death ; and hence we must speak of its value as expressive of the results of an extensive group of investments. The market-value, again, of an absolute reversion is not the value of an individual purchase, but should be expressed as the value that may be granted with a well-founded expectation of the required return of interest in an adequate number of instances.

The student may also be cautioned to refuse the acceptance of stated *percentages* of observations without production of the numbers out of which they have been formed. A disease which destroys one person or one hundred may equally be affirmed to be fatal in fifty per cent. of the cases affected whether the number of persons attacked be two or two hundred. But one out of two will probably be purely accidental, while one hundred in relation to two hundred will be significant of vast importance.

No department of inquiry is so densely strewn with fallacious deductions, and hopeless and mischievous hypotheses constructed on their basis, as that of statistics of every description, in consequence not merely of comparisons between incongruent materials, in whole or part, or failure to observe whether the conditions affecting the several sets of observations compared are fundamentally correspondent, but largely also from the error of adopting the process of the simple arithmetical mean or average in the calculations in place of the genuine and indicative mean.

All observations collected, and facts observed, are unavoidably subject—from the limitations of human faculty, the lapse of attention or mental clearness, the failure of precise accuracy, and the deficiencies or want of delicacy in the instruments employed—to manifold errors. Errors of mal-observation or mis-observation occur ; errors of classification of incompatible materials ; the " personal equation " enters, or the individual inherent rapidity or slowness in sensation, perception, and record of each observer (this peculiarity, however, can be determined in each instance and allowed for : the men of larger physical frame should be found to exhibit the least quickness in this respect, cæteris paribus, since the nervous current, constituting the medium for the transmission of sensation and volition, and travelling at the rate of ninety feet per second, has a longer tract to traverse, and moreover personal interest in the subject of investigation counts considerably) ; mistakes of description happen ; failures in discrimination ; errors of tabulation and transcription ; errors of summation and other arithmetical operations ; oversights in checking results ; and paucity of data either generally or at specific stages.

Before a series of observations can be employed as the basis of life assurance transactions, a method of adjustment or graduation, tending to produce that regularity of progression which the

order and continuity of Nature in every sphere entitle us to anticipate, must be adopted. The specific and legitimate purpose of Graduation may first be noted. By this method we endeavour to disentangle our observations, derived from Nature, from the accidental errors of collection and tabulation (generally) which have become introduced in the process of obtaining them, so that the actually existent facts may be exhibited in their natural purity, and thus be able to disclose clearly the laws or uniformities which they embody. The process is simply that of presenting Nature precisely as Nature exists. This is the fundamental conception of Graduation: a practical necessity also arises in connection with the production of an orderly progressive series of premiums.

The present book is of too elementary a character to include a detailed or critical account of the various systems which have been pursued to effect this object, and a brief summary alone can be attempted.

Among the essentially mathematical schemes may be mentioned the summation of the numbers living at each of five consecutive ages, and the adoption of the one-fifth part as the adjusted central value for each section—a process, it will be noticed, of simple averaging.

Weights may be introduced by the following plan :—

If $P_{x-2}, P_{x-1}, P_x, P_{x+1}, P_{x+2}$ be obtained from the formula (modified for each P) $P_x = \dfrac{p'_{x-2} + p'_{x-1} + p'_x + p'_{x+1} + p'_{x+2}}{5}$,

where p'_x expresses the probability according to the observations of mortality employed of surviving a year at age x, then the adjusted probability of living a year at age x, or p_x, is

$$\frac{P_{x-2} + P_{x-1} + P_x + P_{x+1} + P_{x+2}}{5},$$

where each P is formed in the mode above described. A weight is thus accorded to the unadjusted probabilities at the several ages inversely proportioned to their distances from the age considered.

This purely mathematical plan has been most ingeniously expanded into a formal and rigorous system by Mr. Woolhouse. The numbers living at quinquennial intervals of age, such as 10, 15, 20, etc., to the close of life or observation, are extracted; then

the numbers existing at ages 11, 16; . . .; those living at 12, 17, . . .; those at 13, 18, . . .; and those surviving to 14, 19. . . . By separate interpolation by the ordinary formula in finite differences, the intermediate values for each series are obtained, and the average is then taken of the five completed sets of results. We thus, as it were, draw a curve through the original data at ages 10, 15, . . .; again at 11, 16, . . ., and so on; and thus produce five different curves closely approximating in outline to one another; and at each age the adjusted curve possesses an ordinate which is the arithmetical average of the several ordinates of the various curves; the whole of the curves are thus massed into one which pursues a central course. (It may be noted that, in the original exposition of this arrangement, the expressions "arithmetical average" and "mean" are erroneously employed as though they were synonymous.)

Or we may assume that the observations form the concrete and visible expression of a law of Nature, or (to speak in more precisely scientific language) a Uniformity of Nature. Extracting as far as possible the character of this uniformity from a comparison of different and adequate masses of materials—the numbers exhibited as surviving to successive ages throughout life by various tables possessing reasonably common features of congruency of conditions—it may then be possible to express the regularity of result displayed in a mathematical formula of practicable compass, from which the adjusted numbers may be calculated by applying the formula to the actual data themselves. Mr. Gompertz's conception of the uniformity in Nature in respect of the duration of life supposed the existence of two operative causes producing death—one acting with a constant intensity at all ages, the other of such a character that the power of vitality or the capacity to resist death suffered equal proportions of diminution in equal small periods of time, so that the intensity of mortality would assume the form of an increasing geometrical progression. The construction of his actual formula was restricted to the second supposition only, but Mr. Makeham's extended formula included both—the constant and the geometrically augmenting series.

Let the constant force of mortality operating with uniform intensity at all ages be represented by A, then Bc^x (which should be compared with the following illustration) will express the second

concurrent force, which increases in a geometrical ratio of intensity: hence the force of mortality furnished by Mr. Gompertz is $B\,c^x$; while Mr. Makeham's formula shows the forces in co-operant action, or $A + B\,c^x$, where x denotes the age, measured from birth.

In examining the expression of an equal proportionate reduction in the vital force by which death is resisted, the student should carefully note the distinction between a loss of opposing power by equal successive *por*tions and by equal successive *pro*portions.

If x be the time, the function $a-bx$ (representing, say, the power to oppose death at any epoch) indicates a diminution of equal *por*tions of power in equal small intervals: thus we have (substituting the periods of 0, 1, 2, 3, etc.), a (where at the origin of observation, $x=o$, and therefore $bx=0$), $a-b$, $a-2b$, $a-3b$, etc.: the loss in the first interval is $a-(a-b)$ or b; the diminution in the second instant is $(a-b)-(a-2b)$ or b: and in the third interval $(a-2b)-(a-3b)=b$: that is, the loss at each stage is constant or equal.

The function of the form $a\,b^x$ represents a loss or reduction of equal *pro*portions in corresponding minute spaces of time. Thus, at the origin of observations, the power to resist death is expressed by $a\,b^0$ or a; at the beginning of the second instant by $a\,b$; at the commencement of the third, by $a\,b^2$, and so on. The losses, therefore (substituting numerical values), are, taking the first two elements as examples, $a - ab$, $ab - ab^2$; the proportion of the first loss to the power which is reduced is $\frac{a - a\,b}{a} = 1 - b$; the ratio of the second loss to the value at the beginning of its epoch is $\frac{a\,b - a\,b^2}{a\,b} = 1-b$, or the same proportion as before—that is, $a : a - ab :: a\,b : a\,b - ab^2$, which is obvious.

A concrete and familiar illustration of a proportional loss is afforded by the present value of a sum, a, due at the expiration of a period, x, or $a\,v^x$. In the interval, h, this function loses $av^x - av^{x+h} = a\,v^x\,(1-v^h)$, which bears to the initial value of $a\,v^x$ the ratio $\frac{av^x\,(1-v^h)}{a\,v^x} = 1 - v^h$; that is, the ratio of $1-v^h : 1$, which is independent of the time, x, or, $a\,v^x\,(1-v^h) : a\,v^x :: 1-v^h : 1$

Again, a graphical method may be adopted which has rendered admirable service for many years in the domains of astronomy

and meteorology. This process is sometimes described in scientific treatises as the method of curves.

Having marked along a horizontal line at equal intervals the ages attained by the lives observed either from birth or from a specified age, an ordinate (or perpendicular line) is erected at each age, the length of which (measured on a uniform scale) represents the number of the successive survivors at each sequent age. If a curve be then drawn through the upper extremities of these ordinates, a broken and irregular line will be produced, due to accidental errors or to paucity of observations at some stages, or to both causes in combination, though, on the whole, if the observations have been numerous, exactly obtained, and well distributed, as in a mortality table of adequate compass, the abrupt and capricious deviations from a continuous flow of outline will prove to be less marked: still, imperfection of progression will exist, but by a skilled observer a curve of symmetric form can be drawn by cutting off all small and irregular sinuosities, some lying on the upper side of the fresh curve and others on the lower—practically equivalent to each other on the whole of the range—and thus producing a flowing and sequent line divested of eccentric and tortuous shapes, and regarding its entire extent, excluding projections above the course of its sweep equal in amount to the depressions situated underneath.

The student will notice the distinction between the graphical exhibition, or ocular presentation, of results derived from other modes of experimental observation or calculation, and the graphical process of directly obtaining results from the observations themselves.

The validity of any method of graduation—the definite intention of which is to secure results relieved from the disturbing intrusion of accidental errors and fluctuations, and forming consequently a closer expression of the uniformity of progression which observations in Nature are assumed to involve—must be tested, as in all scientific investigations, by appropriate verification. This is effected by ascertaining, for the several intervals of age, whether the deaths obtained by multiplying the numbers exposed to risk at the commencement of each interval by the rates of mortality furnished by the hypothesis on which the graduation proceeds are equivalent to the total number of deaths which have actually

occurred respectively during those periods. If, moreover, the entire number of deaths thus obtained for *all* ages by application of the method coincides with the aggregate mass of the deaths observed, the accuracy of the adjustment may be finally accepted.

A grave defect in some systems of graduation has consisted in the neglect of what have been termed " weights," so that the same importance has been attached to the numbers living and the number of deaths at ages where the observations have been comparatively few as to those which have occurred more extensively at other ages. Thus the significance of the natural facts (which should essentially govern the form of graduation, seeing that the adjustment is simply a mode of expressing the natural uniformity which the phenomena and their sequences involve or manifest) is not clearly educed. For, in accordance with previous statements, the truths of Nature, and of the human constitution in relation to Nature (that is, the effects shown by inherent vital power, and the concurrent influence of environment, on the probabilities of death), are more likely to become apparent in those instances where masses of numbers are involved, and where deviations in each direction will probably be counterpoised, than in those cases where the observations are sparse and fragmentary and liable therefore to include accidental fluctuations in one direction, it may be, only. A mass will thus more approximately manifest any natural uniformity, and the probability of this result will increase with the augmentation of the numbers : a meagre collection of units, on the other hand, may, and probably will, be entirely insignificant of any genuine uniformity at all. As the basis of all methods of graduation accordingly the comparative aggregates of facts at the different ages should be taken into account.

The purpose, as we have said, of any process of adjustment is the more precise ascertainment of the character and sequence of the facts disclosed in any natural phenomena by the removal of fortuitous deviations, whose sources have been already described. As a basis for the computation of assurance premiums, the progression of numbers obtained should be one of a continuous character without the intrusion of abrupt breaks or discontinuities of succession, and some graduated results, as a consequence of the attainment of this aim, have obscured rather than rendered clear the indications of progression which some conditions or stages of life really present.

It has been ascertained, for example, that, in adult *male* life, the annual rate of mortality per cent., whether it be deduced from the experience of the peerage families, from the observations upon assured lives, or from the statistics of a particular social class, such as the Society of Friends, exceeds during the quinquennial period from ages 20 to 24 the rate prevailing in the succeeding interval from 25 to 29—thus showing, by a consistent uniformity of experience, that at these respective stages the power in the human constitution to oppose destruction does *not* suffer a diminution of equal proportions in equal times. An additional force is thus introduced; and this apparently strange reversal of the general uniformity observable in vital statistics may be explained with reasonable probability. The period from age 20 to 24 is especially critical to male lives. Habits of life—evil or favourable—begin to be definitely formed at the age of 17 or 18, when the youth first obtains his practical acquaintance with the world, and can independently subject his will to virtuous or ignoble motives. Those who succumb to immoral temptations (particularly as the physical frame is at this period in its critical phase of consolidation, and therefore peculiarly impressionable to the effects of an unworthy mode of life) survive to the age of 20 with deteriorated constitutions, and tend more rapidly and numerously to death; while the virtuous, and those who, though subdued by temptation, have possessed force of will to change their course, and the more vigorous again of those physically best competent to resist the destructive power of vice to which they succumbed, pass into the second stage of years (25 to 29) as survivors of the fittest in a physical sense, and thus exhibit the reduced mortality of that interval.

The necessity nevertheless of a scheme of graduation remains as the foundation of the scale of premiums, and the practical suggestion submitted is that in all graduated tables the actual observations should be recorded side by side with the adjusted results, so that an actuary may test by comparison the validity of the method employed; and at the same time the actuary, the statistician, and the anthropologist may perceive from the original facts any indications of the introduction of a divergence from uniformity (by the intrusion of another force) which the realities of Nature display.

Besides the study of the doctrine of probabilities in its wider scope, the student is counselled to attain a comprehension of the

process which is termed the Method of Least Squares ; and for the purpose of interesting him in the subject it is proposed to furnish two examples of its use and value.

The method in question endeavours to obtain the best mean of a number of observed quantities, or the most probable law or uniformity derived from a number of observations of which some or all are admitted to be more or less imperfect, that is, more or less deficient in expressing the law or uniformity of Nature, which they nevertheless severally tend to embody. The mathematical grounds of its validity cannot here be explained : it is sufficient to notice that the plan proceeds on the assumption that all errors are not equally probable, and that smaller ones occur more frequently than those of a larger magnitude : it is accordingly ascertained, on these rational and experienced suppositions, that, inasmuch as any result is incompetent of being obtained which is devoid of all error, *that* result is the most probable, the assumption of which renders, not the errors themselves but, the sum of their squares of the smallest possible amount or a minimum.

Thus if a quantity increase uniformly, and, being measured at equal intervals of time, is discovered to amount successively to 4, 12, 14, the results clearly do not represent a uniform progression: it is evident, from the conditions mentioned, that they should constitute an arithmetical series, but their wide deviation from a succession of this nature is obvious. What arithmetical progression then do they *most probably* express ? Since several arithmetical series may be assumed which approximate to the disclosed course of observation—such as, 4, 9, 14 : 6, 10, 14 : 5, 10, 15—which of these series most nearly corresponds to the truth ? Taking the difference between each term in each of the three progressions and the corresponding term in the original series, we obtain errors or defects respectively of 0, 3, 0 ; 2, 2, 0 ; and 1, 2, 1. The sums of each set of errors are 3, 4, and 4 ; and the sum of their squares 9, 8, and 6. Although the first assumed series exhibits the sum of the errors less than the others, the third series gives the sum of the squares of the errors at the lowest amount ; and hence, according to the proposition on which this method depends, the third series constitutes the most probable form of progression of the three.

The fundamental, and therefore the necessary, nature of the method is perceived when we notice that it is inherent in the ordinary

or arithmetic mean. Let three observations furnish 93, 94, and 98 : the average is $\frac{93 + 94 + 98}{3} = 95$. If this result be assumed to be true, it is equally assumed that the errors (deducting the several quantities from 95) are 2, 1, and 3. The sum of the squares of these differences (or 4 + 1 + 9) is 14 ; and this is the least possible sum that can be thus obtained. If, for example, we suppose any other quantity than 95—say 95·1—the reputed errors are 2·1, 1·1, and 2·9 : the squares are 4·41, 1·21, and 8·41, and their sum amounts to 14·03, or an excess beyond the 14 arrived at on the former assumption.

The subject accordingly is deserving of special attention from its simplicity and its fundamental character ; and a very ingenious practical application of its process was made many years ago, which should further interest the student in the principle. In the course of a discussion upon the pressure of expenses upon premium income the accounts of a certain Company were examined, showing for each year of a series the new annual premiums, the renewal and single premiums, the expenses of management including commission, and the general percentage of expenditure upon the total premium income. Assuming x per cent. of the new premiums and y per cent. of the renewals to be absorbed in expenditure, each set of two years naturally disclosed discrepant results, ranging from $x = 95{\cdot}8$ and $y = 2{\cdot}25$ per cent. to $x =$ —35·2 per cent. and $y =$ 18·89. The question arose, which of these diverse percentages might be accepted as the most probable ; and the Method of Least squares was employed, where, of course, for the ultimate determination of the values of x and y, two years alone were not selected, but the whole of the equations for fifteen years (each equation consisting of the new premiums of its year multiplied by x + the renewal premiums of such year multiplied by y = the actual expenses and commission of that year) were incorporated. The fifteen separate equations were formed in the mode just symbolically shown. The first equation was then multiplied by the new premiums of the first year divided by 100 (to obtain percentages) : the second by the new premiums in the second year similarly divided, and so on—each equation being thus multiplied by the coefficient of x in its own first term. The fifteen equations were then summed, and an aggregate equation thus obtained in x and y, the right-hand member being the sum of the products

severally produced by multiplying the amount of expenses in each equation by the coefficient of x therein and dividing by 100. The first equation was then multiplied by its own renewal premium income divided by 100; the second by its renewal premium income similarly divided; and the remaining equations were identically treated; so that each equation being thus multiplied by the coefficient of y in its own second term, and the fifteen equations being added together, a summary equation was again obtained in a form correspondent to the aggregate equation above furnished.

The values of x and y consequently were readily extracted from these two final equations, and were found to be $x = 49$ and $y = 6{\cdot}7$—thus furnishing the best and most probable values that could be deduced from the original fifteen equations. If these final values be substituted for x and y in each of the original equations, they will not exactly satisfy any one of them, yet they will more or less precisely satisfy all, and the deviations will be sometimes on the one side and sometimes on the other. It will then be found, on trial, that the sum of the squares of these deviations (or residual quantities, that is to say, the excesses or deficiencies compared with zero) will be *less* than the sum of the squares of the residuals resulting from the substitution of any other values of x and y.

It is well, however, to impress the lesson again upon the student that he who simply follows the rules deduced from mathematical investigations without steady comprehension of those investigations themselves, will remain a mere mechanical and unintelligent Actuary; and will thus forfeit the mental discipline, and the mastery of processes with the consequent conviction of sureness in work, which this apprehension alone bestows.

CHAPTER II

The Risk and its Limitation; with Explanations and Discussions of Cognate Subjects

In the doctrine of probabilities, the risk of loss or gain is usually defined as that fraction of the sum to be lost or acquired which expresses the chance of losing or gaining it. Thus an even chance of losing £40 is considered a positive loss of half of £40, or £20; and the odds of two to one for gaining £60 are counted as two-thirds of £60 or £40. If both these risks were encountered at the same time, the entire transaction would be considered as a gain of £40 − £20 = £20, since this is the sum which would be obtained, in the long run, and taking one with another, by every compound operation of this nature. The mode of ascertaining the effect of the division of risks in probabilities is this: let there be an adventure in which the chance of success is p, and consequently of failure, $1-p$; let failure produce a loss of £n and success a gain of £m: then, $pm-(1-p)\ n$ expresses the result of every such transaction, massing all the events together.

In life assurance the risk is measured by the chance of payment during any given year; the amount at risk at the origin of the contract is expressed by the sum assured, and at subsequent dates by the amount in question plus the attached bonuses and diminished by the reserve; and the value of the risk at any epoch consists of the amount at risk multiplied by the probability of its occurrence, or q, and reduced by the function v suitably formed.

The chance of the advent of the event increases naturally with every advance in age; but concurrently the amount at risk diminishes in consequence of the augmenting reserves which are created out of past accumulated premiums, so that at any moment the sum at risk is $1-V$ if 1 represent the original amount assured with the annexed reversionary bonuses and V the reserve-value possessed in hand in connexion with the policy.

The amount to be retained at risk upon a single contingency—or, as it is usually technically termed, the limit of risk—will

depend upon the expectation of the Company, from the anticipated extent of its business and the social and commercial position of the public to whom it principally appeals and from whom it is likely to secure assurances, of the number of policies it will probably receive each year for sums equalling or exceeding the amount that may be provisionally determined as the limit of retention. Companies usually commence with a maximum of £2,000 to £3,000; and this they subsequently are enabled to increase upon a given life (1) by reason of the reserves successively formed, and (2) by success in obtaining every year a sufficient number of assurances of a higher amount than the sum originally settled as the limit. If the maximum be £3,000, and, after a certain lapse of time, the reserve-value of the policy advance to £1,000, the amount at risk is reduced from £3,000 to £2,000, so that the Company can then, independently of the prospect of securing a superior range of business in respect of individual amount, accept an additional sum of £1,000 upon the same life at his increased age without exceeding the limit primarily adopted.

The question confronting the Company at its establishment, and at any subsequent stage, is the maximum amount that may be safely retained upon an individual life without the probability of inconvenient fluctuations in the incidence of its claims.

The subject has been excellently discussed, and we merely propose to simplify, if possible, and expound the considerations which affect the solution of the problem.

The assertion furnishes no helpful contribution to the settlement of the question that it is inexpedient to incur a risk whose discharge would prove to be embarrassing, impossible or ruinous to meet. A company, administered with the soundest prudence, must, from the special nature of its business, sustain liabilities which, if occurrent simultaneously, its resources could not possibly satisfy. If the whole of its policies, or a considerable proportion, became concurrently due in consequence of an extraordinary death-rate, the Company would obviously succumb. This possibility, however, must be incurred or the scheme of life assurance would prove impracticable of execution; and obviously the capacity of a Company to confront this formidable possibility with a confidence approaching to certainty of assurance is dependent upon the stability of results of extensive numbers and the

predictive character of future events based upon a wide and congruent experience in the past.

An illustration will be serviceable. If the rate of mortality, or the chance of dying in a year (at age 39) be ·01, (the probability of surviving the year being ·99), and a Company had issued a policy for £10,000 upon a single life, the chance of payment would be ·01 or 1 in 100; if there existed 10 policies of £1,000 each upon different lives (constituting the same total sum assured and at the same age) the probability of the whole of them becoming claims immediately would be $·01^{10}$ or, say, 1 in 100 trillions of chances; while if the entire amount assured of £10,000 were distributed equally over 100 policies the similar probability would be $·01^{100}$ or a fraction consisting of 199 ciphers following the decimal point; or, to present the subject more simply, let there be one policy, and again two policies on different lives, the chance of the one failing to become a claim is ·99 or 99 out of 100 chances, while the chance of both not falling due is $1-(·01)^2$ or $·99 \times ·99 + ·01 \times ·99 + ·01 \times ·99$, or 9,999 in 10,000.

It is thus observed that as the area of distribution of risk is widened, the prospect of an immediate loss decreases, while concurrently the chance of the absence of occurrence also diminishes.

These considerations, however, do not practically affect the technical question requiring settlement by the Company, since the possibility of the whole of the contracts maturing at once by death may be reasonably dismissed as chimerical.

A closer approach to a solution will be gained if the chance be determined that the loss occurring during the course of a year shall not prove less than a given amount—for example, £10,000. If there exist 10 policies of £10,000 each, the probability of exemption from any claim in a year is $·99^{10}$, or ·9044: the chance accordingly of at least one loss is 1—·9044, or ·0956, which expresses the probability that the Company will be required to provide £10,000 or more. If the policies number 100 for £1,000 each, the probability of the absence of loss is ·366 or over 1 in 3; the probability of one loss at least is ·37, or upwards of 1 in 3; the probability of 5 losses is ·003 or 1 in about 330; while the probability of 9 payments is ·0000007, or 1 in upwards of 1,428,000. The sum of the probabilities of 0 loss, 1 loss, . . . 9 losses is ·9999997, which constitutes the chance of losses being sustained for a total amount

falling short of £10,000 in all, so that the probability of the losses amounting to £10,000 or upwards is unity minus this fraction or ·0000003, being 1 chance in upwards of 3,000,000.

It thus appears that when the number of policies is increased, the sum assured remaining constant, (1) the probability of a serious loss diminishes with rapidity, while (2) the chance of the absence of loss or a minor loss on the other hand, is also successively reduced. For example, in respect of a single policy, the chance of the non-occurrence of loss is ·99; in respect of 10 policies, the probability is $\cdot 99^{10}$ or ·9044; where 100 policies are involved, the chance becomes ·366; and in the case of 1,000 policies, the probability diminishes to ·000043.

The increased distribution of the amount at risk produces a reduction in the probability of any considerable deviation from the average in each direction, and the chance of fortuitous fluctuations in the number of deaths decreases with the augmentation in the number of contracts.

A Company's decision in the settlement of the extent of limit involves the precautionary principle that a very considerable risk upon an individual life should be avoided on account of the probability of a heavy loss occurring which would exceed the bounds of prudential administration, and the readiness to entertain the risk of an immense aggregate sum distributed among 100 or 1,000 different lives is consequent upon the reduced chance of a serious range of claims and the circumstance that the consideration required for the transaction is adequate to the value of the liability.

A limit then being necessary, the question of its amount occurs. We have already adverted to the fact that in this decision the probable extent of the assurances to be acquired upon individual lives (determined largely by the social condition of the persons from whom the Company solicits business) enters in the principal degree into the consideration. A limit of £2,000 or £3,000 where the policies are expected generally to range from £250 to £1,000 would be inexpedient, and would tend to introduce adverse and inconvenient fluctuations in the incidence of claims. The criterion accordingly has been proposed by a well-known author (whose views we proceed to expound) that the premiums upon the assurances of maximum amount should be regarded as constituting the fund from which the claims upon those policies should be discharged, so that the amount of premiums annually receivable

upon such assurances should prove sufficient for the satisfaction of their demands by death. A guide to the determination of the limit is thus provided. Let it be assumed that the rate of mortality in a year, or the chance of death, is 1 per cent., and that a Company possesses only a few policies of £10,000 each and a considerable number of assurances of £100 each : let the pure premium of ·01 per unit assured, or 1 per cent., be " loaded " (or increased) for possible fluctuations and other requirements by 25 per cent. : then the premium received on each of the large policies will be £125, and on each of the £100 policies, £1·25. Let it be further supposed that one of the maximum assurances becomes payable ; its discharge will then absorb the whole of the year's premiums received upon 80 policies of this amount (£125 × 80 = £10,000), and if a less number than 80 assurances of £10,000 each exist upon the books, the deficiency must be supplied by the premiums paid upon the smaller policies ; if there be 40 policies only of £10,000 each, their separate premiums will be sufficient to provide the sum of £5,000, and the Company consequently must trench upon the premiums of 4,000 policies of £100 each in order to produce the necessary balance of £5,000. The premiums, however, on these policies of minor extent are required for the settlement of their own claims, and cannot therefore (except as regards the margin of 25 per cent.) be abstracted for the purpose of satisfying the demands upon the assurances of maximum amount. Hence the author (just referred to) concludes that the policies effected for an amount in each instance equal to the settled limit must be considered as constituting a distinct class and must prove adequate in number to provide their own claims from the source of their separate premiums. A reasonable position of stable equilibrium can only be secured if this condition be fulfilled.

Taking an average premium of 3 per cent. (at about age 40) the smallest number of policies of maximum amount should accordingly be 33 (3 × 33 = say, 100), and inconvenient fluctuations will in all probability occur unless the number exceed this figure.

Seeing that, when the limit is determined, it cannot be expected that the necessary number of assurances of maximum sum can be acquired at once, the question of the lowest number of such policies with which it will be prudent to commence is answered by the consideration of the element of selection. The rate of mortality

among lives recently accepted, as will be shown in Chapter III., is considerably inferior to that prevailing in an ordinary community consisting of healthy and unsound lives of corresponding ages, and this favourable feature is substantially exhibited among assured lives for a period of at least five years from the date of admission: hence the Company should be in a position to anticipate—if its limit be reasonably and prudently fixed, looking to the general scope and class of the business which it may confidently expect—the acquisition of this adequate number of assurances within that interval—that is to say, at least 6 or 7 per annum: if, on the other hand, the prospects of the office, from the range and character of its operations, will not probably be fulfilled in this respect, the contemplated (and, at a future stage, if its conditions of business should be modified adversely in this aspect, the settled) limit will require to be reduced.

Thus it is evident, from these considerations, that the basis for assessment of the limit of risk does not consist of the extent of the premium revenue or the magnitude of the assets which the Company may possess, but solely relates to the number of policies of maximum amount which the nature and range of its business entitle it to expect to secure. The preceding conclusions, however, will demand modification where the business of a company is very considerable. If the total loading of the annual premiums prove sufficient, after discharging the expenses and commission, to meet the payment of a claim of maximum amount, it is not necessary that the number of the largest Assurances should be numerous. With a business of very extensive dimension, a company might be justified, in prudence, in issuing a single Policy greatly in excess of any other of its existing Assurances.

The preceding exposition embodies the doctrine upon the subject of a former master in Actuarial investigation, both theoretical and practical; but notwithstanding that a double statement was introduced (in the first edition) to this effect, a general misapprehension has existed that the views presented consisted of those of the present author. It may be useful, therefore, to express briefly his own judgment upon the question. (1) In the first place, we do not perceive any connexion between the number of policies capable of acquisition of the maximum amount and the extent of the limit to be determined, nor the necessity or validity of the condition that

such Assurances should prove sufficient to discharge their own claims from their segregated premiums. As we have insisted throughout, the general mass of the business as a whole, its width of area, and its distribution, must be preserved in view in deciding upon the various problems that may occur for practical settlement. Indeed, if we carried the principle implied in the doctrine previously expounded to its legitimate conclusion, risks of *any* special class would require to be avoided unless the company were reasonably sure of securing an adequate number of that character. The doctrine in question suggests the treatment of all maximum policies as a *distinct* class, and involves, in principle, a *general* teaching naturally applicable to any other specific section of risk. Anticipating for a moment our subsequent illustration of a negro life (employed there simply as the type of a particular category of risk) the doctrine would bar the acceptance of such a case if it were confined to a single instance. But, as we shall see, this suggestion is erroneous and a risk of that kind can be properly massed with the remaining business of the company. A similar remark can be applied to any other description of special risk; and hence it is clear that the general doctrine itself is insecure.

An ingenious illustration (which we somewhat modify) has been adduced by an able American friend. Assume that a company possesses a business composed entirely of large individual risks sufficiently extensive to prevent serious fluctuations; suppose—for the purpose of gaining the clearness of a concrete example—that the company's assurance contracts number 10,000 of £2,000 each; if the anticipated rate of mortality be 1 per cent. per annum, 100 policies, with sums assured of £200,000, are expected to become claims within the year; according to the formula hereafter furnished the average deviation $\pm$ from the number of anticipated deaths (or the extent of fluctuation) will be expressed by $\pm \cdot 8\sqrt{100}$, or 8 lives with £16,000 (8 × £2,000) assured; that is, taking the positive sign, an excess sum of £16,000, for which provision must be made. This deviation will not, in a well-established business, materially affect the expectation of the office in respect of bonus. Let the company now retain only one of its policies for £2,000, and, for the remainder, substitute 99,990 Assurances for £200 each: an equality of aggregate sum assured is thus preserved and consequently the same premium revenue exists; the former expected rate of death

of 1 per cent. (the ages of the substituted policies being identical with the previous ages) prevails ; and the capacity of the company for realising the same anticipated profit continues ; but the range of fluctuation is reduced to about one-third of the prior amount ; that is to say, instead of preparing for a probable fluctuation of £16,000, the company will be required, in consequence of the extension of the area of risk, to expect a fluctuation of about £5,000 only. Thus : the anticipated number of deaths being (approximately) 1,000, the formula $\pm \cdot 8\sqrt{1000}$ gives the probable fluctuation of 25 lives, with consequently a total sum assured of £5,000, or about one-third of the former amount.*

And supposing even that the single policy for £2,000 should form one of the claims, the excess fluctuation would be merely about £7,000. The doctrine cited would demand the multiplication of this isolated heavy risk many times for the purpose of ensuring freedom from inconvenient variations of experience, while it is evident from the preceding considerations that even with a solitary large risk upon the books the range of adverse fluctuation is appreciably reduced. (2) The extent also of the general resources of the company in relation to its aggregate liabilities necessarily enters as a factor in the determination of the limit, contrary to the teaching of the doctrine which we are criticising. The student will find the grounds of this consideration in works upon Probability. A person with a very ample capital, soundly proportioned to his obligations, and with a large command of credit, may clearly and legitimately enter into ventures which would be fatal to an enterprise possessed of limited resources ; and a company with a widely-spread and extensive business, supported by reserves of great magnitude, and a considerable unvalued margin in its premiums, may obviously contract engagements which a less powerful financial position would

* We obtain the same relative results if we adopt another form of estimate which proceeds on the principle that the percentage of fluctuation, for which there exists a given chance, varies inversely as the square root of the entire number of trials. The probable extent of deviation from the number expected to die in one year (100) in our first calculation is 8 per cent., which amounts to ·08 per cent., upon the total mass of 10,000 lives : we then proceed to the computation where practically 10 times the number of lives are involved, and thus obtain

$$\cdot 08 : x \text{ (the percentage of fluctuation where 99,991 lives are at risk)} = \sqrt{99{,}991} : \sqrt{10{,}000}$$

whence $x = \cdot 025$ per cent., or 25 on the aggregate of 99,991.

necessarily avoid. And here also enters the question of a combination of risks in the critical appreciation of the doctrine upon which we are commenting. Two companies each possess a policy for £5,000 (the maximum amount) upon the same life: they amalgamate, and hence the united company holds upon that single contingency the sum of £10,000. As we have shown, an extension of the total area of risk produced by the combination has reduced the range of fluctuation which existed when the companies formed separate institutions, and with the conjoined reserves also for the aggregate £10,000, it cannot be denied that the amalgamated office is safe, although its holding upon an individual life is now doubled. (3) It is clear that no mathematical or statistical criterion for the settlement of the limit of risk can be expressed; its standard must depend upon the extent of the general business which comprises all kinds and amounts of Assurances; the scale of resources possessed in comparison with the obligations; the probable effect upon the amount and rate of profit and especially upon the maintenance of a series of bonuses which shall not be retrogressive but, if practicable, increasing; and hence the primary criterion in the decision is the range of fluctuation which it is deemed prudential, in the sound and permanent interests of the company, to incur for the more efficient promotion of those interests. The preceding factors are obviously incompetent of incorporation into a summary general rule, since they vary in intensity and scope in different companies, and the judgment, therefore, in each instance must depend upon the state of facts which exists, and the capacity, experience and skilled foresight of the Actuary. A corollary from this proposition may be mentioned: an office which distributes its profits annually cannot wisely adopt so high a limit as one whose valuations and allotments of bonus occur once in five years, for in the former instance a serious variation in the incidence of the mortality might very adversely affect the ensuing bonus, while, in the latter, the interval of five years would afford a sufficient opportunity for the redress of any particular yearly fluctuation by the influence of the remaining experience. (4) The element for consideration involved in the problem is not that of solvency: companies generally are too securely established to render this feature one of importance in the enquiry, and the specialised powers of circumspect administration which prevail are adequate and alert

enough to avoid the minutest hazard in that direction. The fundamental determining factor, accordingly,—surveying companies as they are at present constituted—is the probable effect upon the scale of bonus and the maintenance of the past rates of distribution in a progressing or, at all events, in an unregressive, form. (5) We thus regard the action of companies upon this question in a two-fold aspect : (*a*) in their early years the limit should be strictly moderate on the grounds (1) that in consequence of their comparatively minor reserves an adverse incidence of claims under maximum policies would produce an appreciable strain upon the general resources and especially upon the expectation of profits, notwithstanding the diminished aggregate rate of mortality due to recent Selection, and (2) that the force of compound interest has then possessed insufficient time to operate with efficiency ; while (*b*) after a few years, the augmented expanse of the area of the business as a whole, and the accretions to reserves afforded by the more productive power of interest, introduce a different condition of affairs and enable the limit to be soundly advanced from time to time. For a company may properly anticipate a more uniform experience of mortality as the duration of its contracts increases and the number of them is enlarged.

(6) At whatever stage, accordingly, of a company's history, early or late, the limit requires to be settled or modified, the ruling principle is the possible influence upon the rate of profit.

(7) It may be added that in assessing the limit of risk, the Actuary will, in every instance, include essentially in his consideration, the amount of claims which may be annually expected according to the Table of mortality employed in the calculation of the reserves, and will be guided in his determination by such a percentage of fluctuation from the anticipated demands as may appear to him, from the nature and extent of the business generally, to be accordant (in relation to profit) with prudential and enlightened prevision.

The usual limits in practice extend from £2,000 to £3,000 up to £10,000, or even sometimes £15,000. A Company chiefly appealing, from its form of constitution, or aims, or the social and financial position of the persons by whom it is composed, to the wealthier ranks of society, whose policies are generally effected for superior amounts, may obviously possess a limit in excess of

that which prudence would dictate to an Office whose enterprise is mainly confined to the general population.

It has already been observed that as a Company advances in age the extent of its risk upon a single life is diminished by reason of the reduction of the strain upon its resources produced by the enhancing reserves, so that, at a subsequent stage or stages, its holding may be augmented without advancing in total beyond the established limit. If the limit of the sum assured be $£x$, and $£y$ at any time be the reserve retained in connexion with the policy, the actual amount at risk is $£ (x-y)$, and the Company accordingly, without modifying its limit, may accept an additional sum of $£y$ upon the life in question.

The practice is sometimes adopted by Companies of retaining the full limit of risk upon sound or average lives and an inferior limit upon lives whose diminished prospects of longevity demand an additional premium. No valid reason appears to exist in justification of this discrimination. A life of the age of 30 "rated up" to age 40, on a careful and exhaustive examination of all the circumstances, is classed as to the expectation of longevity with normal risks of that higher age; and it seems to follow clearly from the general consideration of the adequate basis of life assurance procedure that if an average life of the age of 40, accepted at the ordinary rate, be assured to the extent of the maximum limit, the same course should be pursued in respect of an applicant who, by means of the extra imposed, has been placed in the category of normal lives at the increased age. But the Company, it is urged, may form an erroneous estimate of the appropriate addition to the age; equally, however, its judgment may err in assessing the apparently healthy life of 40, in connexion with whom the full amount is unhesitatingly retained, as a normal risk.

The author's own conclusion from a lengthened practical experience suggested that inferior lives, whose premiums are properly adjusted for diminished vitality, are in the mass desirable risks, where, of course, the deficiency did not proceed from unfavourable personal habits: a sound life will frequently presume upon his constitutional power to resist disease and death and, over-confident in his vigour and strength, will incur exposure to dangers affecting health which a person, conscious of organic or functional defect, will avoid from the instinct of self-preservation quickened by the

experience of physical troubles attending any undue incurrence of adverse influences.

This remark suggests the further observation that it is a prudent maxim, as a general rule, to reject proposers who have been addicted to drink or who occasionally indulge to excess. The reason is founded upon the existence of the instinct of self-conservation in human nature. A person physically deficient and acquainted with his weakness may justly be admitted at an adequately enhanced premium, since this instinct furnishes a protective force in favour of the Company; but this preservative power is imperfect or absent in the man of inferior habits since the prevalence of these habits in itself implies a diminution of the vigour of that instinct in its influence upon volition, and in the extreme case involves a complete suppression of the will, as the regulative guide of life, due to the practical abolition of the instinct itself.

It need not be pointed out that the system of reassurance, or the transfer to other Companies of the excess assurance beyond the limit, enables an Office to issue its own policy for the entire amount, —the arranged disposal of the surplus forming a private transaction of which the policy-holder possesses no cognisance.

Many interesting problems occur in connexion with the propriety and soundness of entertaining special risks or contingencies of an apparently heterogeneous character, and of the validity of their classification with the mass of assurance obligations of the more ordinary nature. Into many of these questions enters the new element of the value of the obligation as measured by the amount of the consideration received as recompense for its incurrence. A Company may be desired to promise payment of a considerable sum on the happening of an extremely improbable contingency—a policy, for example, of exceptional amount to be payable only in the event of a young person dying on a remote birthday, or of the whole of a specified group of persons dying during the course of a single year in a defined order of death. The reply to this question does not depend upon the practical unlikelihood of securing a large number of risks of this special kind sufficient to permit the occurrence of every fluctuation and the consequent establishment of uniformity in the chances of death, with the guarantee of average results: the answer contemplates the possible profit, and affirms that notwithstanding the fact that the contingency verges upon the

impossible, the venturer could not obtain from the public an adequate compensation in the form of a premium which, from the point of view of profit, would render it worth while to undertake the hazard, remote though it be. The calculated chance of occurrence is so insignificant in these extreme instances that even with a substantial "loading" the consideration-money which could be secured would, from the commercial aspect, yield a profit too immaterial to justify the acceptance of the microscopic possibility of claim.* In life assurance, equally as in the affairs of commerce, an adequate return must determine the prudence and expediency of departing from the customary range of obligations.

An eminent scientist—inferior only to Newton, and, in the judgment of competent assessors, only slightly inferior—one of the subtlest, most versatile, and penetrating minds that ever, though in a purely theoretical sense, discussed these problems,—wisely and conclusively dealt with other apparently exceptionable contingencies of a more practical character. We proceed to expound his views.

It is urged that if a Company accept a risk of any special description, such as the assurance of a negro's life, its action fails in judgment inasmuch as it cannot expect to obtain compensation from other risks of a *like* nature—assuming, for example, that the Company is contemplating the admission of a few negroes only, and not their assurance in a selected mass, upon a suitably calculated table of mortality. To this objection it was justly retorted that there existed the same chance of securing compensation from risks of a *different* kind, and that even if this were not the case the Company would acquire an adequate price for the risk since the premium would be increased beyond the

* As an extreme illustration of an exceptional contingency, consider a contract for payment of an amount on two lives both dying at the same instant of time. The number of instants in the possible continuance of existence of each is greater than any that can be assigned: and since the failure of both must occur at an identical instant, the probability of the compound event is expressed by a fraction where unity is the numerator, and whose denominator consists of a number larger than any that can be assigned, or, speaking popularly, the chance is null. Measured theoretically, therefore, the probability of the risk practically verges upon zero: in the order of Nature such an event appears perfectly natural when it occurs, though the *a priori* chance against its happening be infinite. No Company however, could accept a risk of this character, as the pecuniary value (duly increased) which might entitle the proposition to consideration would to the public appear to be utterly disproportioned to its intrinsic nature.

usual scale for ordinary lives, in proportion to the appropriate rate of mortality. Common sense and sagacity of judgment, it might be added, assess the value of isolated and exceptional contingencies occurring in the customary affairs of life in a corresponding manner. All treatises on Rhetoric devote special consideration to the intensity of an impression produced on the mind by any singular object or circumstance widely divergent from habitual experience; and it is this intellectual disturbance—the strange and startling effect impressed upon a mind, which is incompetent to contemplate events without the intrusion of feeling, by any striking departure from its range of experience and thought —which is alone responsible for a conclusion which is neglectful of *facts* and indicative of mental inertia in the vivid perception of novel truths. If the consideration received for these special risks be proportioned to their exceptional character the stability of the general results is not affected whether the contingencies involving the probabilities of human life prove to be of the same or dissimilar kinds. Moreover, no one has yet essayed to assign by calculation the nature of the precise disadvantage which is thus assumed to attach to singularities of human risk, or to define the extent or degree of singularity which renders a contingency of this type ineligible. It cannot be determined, for example, whether or not an assurance upon a selected negro at an adequate premium constitutes a different *kind* of risk from an assurance upon the life of a European, and even if this proposition should be asserted—that the contingencies possess a diverse nature—it would in proportion be difficult to prove, following analogy, that a single policy from a specified town or country ought not to constitute an objectionable risk for a similar reason, or rather the absence of a reason. A Company equally accepts an assurance upon a master butcher as upon a clergyman.

The distinguished scientist confirms his valid reasoning by stating that even were it feasible to discriminate every imaginable diversity in the kinds of risk, with any approximate precision, so as, for example, to present 1,000 descriptions: then if 1,000 equal risks of each of the separate kinds were at first accepted by 1,000 different Companies at proportional premiums: one kind by Company A, another kind by Company B, and so on, each confining its transactions to a single kind; and assuming each Company, when

restricting itself to its specialized class of risks, to be solvent—a condition which would exist since each office would demand an adequate premium—it is consequently evident that if each of the 1,000 Companies, instead of limiting its operations to its individual description of risk, exchanged one of those risks with each of the remaining 999 offices, so that each now possessed a heterogeneous body of contingencies in place of the original homogeneity, then the profits and losses of each Company being supposed to be fairly balanced when they each confined their range to their own order of risks, such profits and losses will continue equally balanced under the assumed redistribution.

Risks, accordingly, dependent upon human life may appropriately be classed together subject only to adequate compensation being received in each instance; a negro of the age of 30 paying the premium charged to a European of $30 + x$ years of age according to the character of the risk.

The ground of validity of the classification of a European of age 40 with a negro of that age but "rated-up" (to produce equivalence of risk) to $40 + x$ is afforded by the fact that a fundamentally natural scale of measurement can be employed for both risks under the law of human mortality, with the sole variation that different parts of that scale will apply; while Fire risks, for example, are not capable of being massed in the same category with life contingencies since the standards of measurement are inherently discrepant.

Whether the amounts assured by a Company consist of a single policy of £100,000, or of 10 policies of £10,000 each, or of 100 of £1,000 each, or 1,000 of £100 each, the expectation of loss is identical—being in each instance, on the suppositions mentioned, £1,000. (The student will remember that in the doctrine of probabilities the expectation of loss is obtained by multiplying the amount of loss by the chance of its occurrence.) When there exists the single policy for £100,000, the probability of loss is ·01 (adopting the same value of "q" as in a former illustration), and the expectation of loss is consequently $\cdot 01 \times 100{,}000 = 1{,}000$; and, generally, if the aggregate sum assured be S under any number n of policies, and the chance of any one of them becoming a claim be x, the possible losses are (each policy assuring $\frac{S}{n}$)

$\frac{n}{n}S$, $\frac{n-1}{n}S$, $\frac{n-2}{n}S$, . . . $\frac{2}{n}S$, $\frac{S}{n}$, *o*. The several probabilities of these losses are—

(1) x^n . . . (that is, the chance of the whole of the policies maturing by death):

(2) $nx^{n-1}(1-x)$ . . . (that is, the chance of all except one becoming claims: thus, if there be four lives, *A*, *B*, *C* and *D*; *A*, *B*, and *C* may die and *D* remain, the probability of which is $x^3(1-x)$; *B*, *C* and *D* may die and *A* survive, whose probability is $x^3(1-x)$; and so for the combinations of *A*, *B*, and *D*, and *A*, *D* and *C*, dying: the sum of the probabilities being $(x^3 + x^3 + x^3 + x^3)(1-x)$ or $4x^3(1-x)$.

(3) $\frac{n.\ n-1}{1.2}x^{n-2}(1-x)^2$ . . . (the chance of all dying except two).

(4) Finally, $(1-x)^n$ (or the chance of all surviving).

The total expectation of loss, multiplying the several losses by their respective probabilities of occurrence, is—

$$S\left(x^n + \frac{n-1}{n} \times nx^{n-1}(1-x) \ldots + \frac{nx\,(1-x)^{n-1})}{n}\right)$$: (for the last term of $o \times (1-x) = o$): which is reducible to—$Sx\,(x + (1-x))^{n-1} = Sx \times 1^{n-1} = Sx$, which is independent of *n* or the number of the policies. Thus: when $S = £100{,}000$, and $x = \cdot 01$, the expectation of loss is £1,000, whatever value be assigned to *n*.

Again, the "most probable" loss (which differs from the expectation of loss) may be found. If *S* represent the sum assured under *n* policies of equal amounts, it can be shown that if $(n+1)x$, —where *x* denotes the probability of a claim occurring in a year)—be an integer, $(n+1)x - 1$ and $(n+1)x$ claims are equally probable, and more probable than any other number of claims. With 625 policies for £160 each, $n = 625$, $x = \cdot 01$ (say); and consequently $(n+1)x = (626)(\cdot 01)$ or 6·26, so that in this instance the most probable number of claims is 6, with a loss of £960. In general, the expectation of loss is Sx or $nx \times \frac{S}{n}$, and the most probable loss is $\{$the integral part of $(n+1)x\} \times \frac{S}{n}$, so that if nx be an integer it

must be the integral part of $(n + 1)x$, and in this instance the expectation of loss is identical with the most probable loss.

The problem of the number of lives assured, or the extent of observations, necessary to be secured in order that the general deviation from the expected number of deaths implied in the Table of Mortality employed may fall within certain assigned limits of fluctuation, may here be briefly noted.

The proposition may be expressed in a concrete and familiar form : what is the number of lives required to be under observation so that, if the anticipated deaths be 10, for example, the specified deviation from that number may be confined within $\pm$ 10 per cent., or that the actual experience shall range between 11 and 9 ? If 100 policies be effected for £100 each at the age of 35, the O^m Table shows that the probability of dying in the ensuing year (or q) is ·00738, or that a total payment of claims may be expected of 100 × £100 × ·00738 or £74—the number of deaths being 100 × ·00738 or ·738. If the actual experience should happen to be double, or 1·476, the claims of course would amount to £148, while the aggregate pure premiums received (at 3 per cent.) would be £212, and a heavy burden would thus be entailed upon the Company's resources.

It is important consequently to determine approximately the number of lives exposed to the risk of death which can constitute the basis of reasonable conjecture that the actual claims shall occur within a definite range of fluctuation, in excess or deficiency, from the number involved in the Table of Mortality applied. If the probability of an event occurring at a single trial be q, in n trials it will happen on the whole nq times, and it has been proved by mathematical analysis (with the process of which the student should hereafter become cognisant) that the general magnitude of the deviations from the result above expressed or nq, that is to say, the sum of every possible fluctuation multiplied by the probability of occurrence in each instance, may be represented approximately by the formula $\sqrt{\frac{2}{\pi} npq}$, where π possesses the determined value of 3·14159 . . . ; n expresses the number of trials, or the extent of observations ; p (in this instance) represents the chance of surviving a year at the given age ; and q the probability of dying during the course of that year. Substituting the value of π, we

obtain $\sqrt{\frac{2}{3{\cdot}14159\ldots}}$ $.npq$, or $\sqrt{{\cdot}6366}\ \sqrt{npq}$: extracting the square root, we arrive at ·798 $\sqrt{npq}$, or approximately ·8 $\sqrt{npq}$, or $\pm\frac{4}{5}\sqrt{npq}$.

As an example, assume that 10,000 lives are at risk at age 55, where q on the O^m Table is about ·02: the expected number of deaths is accordingly 200, and the extent by which the actual deaths would on the whole exceed, or be deficient from, this number would be obtained from the formula $\pm\frac{4}{5}\sqrt{10{,}000 \times {\cdot}02 \times {\cdot}98}$, where the fraction ·98 is p or $1-q$. The result is 11·2: that is to say, the deviation, on this assumption of the numbers assured, from the expected number of deaths, or 200, would be 11·2, expressing therefore an actual experience ranging probably between 211 and 189.

(It may be observed that since $\sqrt{p}$ or $\sqrt{1-q}$ is nearly unity for all, except extreme, ages, the expression $\pm\frac{4}{5}\sqrt{npq}$, or $\pm\frac{4}{5}\sqrt{nq\,(1-q)}$ may be approximately written $\pm\frac{4}{5}\sqrt{nq}$ or $\pm\frac{4}{5}\sqrt{\text{the number of expected deaths according to the table}}$. Thus: at the age 25, on the O^m Table, $\sqrt{p}$ is nearly ·997; at 45, ·994; and at 65, ·978.)

Assuming the average age of 42, it is required to ascertain the number of the assured which should be under observation in order to secure a deviation of ± 10 per cent. of the expected deaths,—a range of divergence which may be regarded as reasonable. At that age, q on the O^m Table is nearly ·01. Let x be the required number: then $\frac{4}{5}\sqrt{x \times {\cdot}01 \times {\cdot}99} = x \times {\cdot}01 \times \frac{1}{10}$: or $\frac{4}{5}\sqrt{x \times {\cdot}0099} = x\,({\cdot}001)$; and we obtain, by squaring, $\frac{16}{25}\,(x \times {\cdot}0099) = x^2\,({\cdot}000001)$, or $\frac{16}{25}\,({\cdot}0099) = x\,({\cdot}000001)$, whence $x = 6336$.

If it is desired that the fluctuation be restricted to ± 1 per cent. of the anticipated number, our formula will be, if x be the necessary

number of lives, $\frac{+}{-}\frac{4}{5}\sqrt{x \times \cdot 01 \times \cdot 99} = x \times \cdot 01 \times \frac{1}{100}$; whence, Proceeding in a similar manner, $x = 633{,}600$.

If the number of lives be 10,000,000, the deviation will be found to be $\frac{+}{-}\frac{4}{5}\sqrt{10{,}000{,}000 \times \cdot 0099}$, or $\frac{+}{-}\frac{4}{5}\sqrt{99000}$, or about $\frac{4}{5}$ of 314·7, or 252.

The expected deaths being 10,000,000 × ·01 or 100,000, the divergence thus amounts to about $\frac{1}{4}$ per cent.

The rapidity with which the range of general fluctuation decreases as the number of observations is augmented will be noticed, so that in the limit no deviation would exist. If the student cares to indulge in a fanciful calculation (true, though impossible in itself) he can apply the formula by inserting ∞ or an infinite number of lives, when he will obtain—if x be the required percentage of deviation—the value of x expressed as a fraction with a finite quantity as the numerator and an infinite number as the denominator, or zero will be assigned as its value, indicative of the fact that the experienced mortality will precisely coincide with the expected result.

It is to be observed that the difference will the more widely fluctuate with the numbers ordinarily at risk in a Company during the currency of a single year compared with the more confirmed stability afforded by the data of a valuation period; and hence the assessment should be based upon the numbers existing at the termination of a valuation interval and at the average ages then disclosed. And, moreover, the results will also depend upon the mode of distribution of the total sum assured—whether any exceptional amounts appertain to certain ages or groups of ages compared with the sums assured at the remaining ages. It may be reasonably stated that, with a view to ensuring a deviation within convenient limits, the aggregate number at risk, assuming a fairly uniform distribution of amounts assured, should be at the lowest 1,000.

If, for example, as a mere illustration of procedure (and not of sufficiency of materials), the expected deaths in a year are D, and it is required that the fluctuations shall not exceed 10 per cent. of the deaths expected in the entire quinquennium, then—

$$\frac{4}{5}\sqrt{5\ D} \text{ must be } < \frac{5\ D}{10};$$

that is—

$$\frac{8}{10}\sqrt{5\ D} < \frac{5\ D}{10}$$

$$\text{or } \frac{64}{100}5\ D < \frac{25\ D^2}{100}$$

$$\frac{64}{100} < \frac{5\ D}{100}$$

$$64 < 5D$$

$$5\ D > 64$$

$$D > 12{\cdot}8;$$

or the deaths (5 D) in the quinquennial period should be 64 at least.

This exposition supplies a practical exemplification of the second proposition involved in the theorem of Bernouilli (considered in Chapter I.) by demonstrating that in an augmented number of trials or observations the probability of a specified event happening within certain assigned limits of divergence can be rendered as near to certainty as we please by multiplication of the experiments.

The question of the number of risks, adequate in extent to permit all fluctuations in different directions to become competent of display, and thus produce in the mass a stable result, is one upon which the student's attention should be carefully concentrated. Instances have occurred where Pension Funds, involving rates of marriage in addition to rates of mortality, have been formally valued although the number of members comprised only 70 or 80; and a case is in print where the financial condition of a Friendly Society was investigated by the usual actuarial methods while the members under observation numbered only 26! The valuation functions employed are deduced from a mass of observations in which the divergencies have produced their several effects on the whole, and the application of these functions necessarily presupposes that any funds or statistics in which they are adopted exhibit, in order to ensure an equivalent accuracy of prediction, a corresponding and adequate range of area, similar (though usually inferior) to that of the observations from which the factors were originally derived. Hence valuations of the nature just described consist simply of empty and futile calculations, and constitute

no trustworthy indication of future probabilities. Funds of this meagre description should be wound up, or transferred to cognate institutions possessing an ampler scale of operation.

The student, however, should bear in mind that the notion is fallacious that even in extensive observations a complete and exactly neutralizing balance exists between diverse individual fluctuations: he will perceive that the result which does occur expresses that, in such instances, the proportion which the fluctuations or deviations bear to the anticipated event is one of diminishing magnitude.

The student is presumed to be acquainted with the mode of constructing the pure premium, or the premium for the risk of death alone—compounded of the probabilities of dying in each successive year and the accumulative force of interest. It will, however, be useful to explain the process by which, when the pure premium has been obtained, its amount is increased for provision of the administrative demands of the business and the formation of surplus.

The addition by which the pure premium is augmented for future expenses, fluctuations and profits is termed the "margin" or "loading,"—an infelicitous expression. The system of life assurance calculation possesses an admirable symbolic language; but its terminology (to employ the expression of scientists) is lamentably defective and inadequate. The terms "gross premium," "mathematical premium," "risk premium," "office premium," and "loading" are indefinite and unscientific; and it is to be hoped that some consonance and symmetry may be adopted hereafter between the descriptive expressions of this science and its excellent analytical speech.

In earlier days, the loading was effected by means of a uniform percentage addition to the pure premium. At age 35, the O^m 3 per cent. pure premium is 2·116 per cent.; at age 55, 4·641: hence a loading of 20 per cent. would produce 8*s.* 6*d.* per cent. in the former instance, and 18*s.* 7*d.* per cent. in the latter. This inequality of assessment is clear: the commission being a percentage is naturally heavier on the higher premium; but the contribution to the ordinary expenses of management should be the same since no greater charge is entailed in the administration of assurances on lives aged 55 than on those where the age is 35.

The mode of determining that portion of the loading which forms the contribution to profits will depend upon the plan of distribution adopted, and hence the universal principle occurs that premiums should be constructed in consistency with the method of valuation and the system of division of profits.

It is customary to select a loading partly consisting of a constant addition for defraying the general administrative cost, and a percentage for commission and profits ; and here it should be noticed that the percentage imposed for commission should not be assessed upon the pure premium, but upon the *entire* premium (to be determined, including the commission) upon which complete amount the commission is charged. Thus, omitting all elements (for simplicity of illustration) except the commission, if the rate be 5 per cent., and the pure premium 2·116, the full premium is thus obtained : let x represent that premium, then $x - \frac{x}{20} = 2{\cdot}116$, or $x = 2{\cdot}227$; and the 5 per cent. being deducted, we have exactly remaining the pure premium in question.

The subject in a book of this limited character need not be pursued ; but it may perhaps be added, in connexion with the constant, that if this amount be x per cent. upon the sum assured a policy for £20,000 is charged 200 times the annual contribution levied upon an assurance for £100 only : that if the constant be related proportionately to the sum assured and be fixed at 5*s*. per cent., the former policy contributes £50 a year and the latter 5*s*. Assurances of reduced amount no doubt entail a greater proportionate trouble and cost, to some extent, than policies for larger sums, but the unreasonable disparity exhibited by this method of forming the constant is evident. The superior policies, so far as expenses, apart from commission, are concerned, merely involve an increased medical fee and a higher stamp, but the cost of their acquisition is usually considerably lighter than that appertaining to smaller assurances, and the administrative charge is in no degree enhanced.

The loading, it is clear, should, as regards expenses (independent of commission) take cognisance of (1) the heavier initial charges attendant upon the completion of the policies, and (2) the diminished cost of the renewal administration. The latter can be ascertained as a basis by comparing the total ordinary expenditure

(apart from commission and preliminary charges) with the aggregate original amount assured. It may be incidentally mentioned that the loading must also serve as compensation for the loss of interest upon the premiums consequent upon the grant of days of grace for their payment. In the construction of the pure premium, the Tables employed assume that accumulation instantly occurs at every recurrent due-date, while, in practice, deferment is permitted for one month. If then the premium Revenue amount to £100,000 per annum (considering annual premiums alone), and if the rate assumed be 3 per cent., a yearly loss in this respect is entailed of £250.

We append a few illustrations of formation for the student's benefit.

If the initial expenditure be represented by 1 per cent. upon the sum assured (the first payment of commission being usually at that rate); the renewal commission and provision for fluctuations of experience by $7\frac{1}{2}$ per cent. upon the premiums, or ·075 per unit; and the contribution to the uniform expenses of management by a constant of 2*s.* 6*d.* per cent. on the sum assured, or ·00125 per unit, the contract premium has thus been constructed in one instance (where π_x is the pure premium)—P_x (or the non-profit office premium)

$$= \pi_x + \cdot 075\ \pi_x + \cdot 00125 + \cdot 00125\ (\cdot 075) + \frac{\cdot 01}{1 + a_x} + \frac{\cdot 01}{1 + a_x}(\cdot 075):$$

(the ·01 is, by division by the annuity-value, distributed over the entire duration of the policy): that is—

$$P_x = \pi_x\ (1 \cdot 075) + \cdot 00125\ (1 \cdot 075) + \frac{\cdot 01}{1 + a_x}\ (1 \cdot 075).$$

Hence—

$$P_x = 1 \cdot 075\left(\pi_x + \cdot 00125 + \frac{\cdot 01}{1 + a_x}\right) \text{ or } P_x = \left(\pi_x + \cdot 00125 + \frac{\cdot 01}{1 + a_x}\right)\frac{1}{\cdot 93}$$

where $\frac{1}{\cdot 93} = 1 \cdot 075$.

The objections to this mode of formation are (1) that the percentage of $7\frac{1}{2}$ is imposed also upon the constants, but since these have been assumed to be adequate in themselves for their express purposes, the adoption of this method renders these items excessive or beyond the supposed requirements, and (2) the percentage of $7\frac{1}{2}$ is assessed upon the pure premium, but this

percentage forms a deduction from the contract premium, and consequently the addition in question is insufficient. Hence if the data adopted be correct (and an examination of the Company's accounts should constitute the foundation of the estimate) the proper formula should be—

$$P_x = \pi_x + \cdot 075\ P_x + \cdot 00125 + \frac{\cdot 01}{1 + a_x}; \quad \therefore P_x(1 - \cdot 075) = \pi_x + \cdot 00125 + \frac{\cdot 01}{1 + a_x} \text{ or } P_x = \frac{1}{\cdot 925}\left(\pi_x + \cdot 00125 + \frac{\cdot 01}{1 + a_x}\right)$$

The preceding inquiry relates only to non-participating assurances. These premiums are in no way suggested as practical ones; they exhibit simply the mode of formation: however scientifically a scale of premiums may be framed they must be viewed judiciously in relation to current competitive rates.

In the calculation of premiums, and particularly in complex problems, the student is recommended to adopt the excellent method of determining the values of the Benefit and Payment sides of the transaction; and it is hardly necessary to add that where the arrangement comprises a return of the Office or Contract premiums received, under certain contingencies, the loaded premium must be introduced into the Benefit side of the account. The approximate method of assessing the value in very complicated problems by the application of the Calculus is necessarily omitted in this book.

Apart from the loading for expenses and commission, let us consider the addition to be imposed upon the pure premium for provision of a compound reversionary bonus at the rate of £b per unit per annum upon the original sum assured and upon all bonuses existing at the beginning of a quinquennial period in respect of each year completed during that term.

During the first year of the first valuation period the bonus to be allotted will be b: in the second year, the total bonus will be $2b$; $3b$ in the third year, and so on; thus, at the close of the first valuation period the sum of the bonuses to be assigned will be $5b$. At the expiration of the first quinquennium, the bonus for the ensuing year will consist of b assessed upon the sum assured and b ($5b$) in respect of the existing Bonuses or together b ($1 + 5b$), making with the bonus allotted during the first stage $5b + b$

$(1 + 5b)$, and so on. If $\frac{R_x}{D_x}$, as usual, be the value of an assurance, increasing by 1 per annum during life, $\frac{M_x}{D_x}$ being the value of the original amount assured, we obtain as the annual premium loaded for this description of bonus—

$\frac{M_x + \Sigma b}{N_{x-1}}$, where Σb represents $b\ (R_{x+1} - R_{x+6})$ (or the value of the bonus for the first five years) $+\ b\ (1 + 5b)\ (R_{x+6} - R_{x+11})$ (or the value of the bonus for the second five years) $+\ b\ (1 + 5b)^2\ (R_{x+11} - R_{x+16}) + \ldots$ to the close of life.

It may be useful to explain the mode in which the portion of this expression applicable to the third quinquennial term, or $b\ (1 + 5b)^2$ is obtained. If the student will tabulate the series of increments by bonus stage by stage, he will find that in the tenth year (immediately prior to the origin of the third Valuation period) the sum assured and the aggregate attached Bonuses amount to $(1 + 5b) + 5b\ (1 + 5b)$ or $(1 + 5b)\ (1 + 5b) = (1 + 5b)^2$; and the bonus to be granted in the eleventh year upon this total sum will be $b\ (1 + 5b)^2$.

The question has occurred whether a loading of a given percentage upon the pure premium will provide for an additional mortality of the same percentage upon the normal rate of mortality. Now the relation existing between variations of premium and mortality involves the rate of interest employed. The increase in the value of the liability upon existing policies attendant upon a higher rate of mortality is not produced by a greater number of claims to be discharged but by the fact that the claims assumed in the premiums occur at an earlier date than was contemplated. The value consequently will vary with the rate of interest adopted, and any assumed relation between the preceding elements which neglects the factor of interest cannot be that which actually prevails. Regard an extreme instance. The rate of mortality at age 98 (on the O^m Table) is ·5. An increase of 100 per cent. renders death a certainty during the ensuing year, and the value of an assurance at age 98, or A_{98}, is consequently reduced to v, or at 3 per cent. ·9709.

Subject to the normal mortality, and at 3 per cent., A_{98} = ·94685 (O^m 3) and π_{98} = ·51838. Augmenting these amounts in

the same ratio as that in which the mortality has been increased, or 100 per cent., we obtain 1·8937 and 1·03676. But on the assumed relation, each result should be equal to ·9709, and hence it follows that the relation in question does not exist.

It has been recalled to my remembrance that an empirical generalisation has been obtained in connexion with this relation, namely, that if the rate of mortality throughout life be increased k per cent. the uniform whole-life annual premium will be augmented $\frac{1}{2}$ k per cent. This rule is very approximately coincident with facts, but while the shorthand of generalisations like this is serviceable to the experienced Actuary, who can estimate (from matured knowledge) the appropriateness of their utilisation in any case, the student is earnestly counselled to examine and understand the original processes themselves from which such rules are approximately deduced.

Attention may be briefly directed to what are termed the level, or uniform, and the assessment, scales of premium.

The uniform plan adopts the same premium for any stated age at entry throughout the entire period of life. At age 35, for example, the H^m 3 per cent. annual pure premium is 2·193 per cent.; the premium for an assurance during the term of one year succeeding that age is ·852 : for an assurance enduring only for five years, ·914 per cent. per annum : while at the age of 60 the pure premium for one year's risk is 2·881 per cent., and for an assurance continuing for five years only, 3·35 per cent. per annum. Hence, regarding the period of one year only, the uniform pure premium involves the sum of 2·193 — ·852, or 1·341 in excess of the actual value of the risk of death at that age ; while at age 60 the position is reversed, and the policy-holder pays, according to the level plan, 2·881 — 2·193 or ·688 in deficiency. The surpluses consequently received by the Company during the earlier stages of the assurance must be accumulated at interest for the purpose of compensating the deficiencies which occur at the ulterior periods, so that during the entire range of duration the uniform premium exactly provides the increasing risk of death. The assured in this scheme possesses the advantage, not merely of bearing a uniform (in place of an augmenting) burden but, of knowing also at every epoch the amount he is required to provide.

The assessment system, on the contrary, exacts from each member in each successive year a subscription towards the payment of the

claims (or the Mortuary Calls, as they are termed) which have actually occurred, according to the age attained at the date of the levy ; and the ratios of his contributions are in proportion to the probability of dying in each sequent year (q_x) in accordance with the Table of Mortality employed. Limiting our consideration of the scheme to its principle, without entering into details, and omitting the plan (forming a violation of the fundamental notion) of increasing the periodical assessments with a view to reducing the extent of future contributions, it is obvious that if the experienced rate of mortality coincide with that indicated by the Table or prove to be inferior, the assured for many years enjoys the advantage of a diminished burden by payment only of the premium for a temporary policy ; but, as will have been observed from our examples, the annual premium at a certain subsequent date (apart from fluctuations in the rate of mortality) will exceed the uniform premium and these excesses will progressively increase. An augmenting liability is thus entailed upon the policy-holder (contrasted with the uniform plan) at a period of life when practical retirement from active labour and the natural desire for reduction of expenditure render the advancing responsibility one of inconvenient and serious pressure. It is true that if the average age of the members existing from time to time can be maintained at a diminished level by the continuous advent of fresh entrants of young ages, the rate of mortality at the older ages can to an extent be reduced below the normal increase. But the practical difficulty arises whether, when the heavy contributions in later years are perceived, this alleviating element can be expected to occur ; or whether, rather, this ulterior aspect will not deter future applicants and thus entirely defeat the possibility of the compensation to which we have referred.

The pure level, or uniform, premium, it need hardly be pointed out, consists simply of the present value of these Mortuary Calls distributed into equalized annual instalments throughout the whole duration of life.

The Tontine scheme generally may be shortly explained, and in its simple form consisted of a fund created by the subscriptions of many members, where the interest derived from its investment was divided equally among the successively surviving holders and the last survivor received the entire fund.

Attempts have been made from time to time, but generally unsuccessfully, to purchase extensive properties upon this basis. An estate, we will assume, valued at £20,000 and yielding an income of £1,200 a year, is secured by a syndicate. For simplicity of illustration, let the subscribers number twenty, so that each contributes £1,000. The practical execution of the arrangement consists in each member nominating a life (his own or that of some selected person) and effecting an assurance with a Company for the amount of his capital of £1,000, to be payable on the death of the nominee.

The persons chosen for assurance were either the contributors themselves, or their relatives, or individuals in prominent financial, social, or commercial positions (to whom of course the assurance was unknown) so that proof of death might easily be obtained. (With an extensive number of nominees, and the protective condition that only superior lives would be selected in view of each subscriber's desire to secure the estate ultimately, medical examination could be dispensed with on payment of a moderate extra premium for the possibility of vital defects.) Regard then a single contributor : he assures his nominee to the extent of his risk of £1,000 at a premium of £30 : his portion of the income of £1,200, or £60, is sufficient to return to him 3 per cent. (or £30) upon his investment, and to discharge the annual premium of £30. He thus receives remuneration upon his capital during the nominee's lifetime, and at the latter's death the return of the investment from the Assurance Company, while, should his nominee be the last of the number to die, the investor would realize the entire property of £20,000.

A few observations may appropriately be inserted upon the acceptance of lives for assurance in general. In the primitive history of assurance the process of selection was exceedingly simple : no medical test at entry was imposed, the medical attendant being merely questioned by letter if the applicant possessed one : the proposer appeared before the Board of Directors and was subjected to verbal inquiries of any and every kind. This method was efficient at that early stage since (1) companies were not, as unhappily they are at present, inordinately eager for new business : admission into an assurance institution was regarded as a privilege conceded by the Company rather than a favour urgently desired from the public ; (2) the applicant was introduced

by responsible persons whose character and position constituted a kind of voucher for his respectability and fitness ; (3) the absence of the existing demand for an extensive amount of business afforded a diminished scope and opportunity for hasty acceptances and fraud ; and (4) the system of assurance was then more limited in its range and was confined largely to the superior classes in life and social circumstances, with the attendant advantage of higher chances of longevity.

In the admission of candidates for assurance, the doctrine of heredity has enforced the paramount necessity of an equal attention being devoted to the records of family history as to the personal vigour of the applicant. Modern investigation, however, has modified former medical conclusions, especially upon the subject of phthisis, by its recognition of the beneficial effects of isolating from his early years any person, descended from a consumptive stock, from the society of infected persons. Moreover, the subject may be regarded roughly from the point of view of probabilities. Phthisis being consequent upon the ravages of a micro-organism which demands appropriate nutriment for culture—a suitable *nidus*—we perceive the chances, though at present they do not admit of calculation, (1) that although a person may possess the requisite element, the bacillus may fail to enter, or (2) the bacillus may intrude, but the appropriate *pabulum* may not be inherited.

And changes of practice based upon modern doctrines have also altered materially in other directions the character of medical and surgical deductions of remoter years. One of the most striking incidents in the experience of the author was that of a comparatively young man with extensive malformation of the heart which was diagnosed as congenital. He was accepted with an addition of twenty years ; and the prevision of the examiner (a man of bold but sagacious judgment) was confirmed by the fact that the proposer died of bronchitis after paying premiums upon a non-participating policy exceeding, without interest, the sum assured.

The customary plan of what is generally termed the " rating-up " of inferior lives consists in the imposition of a number of years to the actual age with a view (conformably in our judgment with the true doctrine) to classing him with lives possessing the normal prospect of longevity of the advanced age in question. Some tendencies to disease are practically constant, and a uniform extra for life

fairly affords the necessary compensation. Others increase in intensity with age, where the extra should be theoretically an augmenting one : this course, for obvious reasons, is impracticable if only on the ground that in particular cases a change of physical conditions or surroundings or mode of life might abate the intensity so that the life might thereafter become assurable elsewhere at a reduced rate : the equalized uniform extra should accordingly be based upon a substantial excess beyond the actual risk in earlier years with the object of balancing the deficiency as the tendency is expected to develop. Some tendencies diminish with years, and here the uniform extra should include cognisance of the lighter chances of death at subsequent periods.

An addition to the age, however, in the usual mode strictly implies the assumption that the extra risk of mortality increases annually with the duration of life—that is, that the aggregate enhanced risk is distributed in an *increasing* progression. This assumption applies to many forms of deterioration, and hence justifies the addition in this shape. In other instances, however, as we have pointed out, the additional risk may be distributed *uniformly* over the whole period of existence, or may assume the form of a *decreasing* progression, where the customary mode of surcharge would not apply.

Particular care should be exercised in the imposition of an addition in connexion with term policies, since a comparatively substantial addition to the age may produce an insignificant pecuniary extra. Thus, at age 35, where the pure premium (H^m 3) for a temporary assurance continuing for five years is .914 per cent. per annum, an addition of five years to the age would yield a premium for the same term of 1·05 per cent., or an extra of 2*s*. 9*d*. per cent. only, which probably would prove to be entirely useless. Hence in these instances, the addition to the age, based upon its financial equivalent, should be substantial. It is further to be noted that if the effects of deterioration be truly expressed by the addition of a certain number of years to the age (which presumes the distribution of the aggregate extra risk as a progression advancing with the age), the sub-normal life, ineligible for the entire period of life without a considerable extra premium, may, without any serious increase of risk, be accepted under an endowment assurance policy for a suitable term at the ordinary rate. If the

pure single premium for a whole-life Policy of £100 payable at the death of a person aged 30 be (say) £31·34, and £38·18 be the corresponding premium at the age of 40, the addition is nearly 22 per cent. for provision of the augmented risk as the equivalent of an addition of ten years to the age. But if an endowment assurance be effected, payable at age 50, or earlier death, the single premium at age 30 is (say) £50·31, while at age 40 (payable at 60) the single premium amounts (say) to £51·61. The addition consequently of about 2½ per cent. is sufficient to cover the enhanced risk corresponding to the same addition of ten years to the age. The explanation of these comparative results is furnished by a consideration of the two distinct parts of which an Endowment Assurance is composed, with its combination of the two separate premiums for the different risks of a temporary assurance and a pure endowment. By the assumption previously suggested respecting the distribution of the additional liability, the portion of the premium required for the temporary assurance, though increased to some extent, is not in any way increased so largely as would occur under a policy for the duration of life. The remaining portion of the combined premium, however—that which is necessary for the chance of survivance—is *diminished* by the presumed increase of mortality, and although these two opposite causes do not exactly balance, they produce the effect in many instances of so reducing the balance that the resulting amount may be neglected without appreciable hazard.

A Company is occasionally required to remove or reduce an extra charge on renewed medical examination after a certain period of time has elapsed.

Theoretically and strictly, this course is inexpedient and unjust, since extra premiums, like ordinary premiums, form average amounts (assuming that the extra has been assessed with judgment, though here it may well occur that medical sagacity and foresight were at fault in certain obscure instances), where those which are proved by the subsequent history of "rated-up" lives to have exceeded the value of the special risk involved must afford compensation for the cases where the rate of experienced deterioration has surpassed the expectation of the future which the additional premium was designed to measure. This course, however, might fairly be applied to those instances where the tendency to disease

diminishes with advance in age, though here again, contemplating the entire mass of "rated-up" lives, valid objections might be urged. Competition, however, compels the concession *occasionally* of reductions in special instances; and moreover the sound and expedient consideration enters that if in such a case the abolition or abatement were refused, and the life were accepted elsewhere upon reduced terms, proving his comparatively enhanced vigour, a greater injury might be inflicted upon the general body of the assured (in respect of the rate of mortality) by this permission of the abstraction of a desirable member than harm would be imposed by the cancelment of the extra charge.

Rules for the conduct of business affairs, depending as they do upon an acquired experience which cannot comprehend every conceivable mode and combination of contingency which may hereafter occur, do not consist of mechanical commands, but simply embody memoranda for future guidance which judgment and a wise expediency may modify in adaptation to novel or diversely conditioned problems.

Ingenious methods have been devised for the purpose of removing objections urged by an applicant to the exaction of an extra rate in the customary form of an enhanced premium. The extra, it will be observed, is still imposed, but in the more obnoxious shape of a diminution of the assurance benefit. Assume that a person aged 30 presents the diminished prospects of life appertaining to the age of 40: the premium for age 40 is accordingly fixed for the assurance, and the difference between its annual amount and that for age 30 is multiplied by the number of years contained in the expectation of life furnished by the Mortality Table at the actual age. If death occur in the first year, the sum assured is diminished by the total product just mentioned; if the assured die during the second year, the diminution consists of the same product less one year's difference of premiums; so that the reduction annually and regularly decreases until, if the policy-holder survive the expectation of life, the *full* original amount assured becomes payable at his subsequent death. This scheme, it will be observed, essentially consists in the grant of an *increasing* assurance at the rate of premium required for a *uniform* assurance equal to the ultimate maximum amount.

The plan is ingenious but unscientific, and it possesses the practical disadvantages (1) that if death occur during the early years of commercial activity, when an adequate provision for the family is especially imperative, the benefit received is comparatively insignificant in amount; (2) policies thus burdened cannot be readily or helpfully utilized as securities in business transactions except to a very limited extent; (3) it has been mathematically proved that the empirical deduction from the sum assured is generally considerably less than the theoretical amount measured in proportion to the actual risk incurred: for example, where the "rating-up" of five years adequately provides for some physical or other defect on a scientific assessment, the method in question assigns an addition of two years only; hence (4) a detrimental effect is entailed upon the ordinary policy-holders in consequence of their being weighted with a heavier proportionate contribution to the settlement of the claims of those whose payments to the fund are on this scale inadequately adjusted to the genuine additional risk.

The mathematical treatment of the problem is simple: for a Policy in this form really comprises two assurances: (1) an immediate life assurance of a fixed amount, and (2) a uniformly increasing Assurance up to a determinate sum on the termination of the expectation of life.

If x be the "rated-up" age, and e the expectation of life at the real age, taken to the nearer integer, then let I denote the annual increment, the ultimate amount assured being unity. Then the pure premium for the immediate assurance is $\pi_x\ (1-eI)$, and that for the augmenting assurance, $I\frac{(R_{x+1}-R_{x+e+1})}{N_{x-1}}$. If r be the addition made to the age, the pure premium payable is π_{x-r}, and consequently $\pi_{x-r} = \pi_x\ (1-eI) + I\frac{(R_{x+1}-R_{x+e+1})}{N_{x-1}}$, whence I can be obtained.

It may be expressed as a general maxim in practice that where real danger of brevity of life exists a small extra involves the disadvantages of usually proving insufficient and of producing irritation in the applicant, who, judging from its trifling nature, is liable to imagine that the addition is an arbitrary and unjustified exaction. Let a substantial extra be required or the ordinary rate be charged.

The question of the segregation of the profits contributed by persons pledged to total abstinence from intoxicating drinks, and their exclusive distribution among the members of that class, suggest many considerations based upon the practical regulations adopted by Companies which pursue this course. These rules, as affecting particular Offices, cannot judiciously be discussed in this book, which aims at impartiality and descriptiveness of general treatment. It may, however, first be mentioned that the Company which originally introduced this segregation, and which transacts the most extensive Assurance business among abstainers from alcohol, has recently published the results of an investigation (extending from 1841 to 1901) into the mortality experienced among its abstaining and non-abstaining members respectively. The numbers under observation in connexion with policies upon healthy male lives alone were—

Class	Number of Policies	Years of life under observation	Deaths
Abstainers	29,094	398,010	5,124
Non-abstainers	31,776	466,943	8,947

The mortality of the non-abstainers agreed with remarkable closeness with the O^m table, and thus proved that this class consisted of average lives. Comparing the mortality of the two sections, the results entitled the Company to submit (1) that, tested from every aspect, the rate of mortality prevailing among the abstainers presented a markedly favourable superiority to that of the non-abstainers throughout the entire working years of life in respect of every description of policy and for both sexes, and (2) that this superiority was not produced by the operation of the "Transfers" between the two branches (that is to say, the transfer to the abstaining section of members in the non-abstaining class who subsequently became teetotalers, and the contrary transfer from the former branch to the latter of members who at any time ceased to be total abstainers). Regarding the entire period of 61 years, the deaths among males in the abstaining class (at all ages) were 129 and those in the non-abstaining class 192, out of every 10,000 exposed to the risk of death. On the other hand, a very instructive and interesting statement has recently been published of the experience of the General and Temperance sections of a Government Department

of Insurance. In five valuations, the profits able to be allotted to the two sections were equal on two occasions; on one occasion the bonus to the Temperance class was the higher; while at two distributions the General section secured the increased rate of profit. And it was justly observed, in connexion with these curious variations, that the comparative proportion in which endowment assurances entered differently into the two sections affected the results; while the rate of progress in numbers was also found to constitute an additional element of divergence, since the smaller section was necessarily subject to increased fluctuations of mortality.

The general conclusion appears to be that the enlarged duration of life on the whole may not be the product solely of abstinence from intoxicating drinks, but the combined result of those qualities of self-conservation exhibited in the habitual mode of life and the general self-control of such persons, of which general habit teetotalism is simply an indication in this one particular direction.

An Assurance Company, it may be asserted in general terms, should exist as a homogeneous body—a social and commercial commonwealth—with the avoidance, so far as is practicable, of the adoption of different sections or series of membership, in relation to every element involved in assurance administration and aims. If distinctions are introduced into an Assurance community—if, for example, one section obtains exclusively the profits derived from its separate contributions without massing them with the profits produced by the remaining membership, is that section justly entitled to regard the *common* assurance Fund, created by the entire body of the constituents of the Company, as a protection for its claims? If that section sever its connexion with the general community upon the one point, is it equitably at liberty to demand dependence upon the community on another point? If it detach itself from the general interests for a specific and exclusive benefit in respect of *profits*, can it legitimately claim to retain the protection of the common Assurance Fund, established by the contributions of *all* the members, in respect of *security*?

One or two detached subjects of interest may be named.

Up to the age of about 50, females exhibit a higher rate of mortality than that prevailing among males, as a consequence probably of the risks of child-bearing and its frequent sequelæ; after that

age the excess of their expectation of life so far surpasses the corresponding vitality of males that this ultimate surplus of physical power is sufficient to counterbalance the deficiency of vital persistence during preceding ages, and thus to apportion to females a more extensive expectation throughout the entire duration of existence. An Assurance Company, many years ago, regarding only the complete comparison between the two sexes, and omitting to observe the altered condition which was exhibited at earlier periods of age, accepted females at reduced premiums. Assurances, however, are usually effected prior to the age of 50, and an unhappy experience speedily compelled the abandonment of the plan.

The mode of assessing a *uniform* extra premium for a section or class of lives exposed to additional hazard may be mentioned.

Regard as an example the diminished probability of life possessed by females under the age of 50. It was ascertained, in one investigation, that 85 out of 10,000 males at ages ranging between 20 and 44 died during the course of a year, while out of the same number of females exposed to the chance of death during corresponding ages, 111 died. What *general* annual extra premium should be charged to females for the purpose of equalizing their risks to those entailed by the assurance of males? The Company, on the basis of the preceding result of observation, must anticipate (111—85 or) 26 additional deaths. Assume that the amount assured in each instance is £100; provision consequently must be created, in respect of females, for an excess of £2,600 in claims: the annual addition thus demanded, if distributed among the 10,000 females, amounts to $\frac{£2,600}{10,000}$ or ·26 or 5s. per cent. per annum.

The mortality in childbirth of the general population of this country has been ascertained to be about 1 death in 200 confinements. In respect therefore of this peculiar hazard, for every 200 females assured the Company will be required to provide for an additional death: and if each assurance amounts to £100, the extra payment in a year will be £100; hence the special charge for each confinement should be $\frac{£100}{200}$ = ·5 or 10s. per cent.: or assuming that a confinement occurs at intervals of two years, the annual extra rate would be one-half of the preceding amount, or

5*s*. per cent. per annum—thus coinciding with the previous deduction.

An investigation by an Assurance Company disclosed the result that out of 3,529 persons engaged in the occupation of publicans and exposed to the risk of death at all ages, 102 had died. The number of deaths normally expected by the English Life Table No. 2 (for males) was 68·524; hence the extra annual premium necessary for the hazards of the trade was ascertained to be $\frac{102-68\cdot524}{3,529} \times £100 = \cdot948$, or 19*s*. per cent. per annum. Thus, according to the mortality experience of male lives generally throughout the country, the amount of claims in the course of a year, assuming each to be assured for £100, would be £6,852: the actual claims which occurred among these traders amounted to £10,200, and consequently an additional £3,348 had to be provided: distributing this extra compensation among the aggregate lives at risk, $\left(\frac{£3,348}{3,529}\right)$, the appropriate increased premium is obtained.

CHAPTER III

The Meaning and Effect of Selection

The term " selection " is employed in its customary sense of choice ; out of a mass of individuals some may be voluntarily and intentionally detached for the purpose of forming a specially constituted separate body ; or from any aggregate abstractions may be made, equally voluntarily, but with no intention of creating a distinct mass or of deliberately affecting the constitution of the original combination. This two-fold process exemplifies the concurrent action which is continually proceedin gin life assurance business. There occurs (1) the selective and purposive act of the Company in choosing from the applicants for admission those only who, from the results of medical examination and the test of inquiries respecting habits of life and occupation, appear to possess the prospect of longevity normally appertaining to their respective ages, with the object of forming a superior mass in this respect as the area of administrative operations. The purport of this process is the production of equal chances of risk and benefit of membership for all persons assuring at the same ages.

The practical result of the act of rejecting lives exhibiting a reduced probability of life is that the persons selected present for at least a definite term of years a superior vitality, or diminished rate of mortality, to that of the general population, or to that prevailing in a body which comprises healthy and unsound lives of corresponding ages. In the case of a Company newly established this operation permits the accumulation of an adequate mass of membership (as a numerical basis for stable and steady results) before the advent and strain of a heavier reduction by mortality ensue ; and in respect of a company already founded the process keeps down the *average* age of the community as a whole by the commixture of the advancing ages of existent members with the younger ages of persons successively admitted, and consequently tends to maintain the aggregate rate of mortality at a diminished (or practically constant) level through the influence of the lighter mortality of freshly selected entrants upon the increasing mortality of existing lives as their ages proceed. The admission of vigorous lives at the younger ages thus aids in lessening the average age of the

entire membership for valuation purposes, with a consequent proportionate reduction in the amount of the reserve required for the complete contracts, and concurrently conduces to the perpetuation of the company as a permanent corporation.

A special case may be noticed. If a person be not finally inadmissible, but may be reasonably accepted with an addition to his actual age (on account of personal or inherited constitutional defects, real or probable), he will be charged such higher premium as will approximately place him in the class of persons possessing the normal health and vigour belonging to that increased age. This mode of viewing the assessment in these instances appears to be the correct one. The plan is frequently adopted of assuming (taking, for example, the actual age to be 30) that the normal expectation of life is likely to be reduced by, say, seven years; and assessing the increased premium to be charged on the basis of an age whose expectation is the expectation for age 30 minus seven. (A deduction from the expectation, it may be added, does not produce the same result as the addition, equal to that deduction, to the age. Thus, the expectation, by the O^m Table, at age 30 is 35·067 years; if seven years be subtracted we obtain an expectation of 28·067 years, which, in that table, is appropriate to an age between 39 and 40—different accordingly from the age of 37 formed by the imposition of seven years, where the expectation is 29·626). This method, however, is less susceptible of approximately accurate estimate—and all these assessments are necessarily approximate only—than the process of determining that an applicant of 30, showing a sub-normal aspect from all points of view, presents the prospect of life of a normally constituted person of a certain advanced age (37, for example), and fixing the premium on the basis of that addition.

The use of the function termed "the expectation" of life appears to be too loose and indeterminate to constitute a measure of deterioration, and the judgment of the medical man would, it is conceived, act more surely and precisely in comparing the deficient vigour presented by the candidate of 30 with, say, the normal resisting energy of age 37 (diminished, as the latter has become, by augmentation of age). The considerations, moreover, adduced in Chapter I further justify the rejection of the expectation of life as an instrument of measurement.

We have thus briefly described the first process of selection.

When the assurance has been effected, (2) the second and antagonistic force of selection enters into action in the form of lapsing and surrendering policies.

In Chapter IV., on Surrender Values, the mode will be explained in which the operation of this force detrimentally affects the vital quality of the lives remaining in the Company by the abstraction, on the whole, of sound lives, and the adverse financial effects which these withdrawals entail. All that need now be preliminarily stated refers to the distinction between the two forms of selective action. The Company's act is deliberate, for the express purpose of benefiting the whole of its members and according to them an even level (relative to age) of risk and advantage; the procedure of the policy-holders is also deliberate, but, while injuring on the whole the interests of the assured community from which they separate, is not intended to produce this damaging result.

Dealing then with the first process of selection, it has been definitely ascertained, from an exhaustive analysis of manifold observations upon assured lives, that a body of persons who have been recently selected for assurance exhibit a lighter rate of mortality than that which prevails among the assured who have attained the same ages as those of the newly admitted entrants, but whose policies have endured for a longer period—that persons, in other words, who were accepted three years ago, at the age of 40 and are now 43; are subject to a reduced death-rate to that which appears among persons assured five years ago and possessing the same present age of 43, but heavier than that exhibited by persons now first assuring at the age of 43. Thus, adopting the select tables formed from the H^m observations—

I. The probability of dying in a year at age 43 in respect of the assured of three years' standing,—the age at entry having been 40, } is ·0113, or 113 per 10,000;

II. The similar probability where the policy has endured for five years from the age at entry of 38, } is ·0118, or 118 per 10,000;

III. The corresponding probability in respect of lives just accepted at 43, } is ·006, or 60 per 10,000.

The probability of death of ·0118 (abbreviated as to decimals)

is that furnished by the $H^{m(5)}$ Table, where the experience constituting its foundation excludes the more favourable results of mortality prevailing during the first five years of assurance; that is to say, only those lives were admitted into this table whose assurances had continued for five years and upwards, and where, adopting the current phrase, the force of selection had disappeared.

The results expressed in I. and III. are the natural consequences of the efficient process, medical and otherwise, which was applied by the Company when the several groups were respectively assured, and which, though uniformly adopted for each group at the epoch of acceptance, has, by mere effluxion of time (intensified by the adverse effects of withdrawals explained in Chapter IV), lost a portion of its influence as shown in group II. compared with group I.

In the earlier-selected mass of the assured, chronic and acute diseases have had a more enlarged opportunity for occurrence and fatality than in the later admissions; and the more protracted the antecedent period to the present age under contemplation, the wider has been the occasion afforded for the advent of such maladies and the fatal effects they present. This mass, too, will include diseases which, originating in the early years of its selection as acute, have now been able to assume a chronic form. In the more recent admissions, acute and rapid ailments will predominate over other causes of mortality, while in those lives just selected the factors of destruction will only consist of fatal accidents and sudden accessions of disease which pursue a speedy course. Hence each successive mass of accepted lives naturally shows as time proceeds the slighter effects of sudden illness, and also contains the chronic forms of malady into which acute diseases originating during the prior years of selection have thus been afforded a sufficient time for development.

We are thus certainly directed to the conclusion that the rate of mortality among assured lives is a function of two variable quantities—the age at entry and the duration of membership.

The action of the Company in exercising a choice in the admission of normal lives is technically termed "selection"; the lives experiencing a diminished rate of mortality in consequence of this selective process are designated "select lives"; and the advantage conferred upon the assurance community of any Office by the temporarily reduced death-rate and the more extended corporate existence

due to the infusion of sound lives, is described as the "benefit of selection."

No definite term can be precisely assigned to the extent of continuance of this select character; but it is usually assumed in practical investigations and calculations (based upon analyzed and uniform experience) that, for all effective purposes, the benefit practically disappears on the termination of five years from entry. Hence the $H^{m(5)}$ and the $O^{m(5)}$ Tables were constructed on this assumption and excluded from the experience they express the favourable mortality prevailing during the preceding five years, or the efficient period of the maintenance, in any appreciable form, of the benefit derived from selection.

These two tables may thus be regarded as representative of the rate of mortality expected to prevail among lives assured during the remainder of life commencing with the expiry of the term of selection; and the functions deduced from them may be appropriately employed in connexion with policies, which have subsisted for five years and upwards, for the valuation of the sums assured and premiums—the adopted premiums, of course, being those derived not from the $H^{m(5)}$ or the $O^{m(5)}$ Table but from the H^m or O^m observations for the age at entry: the premiums to be valued must necessarily, as *facts*, include the entire experience from admission, while the valuation factors rightly take cognizance of the *fact* of the extinction of selection.

The following table is instructive in its clear exhibition of a comparison between the rates of mortality for different periods of assurance, based upon the experience of healthy lives in the H^m observations:—

Ages in quinquennial groups	Annual mortality per cent. in periods of assurance.				
	Under 5 years	5 years & upwards	Under 10 years	10 years and upwards	Total period of life
(1)	(2)	(3)	(4)	(5)	(6)
25 — 29	·66	1·0	·73	·92	·73
35 — 39	·83	1·1	·93	1·17	·97
45 — 49	1·17	1·44	1·25	1·52	1·36
55 — 59	1·81	2·47	2·1	2·52	2·35
65 — 69	3·63	5·06	4·35	5·11	4·9

It will be observed that the figures in column (3) diverge sensibly from those in column (2), while they tend to close approximation with those in column (5). This comparison indicates with reasonable demonstration that the rate of mortality among lives whose assurances have lasted for less than five years is substantially inferior to that exhibited among lives of similar ages assured for more protracted periods, so that the causes which produce this disparity have practically ceased to operate when the first five years of membership have expired. The $H^{m(5)}$ and the $O^{m(5)}$ Tables, consequently, as we have stated, were based upon these indications as presenting the advent and continuance of the period when the benefit of selection had, for all practical influence, vanished.

(The annual rate of mortality at any age x is the fraction formed by dividing the number of deaths occurring in the year following that age by the number of lives exposed to the risk of death at the commencement of that year—that is, at the age of x; so that if 1,000 assured persons are under observation for a year at age 35, and nine deaths occur between the ages of 35 and 36, the annual rate of mortality is $\frac{9}{1000}$, or ·009 per unit, or q_{35}, or ·9 per cent. or per hundred persons assured at the beginning of the year.)

Tables representing the experience of mortality of assured lives may consequently assume two forms: the one, which may be termed a General or Aggregate, or Mixed Table, and the other which may be designated Select. The tables usually published exhibit the former type of formation, and may be thus described: Assume that our consideration of the table starts at the age of 40: the number stated as living at that age comprises (1) the selected entrants of normal health for that age, newly accepted and subject to a reduced mortality for a term; (2) those who entered under observation at the age of 39 and are still in the select period, though of a less favourable character as regards intensity than that which prevailed at their admission one year ago; (3) the entrants now aged 40 who were accepted at the ages 38, 37 and 36, where the advantageous influence of selection has increasingly disappeared; (4) the assured at age 35, who have now attained the age of 40, where the term of lightened mortality existing at and after entry has practically vanished; and (5) those admitted at ages prior to 35 and now aged 40, in respect of whom the usual epoch of

termination of the definite beneficial effect of selection has become more and more distant. Moreover (6) at every age subsequent to 40, say at 50, for example, the lives newly entering at the latter age are mingled with those who have survived from age 40, and this incorporation of fresh and vigorous elements occurs at each succeeding age until the close of the observations.

The general rate of mortality prevailing in the entire community of the assured of all ages is thus of a compounded character, more largely influenced advantageously by the infusion of fresh lives at the early stages where assurances are chiefly effected; but at subsequent stages, and the more considerably so at advanced ages, the favourable result produced by the introduction of new entrants at those ages is increasingly diminished, since the new assurances, completed at those ulterior dates, being comparatively few in number, bear a decreasing proportion to the existing mass of deteriorated lives, or lives attaining those ages from whom the force of selection has been extinguished. Still, as will be observed hereafter, the inclusion in the table of additional entrants after any given age naturally affects the chances of life and the rate of premium at that age.

A table, termed a Select Table, may also be constructed which avoids this intermixture of rates of mortality associated with different ages at entry and durations of assurance, and which from the outset segregates, so to speak, the specific mortality of the entrants at each separate age. All persons admitted at the exact age of 40, for example (and similarly for each other age), are collected together, and their experience is maintained distinct from that of persons accepted at earlier periods and now aged 40, and from that of persons selected at subsequent ages which the original entrants at 40 successively attain, until the extinction of the entire mass by death (if it were practicable to pursue the inquiry to this extent) or (as is necessarily the practice) until the termination of the observations. A separate table of experience is thus constructed for each age at entry. In this mode of tabular formation we thus possess a distinct continuous record of the rate of mortality prevailing after every age taken by itself, and are enabled, without the intrusion of the disturbing influence of lives surviving from prior ages (and introduced at subsequent ages as just explained) to ascertain the specific death-rate which appertains

to the after-history of members of each age of admission. This mode of construction, it is obvious, would be the genuinely scientific one to adopt where adequate numbers can be secured. In such a table it may be useful to repeat, for convenience and completeness of consideration, that, starting with healthy lives at acceptance and tracing the successive future stages of experience, the causes of death which operate during the first year of assurance (producing a slight result) consist of accidents and acute maladies of rapid course : a somewhat increased number of deaths will occur during the second year, where again acute diseases contracted in the former year have proved fatal during the currency of the second year, and ailments and accidents have supervened, of a corresponding character to those of the first year, since its termination ; and similarly for each subsequent year of the period of selection. It is customarily accepted, as has been stated, founded upon the records of Life Offices, that after a definite term of practically five years, the rate of mortality will thenceforth approximate to that expressed by the $H^{m\,(5)}$ and $O^{m\,(5)}$ Tables where the observations pertain to lives whose membership at the date of their observation had continued for the specified period. Select Tables, accordingly, as usually framed, include detailed cognisance, in respect of each age at entry, of the diminished mortality during each of the years over which the Benefit of Selection appreciably operates, and then (adverting to the H^m Table as a specimen of basis) merge the final ratio of that term of five years into the $H^{m(5)}$ experience where the selective advantage has become virtually extinct.

Special tables of this segregated mode of construction possess many advantages and distinctive uses. They form obviously the appropriate measure of the mortality on which the formation of premiums for assurance at any given age should be based. Besides their practical and relevant aspect, and as the consequence of the process, they are also, from the general scientific point of view, accurately constituted, since they follow the course of statistical congruity of elements by excluding from the data employed for calculations in connexion with lives newly assured the incongruous admission of experience consisting of the observations upon lives accepted at earlier ages, and the observations upon those admitted at later ages which the members entering at any age progressively attain. They thus possess a homogeneous character

compared with the heterogeneous constitution of Mixed Tables. In the Mixed or Aggregate Table, for example, the premiums of young lives are too low, and those for older lives excessive, as compared with the precise measures of risk which premiums deduced from a Select Table involve ; and this consequence follows at once from an intelligent consideration of the elements comprised in the formation of Composite Tables and the mode of the construction of premiums.

The single premium from which the annual premium is derived consists of a series of terms involving the rate of interest and the chances of dying in each successive year. Thus, the single premium at age x, symbolically A_x, is the sum of the series, continued to the end of life, of $\frac{vd_x + v^2d_{x+1} + v^3d_{x+3} + \ldots}{l_x}$, where v, v^2, . . . are the discounted values at interest for one year, two years, and so on ; and d_x, d_{x+1} . . . represent the number of deaths occurring in each year, while in order to obtain the value appertaining to each person living at age x, the sum is divided by the number existing at that age, or l_x. Expressed in an enlarged form, we obtain $\frac{v(l_x - l_{x+1}) + v^2(l_{x+1} - l_{x+2}) + \ldots}{l_x}$

Now in a Mixed Table the l_{x+1}, l_{x+2}, . . . , the numbers living at ages succeeding x, where we assume that the life was first selected, comprise not merely the deteriorated lives existing at the several prior ages and from whom at age x the force of selection has wholly or partially disappeared, but in addition all *subsequent* new entrants into the Company at every higher age who are then subject to select mortality : in the young lives, consequently, this beneficial effect upon the sum of their total chances of life is considerable by reason of the extended space of time, looking to the expectation of life at any young age x, during which these subsequent select admissions will occur (with their temporarily reduced mortality affecting the l_{x+1}, l_{x+2} . . . of the numerator), whose tendency is to counterbalance the heavier mortality experienced by the entrants at x after their period of selection has terminated, and tending moreover to compensate the enhanced death-rate prevailing among the lives existing at age x but assured at earlier dates. In other words, at a young

select age there is a comparatively brief period behind it and a comparatively extended period in front of it, and hence the adverse influence upon the select prospects of lives aged x, due to the heavier death-rate of lives assured at x—5, x—6, and so on is more than counteracted by the select or diminished mortality experienced upon lives admitted at age $x + 1$ and at all ages after x during the lengthened future duration of existence at x.

In respect, on the other hand, of those who assure at older ages, for example at 60, there remains a distinctly shorter period before them in which to secure the beneficial leavening of the rate of mortality through the admission of select lives at later ages (for assurances effected at advanced ages are much less numerous, and consequently the extent of selective benefit upon the premium will be meagre); while, regarding the past term as measured by the present age, the anterior data employed in the calculation of the premium for age 60 include a greatly extended mass of lives accepted at all previous ages from whom the advantage of selection has long been finally exhausted. A definite tendency accordingly exists to depress, and neutralize, the favourable mortality due to selection at 60 and attributable also to the (diminishing) future number of select entrants. Thus, in other terms, the young entrant is benefited in the amount of the premium charged by the constant infusion of superior lives at all ages subsequent to the date of admission; while the assured accepted at advanced ages are burdened by the constantly augmenting mortality of those who have survived to the higher ages in question from previous years: the chances of future duration of life are, in the former instance, improved and consequently, from the more prolonged general period of payment, the amount of each annual premium is reduced (and similarly in the case of single premiums from the more extended term for their accumulation at interest); while the chances of life of older entrants are diminished, and consequently the extent of their annual premiums increased, by the anomalous introduction of the effect of the reduced chances of life of deteriorated survivors from a lengthened anterior period of time. In summary, a young person's prospect of life is exaggerated by the adventitious circumstance that an infusion of "new blood" occurs at every stage of the future through which he is supposed to pass, while, on the contrary, a person assuring at a more mature stage of life suffers

an underestimation of expectation by being classed with a mixed body of lives who have survived from earlier ages.

After proceeding thus far, it appeared to the Author that the student would perceive the relative effect more clearly if the first expression for the single premium were employed, omitting the v for convenience, of $\frac{d_x}{l_x} + \frac{d_{x+1}}{l_x} + \ldots$

In the Mixed Table, where subsequent selection operates, it is obvious that the *d's* will be reduced in quantity by the diminished mortality due to the accession of new members. This advantage will evidently prove the more appreciable for young ages at entry, regarding the longer space over which the fresh acceptances will extend, since assurances are the more numerously effected at early and middle ages. At the advanced ages of entry, the effect will clearly be gravely reduced in intensity on account of the paucity of influence thenceforward exercised by the more scanty introduction of new assurances at these later stages of life.

The following table shows for age 35 the differences of various functions derived from the H^m Mixed Table and the Table of Select Observations deduced—

Table	Probability of living a year	Probability of dying in a year	Value of a life annuity at 3 per cent	Annual pure premium at 3 per cent.	Reserve per unit assured at the end of a year at 3 per cent.
Mixed	·9912	·0088	18·587	·0219	·01390
Select	·9951	·0049	18·491	·0222	·01804

The divergencies of results will be subsequently explained.

Besides constituting the accurate and appropriate basis for the calculation of premiums for assurance, Select Tables form an instrument which is capable of application to many novel and very important purposes ; and one of the signal services which they can render is the computation of surrender values, since account is thus competent of being introduced, with approximate accuracy, of the recognized fact that assured lives who resign their membership are in the mass of superior quality on the vital scale, and thus, as a consequence of the cancelment of their contracts, subject the remaining body to an augmented rate of mortality due to the successive depletion of the vigorous elements which it previously contained.

A description of the symbols employed for the functions deduced from Select Tables, in analogy with those expressive of the computed results of a mixed experience, may be briefly furnished.

The inclusion of selection is indicated by the representation of the age of a select life within square brackets: thus $l_{[x]}$ symbolizes a number of lives immediately chosen at the age of x in analogy with the l_x of a mixed table: while the duration of the assurance, that is to say, the period of time which has elapsed since the original selection occurred, is marked by the addition of the term in question to the suffix, but without enclosure in brackets: thus, $l_{[x-1]+1}$ represents the number living at the present age x...$(x-1+1=x)$... who were selected at age $x-1$, one year ago. If 40 be the present age, that is x, we obtain $l_{[40-1]+1}$: the origin of selection being at age 39...(40–1)..., and the admission of the life occurred one year ago as indicated by the positive addend.

$a_{[x]}$ furnishes the value of an annuity during the lifetime of a person now aged x, and just selected, as compared with the a_x of an ordinary table: $a_{[x-2]+2}$ represents the value of a similar annuity at age x...$(x-2+2=x)$... who was selected two years ago (indicated by the 2 outside the bracket) at the age of $x-2$ contained within the bracket.

In the same manner $\pi_{[x-2]+2}$ expresses the annual pure premium for the assurance of a life now aged x whose entry into the Company occurred two years ago at the age of $x-2$, and where consequently the primary force of selection has been deprived of its original intensity to an extent measured by the lapse of that time. Where selection has just been exercised, the appropriate premium is indicated by $\pi_{[x]}$.

The differences may now be explained which appear in the table of the values of functions previously presented.

The probability of death occurring in a year is inferior in Select Tables by reason of the fact that the total chance at age 35 in the Mixed Table is increased by the inclusion of lives with diminished prospects of longevity, who entered at prior ages, and from whom to a large extent, on the whole, the force of selection has disappeared: and it is evident from the nature of the case that this probability, limited as it is to the consideration of one year, cannot be affected by the introduction of newly selected lives at later

stages. In a corresponding manner is explicable the comparative probability of surviving a year.

As the annuity value in the Mixed Table of 18·587 involves the inclusion, in the basis of calculation, of subsequent select lives as a portion of the data upon which the value is constructed, (which the value for the segregated select lives at 35 does not contain), a larger number of payments of the annuity (in consequence of the prolongation of the expectation of life thus created) may therefore be anticipated on the whole, so that their present value is necessarily increased.

The effect produced where select experience is employed in deducing the annuity-value is different. As the future duration of life at any age is not augmented by the subsequent influx of select lives as a beneficial factor, so that the ensuing lifetime becomes shortened, without alleviation, by mere increment of age, the term of payment of the annuity in the mass will be reduced, and hence, fewer instalments being required to be provided, the value of the annuity series is diminished.

The equalized amount of the annual premium in a Mixed Table is depressed by the effect of the consecutive accretions of fresh lives of select character whose experience is included in the materials on which the premium is founded, thus prolonging the entire range of the future duration of life at any age, which does not occur in a body of persons where the observations are retained distinct throughout the complete course of existence. Hence the aggregate π is less than the select π; the premium being an annuity, more payments will be received under the Mixed Table than will occur under the Select Table, and accordingly the amount of each will be diminished.

This comparative result is of course symbolized in the formula for an annual premium : $\frac{1}{d+\pi} - 1 = a$: consequently $\frac{1}{1+a} - d = \pi$, where the d is constant in both instances, and the denominator of the annuity-value being greater in the Mixed Table than in the Select Table, the derived π in the former case is reduced.

We have dealt here with comparatively young lives possessing a lengthened total duration of lifetime before them over which, in a mixed experience, an alleviation of the rate of mortality is substantially effected by the repeated incorporation of sound lives. But now assume the age of 60, where during an extensive anterior

period lives admitted during that term (in a Mixed Table) have passed through a protracted range of deterioration, and where the general future lifetime is greatly abbreviated for the infusion of the favourable effects produced by the advent of select lives. At age 60, the annuity-value and pure premium are respectively (H^m3) 10·236 and ·0599 in the mixed experience, while in the select observations the corresponding amounts are 10·443 and ·0583. Here the conditions are reversed, as we might expect, compared with the table previously furnished.

In a Mixed Table at this advanced age the advantage from future select lives (owing to the paucity of future admissions at the higher ages) is overbalanced by the large accumulation of inferior lives from previous years; and the number of future payments of the annuity being thus diminished—the period during which they will as a whole be receivable being shortened—their present value is necessarily reduced as compared with the requirements of a Select Table, where this cumulated pressure of anterior deterioration is not included. On the other hand, in a Mixed Table, the subsequent term of payment of the premiums being abbreviated—the advent of death on the whole being nearer, and fewer receipts of premiums therefore to be anticipated—the amount of each payment must of necessity be increased as compared with the demands implied in a Select Table. It will be observed that at the enhanced ages, where new assurances are sparsely effected, the rate of mortality in both tables will thenceforth more closely approximate to each other.

The reserve or policy-value, as is elsewhere pointed out, furnishes the basis upon which surrender-values must be assessed.

Assuming the age at entry to be 35, and the assurance to have endured for three years, the following comparative statement is appended—

1. In a Mixed Table (H^m3) we obtain $A_{38} - \pi_{35}\,(1+a_{38})$—that is, ·45372—·02193 (18·756) or ·0424 as the reserve per unit assured.

2. In the Select Table the *average* value is

$$A_{[38-3]+3} - \pi_{[35]}\,(1+a_{[38-3]+3})$$

—that is, ·46033—·02218 (18·529) or ·0494.

3. While if the life be considered to be still healthy—that is, possessing the normal vitality of a person selected at 38 (which on the whole may be regarded as representing the prospects of longevity

of withdrawing lives) the policy-value will be expressed in a Select Table by $A_{[38]} - \pi_{[35]}(1 + a_{[38]})$—that is, ·45531—·02218 (18·701), or ·0405, which is lower than the reserve furnished by the Mixed Table.

The reserve is augmented, it may be popularly explained, in case No. 2 compared with No. 1 for the reasons already assigned: the general period for the incidence of future claims is briefer in No. 2 on account of the failure of prolongation of the average date of death which would be produced if an influx of the favourable influence of fresh select lives occurred, and hence the sum to be retained in hand for their discharge must be increased: or, to express the position in another mode, fewer deaths have supervened during the past period of three years in a Select Table, and accordingly the survivors (out of an equivalent number in both tables) being proportionately more numerous than in a Mixed Table, an enhanced amount must be maintained as a reserve (that is to say, a larger sum has been saved out of the premiums received) to meet these additional future claims.

The student should examine, on the same plan of reasoning, the comparative results where the assurances are effected at more advanced ages, and also where the more extended durations of the policies, in relation to the benefit of selection, affect the required reserves.

It has been shown by an analysis of the experience of assured lives (the H^m Table) that, where allowance has been introduced into the calculations for the selection exercised by the Company, the annual pure premiums for age 50 and upwards are reduced, and those for age 45 and downwards increased. At the youngest ages, 20 and 25, the addition is about 5 per cent.; from 30 to 60 the difference is practically insignificant between the rates based upon the Mixed Table and one of select character; while for age 65 and higher ages the premiums (involving selection) are considerably lower than those deduced from the Mixed Table in the usual way. Referring to our previous examples, this conclusion is clear: and adopting the age of 67, it appears that the pure premium (at 3 per cent.) on the mixed experience is ·08566 per unit of assurance, and on the Select Table ·08227, so that the latter is about 4 per cent. inferior to the former.

A reference may be desirable to the formula employed in computing the preceding policy-values by adding unity to the annuity-value.

The general formula, $A - \pi\ (1 + a)$, assumes that the next annual premium is immediately due. Hence, in expressing the value of the policy where the example is limited to an individual instance the addition of unity is appropriate: where the policies are sufficient in number to admit of valuation in classes, as explained in Chapter V, the annuity-value is properly augmented by ·5 only. The student, it need scarcely be mentioned, should observe that the single or isolated example in life assurance is to be accepted in the sense of a representative quantity. These remarks apply throughout the book where individual illustrations are presented.

The employment of Select Tables enables compensation to be directly provided for the detrimental influence exercised upon the general vitality of continuing lives in a Company by the withdrawal of healthy members, and this aspect of selection we shall discuss in the chapter on surrender values.

CHAPTER IV

Surrender Values

There is probably no question in the administration of life assurance which is so apt to be misapprehended by the public as that of the values granted on the surrender of policies. The universal criticism upon the quotation is the inadequacy of the amount; and this contention is almost invariably supported by the assertion that since the applicant has entailed no loss by death upon the Company he should receive, at all events, the whole of the premiums he has contributed—occasionally the addition of interest is exuberantly included in the demand.

For a proper understanding of the meaning and range of a surrender value, attention is directed to the general considerations adduced in Chapter I in connexion with the basis of the assurance system, with the detailed elements it involves; and a brief discussion of this particular subject in the light of those observations will be serviceable.

The premium paid by the policy-holder is composed, as has been shown, of two distinct parts: the one constituting the annual fund from which (with its accumulation at interest) the claims by death are discharged; the other consisting of the "loading," or addition for defraying the necessary expenses of conducting the common business for the benefit of all, for providing against possible fluctuations of future experience from that presented by the past, and securing the additional protection of the proprietary capital. In Mutual Societies the former demands—namely, the administrative cost and the possible advent of adverse experience—alone exist; but the nature of the explanation remains otherwise unaffected. These observations relate to assurances completed without participation in profits; where the policy, however, has been effected on the participating scale, the total premium includes also an annual contribution to the Profit Fund for periodical distribution.

A policy-holder decides to cancel his contract on the expiry of three years, for example; at that date a portion of the one component of his premiums (the element which measures the risk of death) has been absorbed in settlement of the claims which

have occurred during his membership, and a balance remains, which is termed the Reserve-value of his Policy, or the amount saved from his premiums after deduction of his proportionate contribution to actual claims. With regard to the other component, the expenses of administering the business have necessarily been rateably defrayed from it; and since the initial costs of obtaining business, including the particular policy, are considerable, practically no portion of the "loading" received during the period in question remains. And it is to be remembered that surrenders are, so to speak, an inverse function of the duration of membership, and mainly occur during the earlier years when (1) the expenses connected with the acquisition of business reach a maximum, and (2) the retiring members possess on the whole the most vigorous prospects of life. At later stages, when surrender values are applied for, special considerations enter, which will be discussed. Hence at the date of surrender the available fund for payment of values consists of the balance of the premiums (for the risk of death alone) which has been accumulated, or, in other words, the reserve, and this balance constitutes the extreme measure of the extent to which, in exceptional instances, the amount allowed can proceed.

Let the age at entry be 35, the policy effected for £500, and the contract non-participating premium be £2·3 per cent., or £11·5. At the end of three years the reserve will amount to £22·17, or about 70 per cent. of the pure premiums received, assuming the O^m table of mortality with 3 per cent. interest, while the remainder of the pure premiums, or 30 per cent., has been necessarily applied (or the basis of life assurance would disappear and the system become impracticable) in proportionately providing the claims which have arisen, and the "loading" has been utilized in discharging the costs of acquisition and management.

In all its dealings moreover with its constituents the Company must scrupulously and impartially consider the interests of *all* members; those who continue the maintenance of their obligations equally with those who abandon the common trust. Hence the general position must be carefully scrutinized when the cancelment of a contract is proposed with a view to ascertaining that the equivalent interests of the remaining members sustain no injury. The policy-holder should bear in mind that although a right of annulment is vested in him by the practice of Offices, his original

and implied contract was, so far as possible, to continue in the assured community, with its concurrent responsibilities, for the duration of his life, and the adoption of the privilege of departure from this implicit bargain with his co-members equitably demands that any real and recognized detriment to which they may be subjected by his act should be rectified and redressed.

Now, lengthened and consistent experience has proved that the continuing body *does* sustain impairment when withdrawals from the common membership occur ; and adequate, though reasonable, compensation, measured so far as accredited facts render it practicable, is consequently imperative prior to the grant of a surrender-value from the general stock, in order that impartial equity of treatment may be maintained. It will be admitted by the retiring policy-holder himself, on principles accepted in all conjoint commercial enterprises, that since he has entered into a community of interests in which the rights and prospects of all (his own included) should be sedulously regarded, any damage which his withdrawal is proved likely to entail upon the position of those who maintain the performance of the common obligations which they and he contracted should be equitably adjusted. This course he would himself require to see pursued if he it was who retained his connexion with the undertaking and discovered that his just privileges were infringed by the detrimental action of a fellow-member. Can an injury of this nature be demonstrated to exist ? And here—and universally in assurance problems—we must steadily remember that our inquiry and practice must be determined, not by the consideration of *individual* instances, but by preserving the clear and definite conception of a *mass* and the results of reasonably extensive *numbers*. The scheme of assurance, as we have frequently insisted, can only be justly interpreted and successfully surveyed if its basis —adequacy of area and a constant consideration of (what may be loosely termed) average results—be the controlling principle or criterion in every mode of administration and judgment.

Now, the surrender of a policy may in *any isolated* instance prove to be consequent upon ill health, or depressed financial conditions (which also affect physical vigour), into which the applicant has fallen, and which prevent provision of the means of continued discharge of the premiums. In such a contingency a Company might even increase its customary scale of surrender-value for the

purpose of benefiting the remaining members by the removal of an undesirable risk and the prevention of a speedy claim. But recalling the condition that we must contemplate masses and not units, it has been ascertained, as an undoubted result of experience, that, *on the whole* and in the long run, the aggregate of members who withdraw (by lapse and surrender) possess the normal state of health, and not improbably indeed a superior vital capacity to that existent among the assurance community from which they retire. By surrenders accordingly—again regarding the mass—the average general vital stability and expectation of longevity prevailing anterior to the withdrawals are reduced in extent and power by the abstraction of these vigorous lives.

Assuming two sets of retiring policy-holders, one in normal health, the other with impaired stamina, and presuming that both sections —not individuals—experience equal difficulty in maintaining their policies, the former will be influenced in their decision by the fact that, as their chances of life continue to be sound, they will be able hereafter (though at a somewhat enhanced premium) again to effect an assurance (if required) when happier times arrive, and hence no restraining or prudential motive in the direction of retained membership exists. The oppositely affected set, finding that their defective health will probably preclude the completion of a fresh assurance at a later date, will naturally exert every effort to preserve their family provision, borrowing upon the security of the policy if necessary, or obtaining the premium from their friends; the graces of human nature are generally so tenuous that relatives and friends will more readily contribute between them the small amount required to provide the current demand than incur the possibility of aiding the family to a wider extent if the assured should die without the possession of a protecting policy. Moreover, if a person be in really doubtful or precarious health, an abundance of speculative buyers are always prepared to purchase the policy at a price (according to circumstances) in excess of the surrender-value on the chance of a profitable return upon the investment. On the whole, consequently, admitting that in occasional individual instances the generalization will not hold, the members who retire possess reasonable prospects of longevity, while those whose difficulty is equally grave in the provision of the premium due, and who now unhappily exhibit a diminished probability of life, will tend to

maintain their contracts. The remaining mass of the assured accordingly are adversely affected on the occasion of surrenders, especially when the cancelments are numerous, by the diminution of the general leaven of vitality produced by the retirement of sound lives, which again is further reduced by the retention of inferior constitutions; so that the prospect of earlier and heavier claims by an augmented mortality has been enhanced, with an additional strain upon the common resources in excess of that anticipated before the surrenders occurred.

It might be conjectured that, assuming a certain average force of vitality (or power of resisting death) to be possessed by a group of 1,000 lives, and an average force of inferior value by another group of the same extent, the abstraction from the former class of 500 (with the retention of the latter at the original number) would produce practically the same decline of average vitality in the combined groups as that which would ensue from the introduction into the aggregate body of 2,000 persons of 1,000 lives exhibiting the inferior degree of quality. But a precise and interesting mode of explanation has been ably presented which we proceed to expound, with the incorporation of illustrative figures, in demonstration of the proposition that the influence induced upon the rate of mortality of the existing members of a company by the retirement of a number of healthy lives is precisely identical with the consequences which would follow if a mass of unsound lives were introduced.

Let 1,000 persons be assured at the same date, and let them be divided into two groups of 500 each. After the expiry of a year, a certain proportion of each set, say 5 per cent. of the original number, or 25 in each class, will have fallen into inferior health and prove no longer eligible at the ordinary rate of premium, while the remaining survivors of each group may be still considered to possess the prospect of longevity appropriate to their age. Assume that on the termination of a year *all* the remaining lives which continue to be assurable at the normal rate in the one body cancel their policies, there will then exist in the one group the 25 uninsurable lives, and in the other the whole of the survivors of the original 500. If these numbers be doubled, the rate of mortality is obviously unaltered (for the rate is a fraction whose numerator is the number of deaths and the denominator the number exposed to the risk of death, so that the fraction is unmodified if the numerator

and denominator be multiplied by 2), and it is apparent that the rate of mortality prevailing among the lives still under observation out of the original 1,000 will be identical with that which would result if, after the expiry of a year, 50 unsound lives were added to the survivors of the 1,000 originally observed. In other words, the withdrawal of vigorous lives increases the proportion which the deteriorated lives bear to the entire number. A numerical illustration will be serviceable.

Procedure	Group I. of 500	Group II. of 500
(I.) Let q be the rate of mortality: then at the close of a year there remain	$500-500\ q$	$500-500\ q$.
(II.) Of these, 25 have become uninsurable, and consequently the sound lives remaining at the end of the year are	$(500-500\ q-25)$ $+25$ *	$(500-500\ q-25)$ $+25$.*
(III.) Assuming that in Group I. the vigorous survivors retire, the constitution of the two groups will then be	$(500-500\ q-25)$ $+25-$ $(500-500\ q-25)$;	$(500-500\ q-25)$ $+25$
Or 25 inferior lives alone remain in group I.		
(IV.) The number of deaths during the second year—indicating by q_1 the probability of death in respect of sound lives, and by $q^1{}_1$ the chance in connexion with unsound lives—is . .	$q^1{}_1\ 25$	$(500-500\ q-25)\ q_1$ $+25\ q^1{}_1$.

[* The 25 outside the brackets represents the inferior lives.]

(V) The deaths in the combined groups are therefore

$$(500-500\ q-25)\ q_1+50\ q^1{}_1.$$

(VI.) and the total rate of mortality for the second year becomes

$$\frac{(500-500\ q-25)\ q_1+50\ q^1{}_1}{(500-500\ q-25)+50;}$$

$$\text{or}\ \frac{475q_1-500\ q\cdot q_1+50\ q^1{}_1}{525-500\ q.}$$

(VII.) This ratio is not altered by doubling the fraction, and we thus obtain the death-rate for the second year of

$$\frac{950\ q_1-1{,}000\ q\cdot q_1+100\ q^1{}_1}{1{,}050-1{,}000\ q.} \quad \ldots\ (A)$$

(VIII.) Now adopt the process of adding 50 unsound lives to the survivors at the end of the first year (the former withdrawals on this assumption continuing within the first group), then the total number at risk at the close of the first year is

$$(1{,}000-1{,}000\ q-50)+50.$$

Incorporate 50 unsound lives, and the number at risk becomes

$$(1{,}000-1{,}000\ q-50)+100.$$

Assuming the same chances of death as those above expressed, we obtain for the deaths in the second year

$$(1{,}000-1{,}000\ q-50)\ q_1+100\ q^1{}_1.$$

(IX.) And consequently the rate of mortality or chance of death for the second year is

$$\frac{(1{,}000-1{,}000\ q-50)\ q_1+100\ q^1{}_1.}{(1{,}000-1{,}000\ q-50)+100,}$$

$$\text{or,}\ \frac{950\ q_1-1{,}000\ q\cdot q_1+100\ q^1{}_1}{1{,}050-1{,}000\ q,} \quad \ldots\ (B)$$

or the same as (A).

The preponderant proportion of retirements occurs within a few years from the date of assurance, and hence, since all are practically of a select character, they determine a deeper effect upon the general vitality of the surviving body. After the lapse of an additional period of time the surrenders become reduced in number, since policies are rarely abandoned when their duration has been prolonged and their value, consequently, materially increased—so that their influence upon the total rate of mortality is less appreciable. At these later stages, however, the survivors of the original

assured are annually growing more deteriorated by their own advance in age and the progressive intrusion of maladies, so that a tendency in itself exists on this account to continue the augmentation of the general mortality. Two forces consequently persist in incessant and opposite operation in determining the rate of mortality among assured lives, and their conjoined result can be confidently employed as an adequate explanation of the varying phenomena disclosed.

A distinct detriment having thus been proved, equity requires that an equivalent compensation, numerically measured in the assessment of surrender-values, should be afforded to the continuing body of policy-holders by retaining in the common fund a portion of the reserves connected with abandoned assurances as a rectifying provision for the augmented mortality generally which will ensue. The influence thus competent of exercise upon the financial position of the remaining members is often termed the " power of selection " of the policy-holders. It need not be added that, in thus describing the adverse effect of voluntary withdrawal, no other implication is intended than that of the results produced.

An attempt has been made to attach a numerical expression to the consequences entailed by the action of retirements. The experience of Assurance Companies (designated the H^m or the death-record of healthy males) was employed, which included the discontinued cases ; and thus an indication was obtained of the mortality which the Companies would have experienced had those who withdrew remained under observation until death and continued subject to the implied rate of mortality. The results of this inquiry for the ages of 30 and 40 respectively showed that—

(1) The rate of mortality at the former age by the combined H^m table and its withdrawals was 7·58 per 1,000 exposed to risk, while the experience of the H^m alone furnished 7·72 ;
(2) The corresponding rates at age 40 were 9·79 and 10·31.

The substantial defect in this mode of investigation is that those persons who withdrew were presumed to continue subject to the average rate of mortality prevailing in the whole body ; the rate of mortality among assured lives is dependent upon two variables —the age at the time and the duration of membership ; and the preceding inquiry takes cognizance of the former element alone, while

in withdrawals the rate of mortality will be reduced,since they mainly occur during the period of selection. The appropriate method of detecting the influence of withdrawals upon the general rate would consist of that which is employed for the purpose ot determining the effect of the recent selection of lives where the customary medical and adjunctive tests are applied. The group of entrants at particular common ages, or at closely approximate common ages, at entry, must be separately traced (apart from lives which have attained that age from prior entry and lives accepted subsequently at ages which these entrants stage by stage reach), and the segregated rates of mortality exhibited by each group must then be deduced, whence the force of selection will become visible.

The subject demands a renewed and exhaustive examination on the basis of the data recently published by the Institute of Actuaries and the Faculty of Actuaries, as its practical bearing upon the permanent interests of Assurance Companies is significant.

But withdrawals exercise a further adverse influence upon the fortunes of the entire body. The assured, as a whole, are not merely a community for creating a provision at death, proportioned to the extent of contributions where, in accordance with the principles of assurance and the individual assent implied in membership, many by length of life subscribe in excess of the sums which their families will receive, while the early deaths of others entail a strain upon the common resources which their contributions do not compensate ; but they also constitute a community for the purpose of conducting their joint undertaking, by discharging the requisite expenses of management. Each member has implicitly contracted to provide his adjusted share of the total costs incurred in the general interests. Hence when policyholders sever themselves from this agreement by surrendering their contracts, a heavier proportionate burden is entailed upon the remaining assured by deprivation of the shares which were previously contributed. Equity requires that adequate compensation should be afforded for this transfer of obligations to those who continue, and hence again the reserve on which the amount of surrender value is based must be modified to meet this loss. Any small balance of loading similarly which may be in hand is applied to the same purpose. For the contract originally entered

into (regarding whole-life policies alone) was one to endure for life—not of course obligatory upon any individual, but understood to be voluntarily intended so far as future circumstances would legitimately permit.

The abstraction, further, of members by surrender (equally as by death) is only counterbalanced by the acquisition of new entrants to supply their place; and this retrieval is necessarily attended by specially increased expenditure, which thus entails an augmented burden upon the community, produced exclusively by the action of withdrawals. Each Company is founded for perpetual existence; and if it is to be maintained as a continuing and prosperous corporation, the basis of adequate numbers on which the integrity of its system of practice essentially depends, the continuous vitality of the general constitution of its members, and the retention of a common vigorous age,—if decrepitude of corporate life, by unchecked advance of age, is to be averted,—must be persistently pursued, although the expense of additional acquisitions of substituted entrants is thus induced beyond the cost of ordinary administration. It is not urged that this condition is importantly involved in ordinary surrenders, but the underlying principle requires to be affirmed in a general discussion of the subject.

The evidence thus adduced, founded not upon *a priori* assumptions, or arbitrary and theoretical speculations, but exemplified by actual experience, is sufficient to prove that the value granted on surrender expresses simply a uniform and equitable regard to the common interests of all concerned—those common interests which every member on entering the corporation implicitly contracted to promote and conserve.

The preceding exposition of the detrimental influence exerted in general by surrenders upon the financial position and prospects of a Company in respect of mortality and contribution to expenses conclusively disposes of the popular misconception that lapses and surrenders constitute a source of profit to the office. The apposite and opposite deduction is apparent, that the permanent interests of a Company are most efficiently maintained when the rate of surrender falls to a minimum.

With regard to surrenders which occur at more advanced stages of duration, the same principles apply, though obviously modified

in practice, in respect of the numerical measurement of adverse effects, by a consideration of the facts: (1) that they are comparatively fewer at the higher ages, and (2) that their occurrence lies beyond the period of selection. In these instances also it is to be noted that a portion of the loading has been absorbed in the creation and distribution of profits in which the retiring policy-holder has participated if his assurance were effected upon the profit scale of premium; while in respect of each class of policy a proportion of this addition, where the security of a proprietary capital fund is afforded, has necessarily been applied as compensation for this protection.

Cases occasionally occur where a just recognition of the position and interests of the body of the assured as a whole, would sometimes sanction the grant on surrender of a sum even in excess of the reserve-value, notwithstanding the considerations already adduced—where, in brief, the withdrawal of a policy on such enhanced terms would compensate the probable loss to the members (by an early death) if the assurance remained, and where this special course would also express an equitable regard for the retiring policy-holder on cancelment of an inferior risk. This statement will be guarded by remarks hereafter. A policy-holder, for example, exhibits the premonitory signs of phthisis; and possibly, from his pecuniary circumstances or the condition of his family connexions, does not deem it to be imperative to continue the assurance. Cash for present uses is preferable, it may be, even to the contemplated speedy payment of a claim to his relations. He submits a bargain to the Company for the grant of terms exceeding the ordinary scale of surrender-value; and the principle of concern for the position of the constituents of the Office generally may confirm the expediency of allowing a considerable sum. The Author recalls an experience of a special instance of this nature: the record of the microscopical analysis of the sputum of a patient in this condition was inspected by him, and from a human point of view the register formed a pathetic history of the alternating contest between the bacteria of consumption and the leucocytes or white corpuscles which resist the microbic attacks. On one day the bacteria would be discovered in diminished numbers and in disintegrated forms: at a succeeding date the vigour of the leucocytes would be discerned, from the

bacteriological examination, to be seriously depressed and the victorious bacteria evident in conquering multitude and strength; and the phases of the weary fight fluctuated from day to day. But recurring to the qualifying remark which has just been made, it should be pointed out that a Company, in its primary care for all beneficiaries, would first suggest a loan upon the security of the policy, or an equivalent paid-up assurance, even in so extreme an instance, rather than the occurrence of the absolute forfeiture of the contract; for it is no futile boast to assert that the action of Assurance Companies does not simply display a severely commercial character, but is actuated also by generous and humane considerations adjusted to every difficult and saddening case that may be presented. A Company, by virtue of its intrinsic constitution as the faithful custodian of *all* interests committed to its impartial trusteeship, allots sedulous, thoughtful, and liberal attention to the circumstances of those who will presumably entail upon it a premature loss, as of those who are likely to contribute long and largely to its common fund.

It may be added that corresponding considerations apply to lapsed policies, where no value on withdrawal is involved. Lapses occur in the early years of assurance, when the lives continue to exhibit a sound prospect of life, and hence the Company is especially affected detrimentally by the twofold operation of (1) the considerable expenditure which has been incurred in the acquisition of these Assurances now abandoned and so recently secured, where consequently no opportunity exists, by the continuance of the policies, for recouping the whole or part of the heavy initial charges by the reduced cost of conducting the renewal business during the remainder of life, and (2) the abstraction of a number of lives in generally vigorous health with the resulting prejudicial leavening of the average resisting power of those who remain.

The usual practice of Companies is the grant of a surrender-value after payment of three years' premiums in ordinary cases, and where two annual premiums have been received and a lapse is imminent, a Company always endeavours to persuade a policy-holder to continue by offering an advance upon the security of the policy towards the discharge of the premium due. The beneficial custom is now becoming universal of maintaining an assurance, which has acquired a surrender-value, and has been allowed to

lapse, by discharging (even outside the knowledge of the retiring policy-holder himself) the successive premiums, so far as the surrender-value at the date of lapse will provide, and paying the amount assured to the representatives if death occur prior to the exhaustion of that value, diminished only by the several advances and interest. This just procedure constitutes one of the marked and beneficent features which characterize the management of these institutions, and is prompted by an equal and undiscriminating attention to the benefit of the entire body of membership.

The observations now submitted amply justify the practice that no surrender-value should be allowed until a certain number of premiums have been paid. This probationary period, as it may be termed, is simply the equitable equivalent, in the general interests of the Company (that is, of the members as a body), for the depletion of the fund by the substantial preliminary charges incidental to newly-obtained business, which thus promptly disappears by lapse, and the injurious influence also produced upon the mortality prospects of the members.

A practical question may here be considered. If a life is " rated-up " for inferior health or constitution by the imposition of an addition to the normal premium at entry, should the surrender-value be calculated upon the actual age at the date of discontinuance (upon either the premiums paid or the reserve, according to the custom of the Office), or upon the increased age corresponding to the premium charged ? The former course would naturally produce a smaller allowance. The considerations which have been urged—and particularly the effect of deterioration occasioned to the average vitality of the remaining mass by the deprivation of sound lives—suggest and enforce clearly that the value should be assessed upon the reduced reserve, arrived at by inclusion of the actual age. Viewing, as we are bound to do, the aggregate instances of withdrawal, independent of isolated exceptions, the persons thus abandoning their contracts may be justly considered to exhibit on retirement an enhanced prospect of longevity beyond that which was originally anticipated and which was attempted to be compensated when the addition to the premium was imposed : that is to say, the chances of life have proved by lapse of time (through a beneficial change, for example, in the conditions of life) to be superior to those which the medical

examination contemplated. Seeing that extra premiums for degenerated health are average assessments, like the ordinary premium, in such an instance as that under review the amount of extra in excess of its demonstrated measure is required for compensation of those cases where future experience may show, by a more rapid course of deterioration, that the original addition expressed an insufficient appraisement of the risk. In every scale of extra charge, as in ordinary assurance, the surplus beyond anticipation and the loss in excess of anticipation must in the long run constitute a balance and equipoise. If then the surrender value were based upon the reserve computed for the enhanced age, this compensatory element for concurrent deficiencies in other instances would be sacrificed. Medical prevision, in the estimate of under-average conditions, has not yet acquired the power of exact prediction, and even did this accuracy of precise measurement exist, it will generally, and must in most instances, be limited to the assumption of certain habitudes of life affecting the disease: these conditions, however, may be advantageously modified in course of time, or a more favourable physical or moral environment may intervene, so that a beneficial effect may thus be produced in the arrest or abatement of the tendency to disease. If a policy-holder subject to an extra charge apply for surrender, primâ facie evidence is afforded that the prospects of life have proved superior to those which the addition was designed to measure (or, at least, that the applicant considers this to be the case), and this presumption emphasizes the justice of forming the surrender-value upon the diminished reserve.

In place of a value in cash, the Company will issue a paid-up policy calculated upon the value, and thus perpetuate some portion of the original assurance, disburdened of future payments of premium. The principles already enunciated will furnish a guide to the settlement of the equitable amount. As the assured is presumably a reasonably sound life—testified by his voluntary resignation of a proportion of his policy—and will thus by his retention of membership aid in maintaining the prevailing rate of mortality at a favourable level, a more liberal procedure is reasonably permissible in fixing the amount of reserve as the basis of computation of the substituted paid-up assurance than that which would be properly applicable to a final cancelment of the contract. In employing

a single premium for the construction of the fresh assurance, a moderate margin for expenses should be included, but the addition for commission should of course be omitted.

Having thus expounded the governing principles affecting the problem of surrender, we proceed to the consideration of some practical modes of application.

In the earlier years of actuarial investigation it was proposed that, with this object in view, the anticipated experience involved in the construction of the premiums, and in the general assumptions of the course of business, should be compared with the experience which had actually prevailed ; and according to the extent of any discovered divergence the reserve, as the foundation of the value to be quoted, should be rectified. Thus—employing the usual symbols—if the assumed elements were π_x, a_x, and ϕ_x, while the proved experience exhibited ${}^1\pi_x$, ${}^1a_x$ and ${}^1\phi_x$ the reserve to be adopted, instead of being $(\pi_{x+n}-\pi_x)\ (1+a_{x+n})$, should be the reduced amount of $({}^1\pi_{x+n} - {}^1\pi_x)\ (1 + {}^1a_{x+n}) - ({}^1\pi_x + {}^1\phi_x - p_x)\ a^n$, where the symbols π_x, ${}^1\pi_x$, a_x, ${}^1a_x$, ϕ_x and ${}^1\phi_x$, bear their customary significance, while p_x is the contract premium charged, and a^n is the amount of an annuity-certain for the period which has elapsed since the commencement of membership.

The expression ${}^1\pi_x + {}^1\phi_x - p_x$ exhibits the annual actual requirements of the past for charges (including the risk of death) and fluctuations, that is, in one event, the actual excess beyond the premium charged of the premium which has been ascertained by experience to be necessary, or ${}^1\pi_x + {}^1\phi_x$, while the expression first mentioned implies a pure premium Reserve on the expected basis. If ${}^1\pi_x + {}^1\phi_x$ be less than p_x the subtractive part will become positive or $+[p_x - ({}^1\pi_x + {}^1\phi_x)]\,a^n$: if the two are equal, then the premium received has proved sufficient for the risk of death, expenses and profits exactly; while if ${}^1\pi_x + {}^1\phi_x$ be the greater of the two, the premium received has not been adequate for these demands and the deficiency should be restored.

If the original contract premium, p_x, were formed of $2 + \cdot4$ (pure premium and loading), while the requirements for the risk and charges have proved to be $2\cdot1 + \cdot5$, the balance of $\cdot2$ must be accumulated for the period, and the reserve on the actual basis of experience correspondingly reduced.

It was suggested further that as a provision for probable diminution in the duration of life among the existing members produced by the surrenders, and the heavier ratio of expenditure which the withdrawals entailed, the Company should retain the value of ϕ^1_x (or the actual annual loading which experience had proved to be requisite) for the entire continuance of life of the resigning assured —that is to say, its accumulated amount at the realized rate of interest from the date of membership to the epoch of cancelment and its present value at that date for the residue of life.

This plan of revision, though founded upon sound principles, is cumbrous, and moreover any divergence between recorded and anticipated experience would have been remedied generally at the preceding valuation, so that the proposed retrospective consideration would not be required. At the time when this proposition was submitted, the subject of the influence of selection exercised by the cancelled policies upon the financial status of the general body was only apprehended within wide limits. It should be added that the approximately exact extent of this adverse force, though now more systematically recognized, has not yet been reduced to a definite and finally acceptable form in numerical shape, and the enterprising student thus possesses a spacious and promising arena for the application of original inquiry and appropriate statistical comparison and deduction of observations.

Additional attempts have since been made in this direction by devoting special attention to the future effects involved in withdrawals, and ascertaining, on the basis of experienced results, the approximate numerical values of the several necessary deductions to be equitably made from the reserve in the computation of surrender-values.

Considering the reserves for various durations of policy-existence in a Company, it was noticed in one investigation that the reserve (where the mortality involved in the calculations consisted of the heavier rate which ensues when the favourable consequences of medical selection have practically become extinct—that is, after the space of five years) exceeded the reserve obtained from a table including in its observations the period of selection itself by about 5 per cent. on the whole of the durations of assurances—ranging from 10·6 per cent. at the expiry of 5 years to 4 per cent. at the close of 20 years, with 2·7 per cent. in a duration of 35 years, and

$2\frac{1}{2}$ per cent. after 45 years. It was then assumed that the force of unfavourable selection by withdrawals might be fairly measured in respect of the aggregate policies by the uniform ratio of 5 per cent., so that on surrender the actual reserve (by a Mixed Table) should be diminished in this proportion as compensation.

With regard to the increased rateable amount of expenses left to be apportioned among a reduced number of assured, it was ascertained that, omitting the exceptional charges for the acquisition of new business, the general cost of administration might be assessed at 7 per cent. upon the total premium revenue; and excluding that portion which would cease to be incurred in respect of abandoned policies (such as stationery, postage, and other expenses of a similar nature), the second deduction on surrender was approximately fixed at $2\frac{1}{2}$ per cent. of P_x $(1 + a_{x+n})$, where P_x is the contract premium.

In Mutual Societies no further deduction would be necessary in respect of the margin of profit included in the loading, but, as regards Companies possessing a proprietary capital, it was proposed (if, for example, the proportion of profits assigned to the shareholders were $\frac{1}{5}$) that an additional diminution should be made from the reserve of $\frac{1}{5}$ $(1 + a_{x+n})$ ϕ where ϕ is the margin of profit included in the contract premium, P_x. The validity of this last proposal, it may be remarked in passing, may be justly disputed. The responsibility of the capital has vanished with the abrogation of the liability; and if the future prospects of profit of shareholders are to be recognized on these occurrences, why not equally the anticipated bonus benefits of the continuing policyholders?—a proceeding which would involve inextricable and embarrassing complications.

The primary defect of this investigation is the adoption of a uniform percentage of deduction in connexion with the resulting increased mortality in place of taking into account in each instance the actual duration of the cancelled policy. The assurances of extended standing are thus improperly fined for the benefit of early abandonments which entail the more serious consequences. It is obvious that if policies are annulled during the term of selection a more divergent effect is produced upon the general scale of mortality than would occur if the surrender were deferred to a later date, when the benefit of selection had become extinguished, and

when consequently the withdrawals would exercise a diminished influence upon the aggregate rate.

In another investigation the same leading principles were employed; but the preliminary expenditure incurred in procuring the business was justly involved. (On the other hand, it might be added that, seeing that this special charge is presumably undertaken in the interests of the entire body, that body should to an extent suffer the penalty of the financial effect of losing business which it had, perhaps unwisely, sought to secure, and thus be guided in its efforts of extension in the future by its painful experience of the past. The imposition of the burden involved in the excessive costs of obtaining business does not exclusively rest, it would seem evident, upon the retiring policy-holders, and the entire pecuniary detriment, it may accordingly and fairly be contended, should not be levied upon the values they receive. It is reasonable to add, however, that this criticism is restricted to that surplus of new business which extends beyond the imperative requirement of repairing "wear and tear" by deaths and cancelments, and preserving the *profitably* progressive course of the Company.)

The deduction from the reserve for expenses proposed in this scheme was $(1 + a_{x+n})\ (r + s)$, where r is the loading for initial expenditure, and s the addition for provision of the general administrative charges.

It was ascertained from an examination of the accounts of various Offices that a sum slightly in excess of £2 per cent. upon the amount assured represented the primary expenditure incurred in securing business, while $2\frac{1}{2}$ per cent. of the total premium income might be regarded as applicable to the ordinary administration. Thus from the reserve for a whole-life policy at age $x+n$ would be subtracted on this account a capitalized sum of

$$(1 + a_{[x+n]})\ (r_{[x]} + s_x),$$

$$\text{where } r_{[x]} = \frac{2 \cdot 1}{1 + a_{[x]}},$$

(the special preliminary charge being thus distributed, by means of the annuity-value, over the entire continuance of the assurance), and $s_x = 2 \cdot 5\ P^1{}_x$ where $P^1{}_x$ is the contract premium. And further it was proposed that the reserve value to be adopted (and from which these deductions should be made) should consist of the reserve for lives assumed to be still select.

On the subject of the adequate compensation for the effect of adverse mortality, the proposition just described contained, it would be observed, the sound suggestion that the reserve to be employed should be derived from a Table of Mortality where the lives entering at each individual age are traced separately throughout their duration, in substitution for a General Table where the data at any specified age comprise the select or lighter mortality of persons newly assured, the heavier rate experienced by members who, admitted at prior ages, have reached the age in question, and from whom the advantageous force of selection has either partially or entirely disappeared; and the favourable mortality affecting those who enter at each subsequent age which the persons of the original stated age progressively attain. Thus, at age 40 at entry, the reserve by a Select Table of Mortality is, at the end of 10 years, 96 per cent. approximately of the Reserve under a Mixed or General Table. It is thus suggested that the reduced reserve should be adopted in the computation of the surrender value (assigning direct effect to the adverse influence of withdrawals upon the aggregate vital capacity), from which would be deducted, as we have stated, the capitalized value of the proportion of loading already mentioned in relief of the more onerous rateable charge imposed upon the resources of the Company. This method of treating the problem from the aspect of enhanced mortality is superior to the plan previously described, since it rightly includes cognisance of the specific ages and durations of assurance appertaining to the retiring members in place of producing partial results by the application of an average assessment of deduction (based upon collective ages and terms) without just discrimination of the diversity of consequence incident to the epoch when the cause is operative.

It is to be observed that with the prolonged existence of a Company, the new or select business will form a smaller proportion of the aggregate amount assured; so that the average vital power of the mass will in most instances continue to diminish by advance in the general age alone; the withdrawals consequently of select lives (unless they occur in considerable numbers) will exert a decreasing influence upon the total mortality. On the other hand, the reserves will have become substantially augmented, and the resulting strain upon the Company's resources (the excess, namely,

of the sum assured and bonuses beyond the reserve) will be gradually lightened. Hence, this method of recognizing the incident effect upon mortality is to be preferred. But, as has been already suggested, the entire subject is capable of wider and more minute investigation before a scheme of adjustment, very approximately equitable and consonant with the actual facts, can be devised.

In Chapter III a reference will be found to the subject generally, based upon the introduction of select values.

The summarized considerations discussed may be presented in the following form :—

I. Withdrawals exercise an unfavourable influence (by way of increase) upon the rate of mortality experienced thereafter by the assured, whose contracts are maintained in existence, and this force is most potent when the surrenders occur during the early or select period of duration, and again operates adversely (if the cancelments be numerous) as the general age of the Assured community increases.

II. The total burden of expenses presses the more severely upon the several constituents of the Company as contributors retire.

III. The Reserve-value of the policy is the superior limit of the allowance on surrender. The full amount, however, may be conceded in exceptional instances, and even a larger sum where the resulting benefit to the policy-holders generally would be advanced by such an act ; but in all ordinary cases—and the exceptions will be rare—the reserve, if deduced from an Aggregate Table, must be diminished by the provision of compensation in the interests of those who retain their membership.

The power of cancelment, it should also be remembered, is entirely one-sided, for the community as a corporate body possesses no right of protecting its endangered interests by the compulsory ejection of inferior lives, which of course would be a proceeding fundamentally antagonistic to the principles of assurance. Seeing, however, that withdrawal constitutes a voluntary act, justice demands that a body conjoined by common ties and devoted to common ends, should be entrusted with an adequate capacity of redress where the general execution of its trust is likely to be adversely affected by the decisions of any members embraced within its scope.

In the practice of Companies the scale of surrender value ranges from 25 to 33 per cent. of the premiums received in connexion with non-participating policies, and from 40 per cent. where the assurances share in profits. This percentage is inclusive of the value of any bonuses which remain attached; and where a portion or the whole of the bonuses declared have already been commuted for cash payments their amount is deducted from the percentage of the premiums paid. A general scale of this description obviously does not sufficiently discriminate between the special merits of each case, but a uniform and simple plan possesses attractions to the public which probably justify the sacrifice of a preciser accuracy to the explicitness and readiness of an average and simplified rule. As the policies advance in duration, the value would be more systematically and appropriately assessed upon the reserve.

Extra premiums for special hazards incident to foreign residence and dangerous occupations do not possess a surrender value, since they are calculated in equivalence to the risk annually incurred and become exhausted in the additional mortality which yearly is entailed. Moreover, if in any instances a surplus remained after the specific risk had been abandoned, its amount would be required to provide for the deterioration of health (and premature advent of death) which the stress of these hazards will produce in other cases less adapted to their successful endurance.

The practical importance of the subject may be measured by the fact that the surrender values granted by British Life Offices (excluding Companies transacting the industrial class of business) exceed £1,125,000 a year.

No popular misunderstandings on the theory and practice of life assurance are more prevalent and inveterate than those which relate to the fundamental conception of the scheme in general and the connected special subject of surrender values. The former perplexity has been effectually disposed of by the analysis in Chapter I of the theorem on which both the doctrine of probabilities and the system of assurance in which it is embodied in a particular form depend. Contrary events in the mass of observations, it has been seen, counterpoise each other in the completed range; and if a brief sequence of events of one particular order (for example, a shortened duration of life) occur, the inequality is

redressed by a more prolonged series of the same kind of events (the lengthened continuance of life, for example) in order that the uniformity of Nature may be displayed when a sufficient area has been allotted and surveyed.

The general remarks already submitted similarly dispel the illusion that a policy-holder is entitled on surrender to a refund at least of his contributions, on the alleged ground that existence is synonymous with exempted liability, where the vision of the unit is dissociated from the contemplation of the mass.

A numerical illustration will more impressively convey the fact with demonstration to the man of business that the necessary progress of a Life Fund inevitably prevents the withdrawal of the entire subscriptions from the common stock.

Assume that an Assurance Society is composed of 1,000 members who enter at the age of 40 and assure for £100 each. The annual premium for the risk of future death after that age (the remainder being applied to expenses, fluctuations, and profits) is 2·524 (O^m 3 per cent.); and further assume that at the end of three years the policy-holders as a body desire to dissolve membership.

The actual non-participating premium for age 40, it may be interposed, is about £2·7, and the difference of £·176 is annually exhausted in the charges and incidental demands of administration, so that any saving is restricted to the pure premium alone.

The experience with respect to mortality and interest is assumed to be that expressed by the O^m Table and 3 per cent.

I. At the origin of the fund, the 1,000 members each contribute £2·524, or £2,524 : this sum accumulated to the close of the year is augmented by 3 per cent. interest, or (say) £76, and amounts at that date to . . .	£2,600
II. The deaths in the year, according to that table number (say) $9-q_{40}$ being ·00915—each entailing a payment of £100, or in all	£900
III. The fund consequently at the termination of the year is reduced to	£1,700
IV. During the second year the fund of £1,700 is increased by interest at 3 per cent., or £51, and amounts at the close of this year to	£1,751

V. At the beginning of the year the members surviving (1,000—9, or 991) each discharge the premium of £2·524 or, in total, £2,501: and this sum by the end of the year gains £75 in interest, so that it then stands at £2,576

Making a total amount of assets of. . . . £4,327

VI. The table shows that (say) 9 deaths will again occur (q_{41} = ·00956), and the claims consequently absorb the sum of £900

Leaving the fund on the expiry of the second year at £3,427

VII. The balance of £3,427 will, during the currency of the third year, be augmented by interest to . . £3,530

VIII. The survivors left (1000 —9—9 = 982) contribute their united premiums of £2,479, and, accumulated at interest, these amount at the close of the third year to . £2,553

Forming an aggregate fund of. £6,083

IX. Ten persons die in the course of this year, producing claims of £1,000

X. So that on the expiry of the third year the net Assurance Fund as the provision (with future premiums) for the successive payment of the existing assurances of 972 × £100, or £97,200, will be £5,083

If then the whole of the 972 policy-holders desired to surrender (and the same observation obviously applies to any individual assured), they could only each receive the sum of $\frac{£5,083}{972}$, or £5·2, as their surrender value, since this is the utmost amount which the actual fund will allow, while the contract premiums paid by each member amount (omitting interest) to 2·7 × 3, or £8·1, and the pure premiums for the chance of death to 2·524 × 3, or £7·572. The difference between the contributions received and the several shares of the fund, or the reserves, together with the interest, has necessarily, according to the fundamental conception of life assurance, been absorbed in the discharge of the 28 claims which have meantime been occasioned by death, and the satisfaction of the expenses which the management of the business has entailed.

It may be added that it will prove useful to the student to exhibit parenthetically the coincidence of this realized result with that deduced from the theoretical formula for valuation, thus furnishing a further illustration of the identity of the methods described as the retrospective and prospective plans, which will be presented in Chapter V.

Assuming that at the close of the third year in question, when the survivors have attained the age of 43, a premium is immediately due from each, the formula expresses $A_{43} - \pi_{40}\ (1 + a_{43})$ per unit assured—that is, supplying the numerical elements, ·49201 — ·02524 (17·441) or ·05180. Since the liabilities at the epoch named amount to 972 policies of £100 each, we obtain ·0518 × £97,200, or £5,035, which coincides with the result already produced after allowing for the slight difference consequent on the rejection of decimals in the q_x employed.

The reasonable conclusion may be stated that, expressed as a percentage upon the reserve, the surrender value should exhibit an increasing ratio instead of forming a constant proportion at all ages and durations.

We have restricted these remarks to the values assigned in respect of the original amount of the policy; for a participating assurance, the present value of the reversionary bonuses which remain attached, or of the future reductions of premium for which any specific bonus had been commuted, would also be granted subject to an equitable diminution based upon the principles which govern the calculation in connexion with mortality effects alone, since in respect of expenses no deduction is requisite inasmuch as, practically, no heavier burden is imposed upon the survivors for costs of administration in relation to the extinction of the bonuses themselves.

CHAPTER V

The Meaning and Process of the Periodical Valuations of the Liabilities and Assets

Assurance premiums, of necessity, were originally experimental measures of the experience which the future might be expected to disclose; and although, with the lapse of the tentative stage, their experimental character has been reduced to a minimum, the possibility of fluctuations divergent from the records of the past must still to some extent be recognized.

As regards the element of mortality involved, the stability and adequacy of premiums may be confidently relied upon at the present time, when we consider the established and uniform rate of mortality among assured lives which exhaustive and appropriate series of data exhibit—the several classes of observations proving minutely confirmatory,—and the practical absence of extensive and severe epidemics. The term "appropriate" is scientifically pertinent, since it is evident that the suitable index to the probable experience of a specified description of risks should be the actual experience which has prevailed in the past among a similarly constituted mass, assuming naturally that the conditions affecting the two series are fundamentally identical, and that the section observed, from which the ratios were deduced, was sufficiently numerous.

The element of future interest, also, employed universally by Companies in the calculation of the premiums to be valued, may be securely admitted, since Offices are generally conservative in this factor of their valuation estimates. A surplus always is found existent between the net return of interest (after deduction of income tax, and allowance for any permanent depreciation of securities) actually realized upon the investments and the future rate on which the valuation is based. Hence a periodical investigation is requisite for the purpose of ascertaining the accumulated amount of this annual surplus which has not been absorbed in the requirements of the business.

The premiums also contain a margin, or "loading," explained in Chapter II, as a provision for the commission and other disbursements expended in procuring assurances, the administrative charges, and the possible fluctuations of future experience from

that expected to prevail. If a comparison between the specific provision included in the premiums for these objects and the expenditure and commission actually incurred disclose an annual balance in favourable excess, it becomes necessary to determine its accumulated amount in the past for judicious utilization in the present and future. It is this third element of expenditure which constitutes the really essential and varying factor in administration and which, especially in connexion with the existing mode and scope of competition, demands the critical and serious attention of the actuary.

In summary words, the surplus of contributions experimentally disbursed by the assured for purposes of complete security is ascertained at a valuation, and an equalization of realized experience with that anticipated is effected by a distribution of profits.

A periodical valuation accordingly subserves the threefold object of (1) discovering the actual equivalence or divergency between the exact results displayed in mortality, interest, and expenditure and the provision created in the construction of the premiums and the formation of prior reserves; (2) determining the amount necessary to be retained in hand, technically termed the reserve, which, accumulated at the rate of interest selected, will, with the prospective valuation premiums similarly compounded, suffice to discharge the claims as they successively mature. (This process also includes the reservation of an adequate annual fund to defray the expected expenditure of the future); and (3) the distribution of the surplus which the valuation discloses among the entitled policy-holders whose contributions (accumulated at interest) have thus been ascertained to exceed the necessary requirements of the past.

But a valuation is in no sense a " stock-taking " in the customary acceptation of that term. In an action tried before the Lord Chief Baron at the Guildhall in 1860 a completely erroneous impression was entertained of the unique character of these investigations. The judge conceived them to form a species of stock-taking as practised by tradesmen. The two operations are essentially distinguished from each other, and differ *ex toto*. In the ordinary stock-taking, the trader is concerned with the value of his articles at the current prices of the period, and his results are obtained by a mechanical arithmetical process competent to average intelligence. In a valuation, however, the probable future requires to be carefully

scrutinized and gauged in the light of the experience acquired both as regards the rate of mortality likely to be exhibited among the members, the rate of interest at which the accumulated fund and prospective premiums can probably be soundly and profitably invested, and the adequacy of the loading reserved for anticipated demands. And it is to be noted that this prevision is not limited to the ensuing valuation period only, but necessarily extends to the entire possible duration of the whole of the existing contracts, comprising a range of time of thirty years and upwards. Sagacity and judgment, in addition to practical knowledge, are intimately involved ; the modes of calculation constitute a specially technical process ; and the skill and professional training of the actuary require also to be exercised in order, by means of what are termed " adjustments," to produce equivalence of conditions between the mathematical functions employed and the actual circumstances of the Company's operations, as will hereafter be explained.

In partial illustration of this specialised work we may cite the distinction of judgment upon the appropriate measures of mortality and interest for the valuation of assurance and annuity obligations respectively. The former are mainly effected upon the lives of males, and hence the functions deduced from the experience of this class should properly be adopted. Annuities, on the other hand, are chiefly purchased upon the lives of females, whose vitality, at the ages at which the purchases principally occur, is considerably superior to that exhibited by males of corresponding ages.

Without speculating upon the possibility of the female constitution (after a defined period of existence) possessing an enhanced intrinsic power to resist death, the difference in the comparative rates of mortality may reasonably be explained by the following considerations :—

(1) At the comparatively advanced ages at which annuities are acquired a power of self-selection is clearly competent of exercise, based upon an extensive individual knowledge of family history and upon a lengthy experience of personal health and strength. Annuities accordingly as a rule are bought only by persons conscious of vigour of constitution, while, from the nature of the case, assurances, where a choice is exercised, are the more eagerly sought by those who, gifted with a diminished vitality,

endeavour in this form to compensate the probability of brevity of active life ; (2) at these higher ages, again, the dangers incidental to child-bearing have been surmounted, and that epoch of life also has terminated which is frequently fatal to women, so that the survivors, who become annuitants, belong to a select class of lives ; and (3) the undoubted reason must be prominently assigned that the modern antiseptic methods of surgery enable internal tumours, to which females are peculiarly subject, to be safely excised before they have assumed a malignant form. In days anterior to this great discovery (vital in a twofold sense), the apprehension, or rather the certainty, of peritonitis (or inflammation of the inner membrane lining the abdomen and investing the contained viscera) barred the possibility of these critical operations, and thus interior growths and tumours developed without check from the innocent to the fatal stage.

It may incidentally be added also that the operation for appendicitis (eminently successful now under antiseptic treatment) was equally impracticable in former years, though of course this disease is not specially restricted to females.

The appended table of the complete expectations of life, or the average durations of future lifetime of annuitants, conclusively illustrates these remarks :—

Age	The experience of Government annuitants (National Debt Office), 1883		The experience of annuitants in Life Assurance Companies, 1893		The experience of the General Annuity Trust Fund (female nominees), 1895
	Expectation of life in years				
	Males	Females	Males	Females	Females
60	14·38	16·26	14·82	17·01	16·6
65	11·68	13·16	12·02	13·82	13·5

(The expectations in the Government and Assurance observations appertain to the analysed (or select) experience, and thus somewhat differ in form from those presented in the Trust Fund, which apply to the general experience.)

The corresponding expectations for males, derived from the records of male assured lives, are 14·07 at age 60 and 11·22 at age

65, showing, therefore, a defect, compared with the respective figures for female annuitants in column 5, of 2·9 and 2·6 years.

Different measures of mortality consequently are imperative for the assessment of assurance and annuity reserves, and it is obvious that the employment of a table for the valuation of contracts dependent upon the lives of male members which contained an appreciable proportion of female lives of select character would, by reason of their intenser vitality reducing the general rate of mortality, produce an inadequate provision for the future. The Carlisle observations, for example, included 55 per cent. of females.

With respect to the comparative rates of interest which may reasonably be assumed in creating the reserves for assurances and annuities respectively, it is to be observed that the average age at which assurances are completed is somewhat under 40, while the most extensive purchases of annuities occur between the ages of 60 and 65. Hence, if deemed judicious, a somewhat higher rate of interest than that adopted for assurance reserves might be justly employed in the valuation of annuity contingencies, since foresight and estimation in the latter case require to be exercised over a briefer period of time, and hence are more competent of approximate prevision. Assuming that the average ages at a valuation were 50 and 70 for assurances and annuities respectively, the general future duration of life in the former instance would be 20·11 years and in the latter 10·38 years, and the condition of the probable course of the money-market could consequently be more adequately gauged over the shorter term.

At a valuation then the records of a Company contain an extensive number of policies; effected at different ages; exhibiting varying periods of duration; involving a multiformity of risks; assuring diverse amounts; some with prior bonuses attached, and others where former bonuses have been wholly or partially commuted for cash payments or equivalent reductions of future premiums; and subject necessarily to differing rates of premium. The mass of the policies have been effected at the ordinary tabular rates for lives possessing the normal prospects of longevity, while in some instances an addition has been imposed upon the age (that is to say, an enhanced rate of premium has been charged beyond that which is naturally appropriate to the actual age) in

order to compensate some defect of family history or personal constitution or both.

The actuary has to ascertain the present sum in cash which, with the future premiums on the valuation basis, will suffice by accumulation at compound interest to discharge the claims as they successively occur.

In explaining the process of valuation, it is proposed, for brevity and simplicity, to omit the consideration of the annuities, as their estimate will proceed on similar principles; all special policies also may expediently be excluded from this explanatory survey, since some of them, endowment assurances,* and contingent contracts, for example, demand specially technical methods. The exposition is accordingly limited to ordinary whole-life assurances on single lives, which still constitute the mass of existing obligations, although their proportion to the entire business is now steadily diminishing in consequence of an unhappy change in the form of popular choice and the preferential selection of endowment assurances.

In the earlier stages of assurance history every policy was separately valued, and in some instances this course was imperative where the profits were divided in proportion to the reserves—a form of distribution which has disappeared.

It is clear that such a method of valuation would be absolutely impracticable (and would prove moreover an idle expenditure of labour and cost) in an extensive business, and hence the actuary adopts the *collective* or *classified* mode of proceeding; and actual calculation, as professional insight would itself confidently surmise, has shown that the results of individual and collective valuations of policies are practically identical. A comparative statement was published many years ago presenting the results of these isolated and classified plans of estimate in connexion with an aggregate amount assured of about £2,700,000, and the totals differed by merely ·026 per cent. of the higher reserve, or one-quarter of a farthing in the pound.

* Note: Endowment assurances in modern times have generally been valued by actuaries on various classificatory plans, where appropriate "weights," based upon the unexpired terms of assurance, have been applied. An admirable and decisive method has recently been elaborated which enables the actuary to effect the valuation of these contracts with a facility and certainty equivalent to those which attend the estimation of ordinary assurance risks.

All policies accordingly with the same *present* age of the assured at the valuation—whatever be the several amounts and varying durations they may show—are classed together under that common age; and, as a sufficient specimen of a sound basis of aggregation, all persons born between the 1st of July and the 31st of December, 1850 (for example), and those born between the 1st of January and the 30th of June, 1851, are treated as born on the 31st of December, 1850, and are assumed therefore at the valuation date of (say) the 31st of December, 1900, to exhibit the common age of 50.

It will be observed that this process reasonably assigns (at the valuation) the *nearer* age of the assured at that epoch. For example, a person born in July, 1850, completes the anniversary of birth or age 50 in July, 1900, and hence his age at the valuation on the 31st of December, 1900, is nearer 50 than 51. It is both impracticable and needless for purposes of accuracy to adopt the actual age attained by each member at the valuation, and in this plan of the nearer age the opposite deviations from the real age practically balance each other in the mass.

A somewhat subtle criticism may here be submitted (with the object of impressing upon the student the value of minute inquiry), though the practical implication is generally immaterial. Where the lives are thus collected, it may occasionally occur that a negative value (that is to say, a result where the value of the future premiums exceeds that of the sum assured) may be produced in connexion with an individual and recent policy. For example, if the assured was born in (say) July, 1850, and effected his policy in December, 1900, the premium at entry would be based on the age to be attained at the ensuing birthday, or age 51, while the valuation age, as above described, would be adopted as 50. Hence the value of the policy per £100 of assurance would be obtained from the customary formula, $100\ (A_{50}) - \pi_{51}\ (\cdot 5 + a_{50})$; or, inserting the values of the functions from the recent tables issued by the Institute of Actuaries and the Faculty of Actuaries, at 3 per cent.—A_{50} being the value at age 50 of £1 payable at death, a_{50} the value of an annuity of £1 during the continuance of life, and ·5 the adjustment hereafter explained—we produce the result of 100 (·562) − 3·892 (14·554) or − ·444, that is to say, a negative quantity of ·444.

The particular policy, accordingly, instead of having a liability

attached, would be erroneously expressed as an asset. Each actuary must judge upon the expediency of removing any such detected cases or permitting them to remain as immaterial in the mass. If the author may submit his personal view, he would adopt the former course in pursuance of a fundamental general principle of valuations that a definite liability should be assigned to every contract.

The mode of classifying "rated-up" lives or persons subject to an additional premium for personal or family defects may here be considered. An assured aged 30 and charged the premium for age 35 attains the *actual* age of 50 at a valuation. The reserve might then be computed either upon the basis of that age or upon the enhanced age of 55; and it does not appear to be material whether the one course or the other be pursued. In the latter proceeding a larger cash reserve is obviously created; but in the former a comparatively increased margin is retained unvalued in the premiums which will be available for providing for the augmented mortality. In other words, where the *actual* age is adopted, the excess between the two rates of premium is specifically appropriated to the heavier demands in the future consequent upon the expected experience of an increased ratio of deaths. The financial aspect of the question will be considered at a later stage.

Hence the policies at the assumed age of 50 are massed together, and their different amounts assured, with the existing bonuses, are summed and treated, so to speak, as an aggregate assurance by a *single* policy at that age.

The premiums to be valued must obviously be those appertaining to the several ages at *entry*—since they represent the definite and distinct risks involved—and being added together are regarded as the total premium revenue of the massed policies just described, and valued as *one* amount. Thus, assuming there exist six policies showing the common valuation age of 50 (this inadequate number being employed for the sake of simplicity only), assuring respectively the sums of £1,000, £900, £800, £700, £600, and £500, and subject to the several valuation premiums for ages at entry of 20, 25, 30, 35, 40, and 45, or £13·06, £13·716, £14·32, £14·812, £15·144, and £15·23 (on the basis of the mortality experience previously mentioned, and 3 per cent. interest), the assurances

would be valued as one policy for £4,500 with the total annual premiums of £86·282; that is to say, the formula of calculation would be $4{,}500\ (A_{50}) - 86{\cdot}282\ (\cdot5 + a_{50})$.

A corresponding process is adopted for assurances exhibiting every other common age similarly ascertained.

The validity of this course is evident; for all the lives in any group possess the same age at the valuation, and hence the same factors of the value of an absolute reversion (or single premium) at that age and of an annuity payable during the lifetime subsequent to that age apply to the aggregate amount assured and the pure premium revenue.

Adopting the present age of 50, and extracting (proportionate) figures from the Board of Trade returns of a certain Company, we find (proportionately modifying the amounts for the purpose of avoiding identification) that the total participating sums assured for the entire period of life, at that valuation age, were £218,520, with existing bonuses of £28,029, and producing annual valuation premiums of £5,275. (The pure premiums are not required to be recorded in the sixth schedule of the Government returns, but an approximate estimate is feasible by ascertaining from the figures in the fifth schedule the percentage of deduction from the contract premiums appertaining to the ordinary whole-life assurances which produces the pure premiums, and applying that percentage to the premiums specified against the particular age in the sixth schedule.)

It is ascertained from the tables (designated the O^m) that the present value of £1 payable at the death of persons aged 50 is ·56154 (presuming the rate of interest adopted to be 3 per cent.), while the present value on the same basis of £1 a year receivable during the lifetime of persons of that age is 14·054. The value consequently of the total £246,549 (sums assured and bonuses) payable at death is $\cdot56154 \times 246{,}549 = £138{,}447$; while the value of the adjusted annuity composed of the future premiums to be discharged by the assured at that age is $14{\cdot}554 \times 5275 = £76{,}772$. The former sum (£138,447) represents the value of the gross or total liability of the Company at this age: the latter sum (£76,772) expresses the value of the obligations of the assured in respect of future premiums; and the difference (£138,447 − £76,772) or £61,675, furnishes the value of the net liability entailed upon the

Company for the entire mass of its contracts existing at this particular age (regardless for the moment of reassurances). This difference is technically termed the reserve, or the provision (with the future premiums) for discharging the claims which will occur under the policies of all members possessing at the valuation the common age of 50.

The preceding reserve would require to be diminished by the equivalent reserves appertaining to the reassurance of surplus risks which the Company has effected with other Offices: the corresponding amount of these reassurances for the age of 50 consisted, in the cited example according to the returns, of £3,200 in sums assured, £830 in bonuses, and subject to pure premiums (adopting the preceding percentage) of £65 a year. Applying the same factors of valuation, we obtain the value of the sums assured and bonuses as £2,263, and of the future premiums as £946, producing therefore a reserve for the reassurances at this age of £1,317. Deducting this amount from the preceding sum of £61,675, we ascertain the value of the final net liability of the Company to be £60,358 in respect of its participating whole-life assurances for this specific age at the valuation.

(It is scarcely needful to point out that the premiums due from the assured are rightly valued as assets as if they would continue to be paid for life, although the policy-holders may abandon their obligations at any moment, since if the policies were cancelled and such assets consequently became null, equally so would become the values of the Company's liabilities.)

A similar process furnishes the reserves at all other ages at the valuation.

The values of the sums assured and bonuses and of the future pure premiums payable for each set of collective ages are then summed: to this total are added the values of the annuities granted by the Company if they do not constitute a separate fund, and the values obtained on similar principles of all assurances of a special character, such as endowment assurances, and contingent policies payable only if one life die during the lifetime of another.

From the aggregate amount are deducted the values of the several reassurances of surplus risks completed by the Company with other Offices, and we thus obtain the total resulting net value of the entire liabilities.

An excellent and simple check upon the valuation of whole-life policies is obtained by grouping them in sets of five consecutive valuation-ages, of 30 to 34 (both inclusive), 35 to 39, and so on : we thus mass the total sums assured and bonuses, contract premiums, and pure premiums for each group, and multiplying these severally by the values of the reversions and annuities for the central age of each set, the sum of the results will be found practically identical with the aggregate reserve. No diversity of result ensues even if the policies included in any group be largely divergent from each other in amount.

The adjustments already referred to, and hereafter explained, are then added—or part of them at least (for example, the modification of the constant included in the annuity-values by which the premiums are multiplied, and the correction of the values of A_x for providing for an early discharge of the claims) may be incorporated in the factors employed in the original calculations instead of being separately expressed—and this process being effected, the final net value of the Company's commitments under all its contracts is presented.

Assuming then that the Life Assurance Fund (accumulated out of past premiums with their investment at interest, and diminished by the claims and expenditure already disbursed) amounts to £6,691,570, and that the sum of the reserves and adjustments calculated as explained stands at £6,049,520, the valuation Balance-sheet discloses the subjoined position—

Dr.	£	Cr.	£
I. To the value of the net liability as expressed in the reserves .	6,049,520	By the Life Assurance Fund	6,691,570
II. To surplus .	642,050		
	6,691,570		6,691,570

The Life Fund will of course have been previously adjusted for any losses or depreciations of value in connexion with the investments and securities.

The surplus accordingly which has resulted from the operations of the Company during the preceding five years (including any undistributed balance brought forward from the last investigation, and presuming the valuation to be quinquennial) is £642,050; and this amount may be traced essentially to one or two or all the elements of a diminished mortality having been experienced compared with that anticipated, a superior rate of interest having been realized over that assumed in the previous valuation, and a reduced ratio of expenditure in comparison with the provision included in the contract premiums.

This example furnishes a brief exhibition of the mode in which a valuation is actually conducted and its result obtained. Had the balance of the valuation account appeared on the credit side —that is, had the value of the net liabilities exceeded the amount of the Assurance Fund—a deficiency would have been indicated. And since a deficiency, equally with a surplus, increases at compound interest, immediate measures would require to be adopted in order to rectify the adverse result and restore financial equilibrium. Had there been discovered an exact or practical equivalence between the debtor and creditor sides of the account—the value of the net liability coinciding with the amount of the fund—the Company would be solvent but no bonus would be distributable.

A deficiency may be compensated in different modes: the calling-up of a corresponding amount of unpaid capital in a Company possessing a proprietary capital not fully paid; the proportionate reduction of the several sums assured; a diminution of future expenditure if the deficiency prove a moderate one; or the supply of funds from some independent source, such as a separately created Investment Reserve Fund. The premiums in a Company with a proprietary capital are not competent of increase since their permanency of amount is guaranteed by the capital stock.

In a Mutual Society, where no share capital exists, the courses are open, for effecting equilibrium, of a transfer from an adequate independent Reserve Fund, if any be available; a reduction of the sums assured; or an augmentation (until solvency be restored) of the premiums payable to the limit, if they have been abated by former appropriations of bonus, of their original amounts.

It will be noticed that the plan above mentioned of balancing the deficiency, if possible, by a diminution of future expenditure is equivalent, in effect, to crediting the valuation account with the capitalised value of a part of the prospective loading (through pledging that part specifically for redemption of position), so that the expectation of future profits is entirely or partially cancelled—a portion of unanticipated loading being obviously, however, retained for satisfaction of the expenses attendant on the conduct of the business. The adoption of any of these modes of rectification presumes that the Company perceives definite hopes of continuance, under such circumstances, as an independent institution without resorting to a transfer to a more vigorous and substantial office.

The question of the proportion of surplus, so ascertained, which may expediently be distributed among the policy-holders becomes the subject of careful inquiry and sagacious judgment, involving considerations of a general character, such as the probable future course of the value of money, and of possibly a more particular nature suggested by the condition and prospects of the Company concerned. For example, an exceptional profit may have been secured during the past quinquennium by an unusual number of purchased reversions having fallen in by premature deaths, and it would tend to deplete the fortunes of the future if the entire balance between the amounts received and the accumulated purchase-monies were at once divided. An adjustment on this account would be arranged by the transfer of an appropriate portion of this surplus to (perhaps) a separate Reserve Fund as a provision for fluctuations of result in similar investments whose realization is deferred.

Certain dominant principles universally apply. In every form and department of administration an equally sedulous and equitable regard should be devoted to the interests both of the present and future time, so that the Company may be sustained as a permanent and prospering Corporation, with a happy continuity of progressive benefits, without the sacrifice of the future to the exigencies and ambitions of the present or the partial eclipse of the present in preferential exaltation of a later time.

While fully conserving the position and privileges of existing members, whose contributions have produced solvency and success,

the equitable treatment of the new entrants, whom the Company invites, should obviously receive attentive care. It is difficult, and indeed impracticable, to indicate the possible considerations or contingencies which will legitimately enter into this review of the condition shown by the valuation; but it may soundly be urged, in purely general terms, that the introduction of any exceptional course, which is not justly and imperatively warranted by facts, by the permanent interests of the entire community of membership, and by reasonable anticipations, is not accordant with the equitable benefits and expectations of the existing assured. If, for example, a rate of interest be adopted for the valuation which is out of judicious relation with financial experience and of rational probabilities—a rate considerably inferior to that which (with a sufficient margin for fluctuations) may be soundly and safely anticipated in the future from a practical business aspect—inequality of treatment may possibly be introduced. Such a rate will yield augmented cash reserves, and thus diminish the surplus disclosed, rendering the Office financially stronger, it is true—stronger, it may be, than is requisite, that is to say, in excess of the genuine necessities of the case—but, by reduction of the realized surplus, the interests of the policy-holders whose assurances are of prolonged duration may be affected unjustly, since many of them must fail to survive for participation in the ulterior advantages which a procedure of this nature will ultimately secure in consequence of the increased annual surplus of interest (between the net return actually obtained and the diminished valuation rate) upon the enhanced reserves.

It is not the purpose of this book—which is mainly descriptive, and only in parts, where criticism appears to be directly suggested by facts, commentatorial—to advocate any particular rate of interest or mortality, or any special modes of administration; and certainly in no way to suggest—rather most earnestly to discourage—any diminution of complete reserves. For the care of the actuary should be steadfastly concentrated in every department of his responsible labour upon the possibilities of the future: present popularity should never for a moment be purchased at the sacrifice of continued stability and augmenting prosperity: he builds not simply for temporary use and benefit, but for the service also of future generations; and his sedulous and

unremitting ambition should unceasingly and strenuously tend to leaving his Company, on his death or cessation of management, in the highest possible supremacy of financial vigour, compared with that which existed on his assumption of command, which his sagacity, trained knowledge, and disciplined judgment can effect. The sole contention here enforced is the necessity of basing all estimates relating to the future as approximately as is feasible upon actual facts, reasonable probabilities, and business conditions. Life assurance is a commercial enterprise, distinctive, however, by the special characteristics that its methods involved are essentially technical and exceptional; that its regards are vitally concerned with a wider horizon than that which circumscribes ordinary industrial ventures; and that the test of its adequacy of prevision is one which annually increases in intensity and rigour of pressure as its obligations become more imminent in consequence of the shortening duration of its commitments.

The premiums to be valued have been described in the preceding sections as "valuation" premiums, and this qualification was intentionally employed. And here may be briefly discussed the various modes in which these premiums may be constructed. It is in no degree necessary that the rates of mortality and interest on which the valuation premiums are founded should be identical with those adopted in their original formation: indeed it would generally be found impracticable to pursue this course. The teachings and indications of experience on this and other elements require to be constantly watched, alertly mastered, and sagaciously pondered, and the range of future provision grounded on their lessons.

The valuation premium is that annual sum which, accumulated at the assumed rate of interest and based upon the expected scale of mortality, will, regarding the assured community as a whole, suffice for payment of the several sums assured at death, supplemented by the reserves created out of previous contributions. The question of an adequate fund for administrative expenses forms a different, though equally essential, consideration.

The valuation is properly termed a pure premium estimate when the premiums capitalized as prospective assets are deduced directly from the Table of Mortality and rate of interest selected as the basis of reserves. The valuation is homogeneous and

consistent when the pure premiums and the factors of multiplication, namely, the values of the reversions for ascertaining the present values of the sums assured and bonuses, and the annuities for obtaining the corresponding values of the future premiums, are founded upon identical assumptions of mortality and interest.

A variation is occasionally introduced when the pure premium is formed at a certain rate of interest, but the multiplying factors (reversions and annuities) are determined on the supposition of a somewhat lower rate. This course will, in comparison with the incorporation of the uniform reduced rate throughout, produce a higher reserve, since a pure premium at 3 per cent., for example, is inferior to one based upon $2\frac{1}{2}$ per cent., but the valuation is thus deprived of its homogeneous character.

As emphasizing the supreme importance of minute scrutiny of facts and results before attempting any process which appears primâ facie to conduce to a stronger reserve, it may be pointed out (as a fragment of history, though the instance is probably exceptional) that with the express object of securing enhanced cash reserves a pure premium by a table expressive of a diminished death-rate may (for example) be capitalised by the (reduced) annuity-value derived from a table exhibiting a heavier measure of mortality. The Carlisle 3 per cent. pure premium at age forty-one is ·02677, that of the H^m 3 per cent. Table, ·02682 : if a valuation be formed five years thence on the H^m 3 per cent. basis, the reserve will obviously be greater if the Carlisle premium be valued by the H^m annuity at age 46 (or 15·26) than would result from the incorporation of the H^m premium. In the special instance under consideration, where the Carlisle pure premiums are valued by the annuities deduced from the Seventeen Offices' experience,* it will be found that at age 40 the Carlisle premium is inferior to that of the Seventeen Offices', while prior to that age the reverse result is observed. At a certain valuation where this combined process may be adopted the average age at entry may be 40 or slightly upwards, and hence the employment of the Carlisle premium with the Seventeen Offices' annuities would yield a heavier reserve : during the course of the sequent quinquennial period, however, the average

* This special experience is included in the illustration as its introduction into practice is nearer chronologically to the Carlisle Table than the data now in general use.

age at entry may have become changed (by an alteration in the incidence of the business) and shifted below the boundary line of age 40 just mentioned: hence the Carlisle premium will exceed that of the Seventeen Offices, and the prudential intention be defeated by the consequent production of a reserve inferior to the result of a homogeneous Seventeen Offices' valuation. The student will also notice, in this process, the anomaly of valuing the premiums computed from one Table of Mortality by annuities derived from another—in other words, the incongruity of the simultaneous application to the same persons, at an identical epoch, of differing probabilities of life.

The practice may sometimes be adopted of avoiding the direct process and obtaining the premiums valued by a deduction from the tabular premiums charged, and the resulting valuation premium may either show (1) an amount approximately equivalent with the pure premium based upon the valuation elements of interest and mortality, in which event no reason exists for the substitution of this indirect mode; or (2) a premium inferior to one obtained in the customary manner, and resulting therefore in an enhanced reserve; or (3) a premium in excess of a genuine pure premium. In the latter event, compared with a recognized pure premium valuation, a portion of the annual loading intended for expenses, fluctuations of experience, and the creation of profits, is treated as an asset against the assurance liabilities, since the difference between the tabular premium and the pure premium as defined is reduced; and the process accordingly involves an anticipation of a part of that margin as a set-off against obligations to which it is not usually designed to apply. So far as this proportion of the loading is thus diverted, to that extent is its specific utilisation annulled. The point is only mentioned for the purpose of explaining the effects of different methods of procedure, since sound reasons may exist for the adoption of the course provided the reserves produced, actual and prospective, be adequately maintained.

The mode of treatment in the valuation of what are termed "rated-up" lives should not be omitted. A person entering the Company at the age of 35 is found to possess a vitality inferior to the (approximately) normal quality appertaining to a healthy life of that age: he exhibits, we will suppose, the diminished prospects

of longevity associated naturally with the reduced physical power of persons aged 45, and is consequently admitted at the higher premium for the latter age. At the valuation ten years subsequent, the reserve may be determined in one of two modes: (1) the "rated-up" age of 45 may be adopted as the basis and the policy consequently valued on the assumption of age 55 (35 + the addition + the term of duration) and a higher cash reserve will be formed; or (2) the actual age at entry of 35 may be employed and the valuation proceed upon the age of 45 (35 + the term expired)—the excess of the tabular premium corresponding to the imposed addition being left unvalued, in the same manner as the loading for expenses and profits, as a surplus annual fund for providing the augmented mortality which is expected to prevail in the mass among persons similarly surcharged at that age.

The tabular non-participating premium for age 45 which is actually charged may be assumed to be £3·3 per cent.: the corresponding pure premiums (employing the O^m Table with 3 per cent.) are 3·046 (for age 45), and 2·116 (for age 35); and the values of the reversions and annuities to be used as factors are, at the valuation ages of 55 and 45 respectively, ·6144 and 12·239, and ·51122 and 15·782. Hence in the former mode the reserve will be ·6144 (100) — 3·046 (13·239) or 21·12; in the latter mode, ·51122 (100) — 2·116 (16·782) or 15·61—the result on the first method of treatment thus showing an excess of upwards of 35 per cent. In the former method, however, the unvalued future annual loading amounts to 3·3 — 3·046 or ·254, while the corresponding sum in the latter is 3·3 — 2·116 or 1·184—which, after allowance for expenditure and profits, is applicable to the increased incidence of mortality. Sound reasons may be adduced for either plan of valuation; and as regards the latter course it may be noted as a practical comment in its favour that in some instances, where the addition is very considerable, the age at a subsequent valuation, in the former process, may exhibit the anomaly of extending beyond the range of the Table of Mortality employed. If the method which involves the actual age be adopted, the annual surplus of premium should be specifically reserved as a fund for discharging the augmented demands from mortality, and implies the assumption that the additional mortality is practically identical from year to year—a supposition different from that on which the extra was originally

calculated. The author's preference inclines distinctly to a valuation upon the basis of the rated-up age.

We now proceed to describe the process of adjusting the results of a valuation founded upon the usual modes of procedure.

The tabular values of the reversions used in finding the present values of the sums assured and bonuses assume that the claims are payable at the termination of the year in which death occurs: practically, therefore, on the whole, at the expiry of six months after the date of death, since on the rough but convenient assumption of a uniform distribution of deaths during the course of any year (a supposition not accordant with the actual incidence of mortality)* the claims in the mass may be presumed to arise in the middle of the year. Hence the tabular factor employed in ascertaining the present value of the sums assured represents that value in defect, and this defect is emphasized by the practice of discharging the claim immediately on proof of death and approval of the legal title to the policy. For the purpose of determining the amount of the requisite adjustment, an examination should be instituted into the experience of the Company during the preceding five years. The interval which has actually elapsed between the date of death and the time of payment should be ascertained year by year in every instance, and the average interval thus discovered (for the five years) should be adopted as the basis of rectification of results. Let the average interval so determined be three months: then it will be sufficient to increase the reserves obtained in the usual manner by three months' interest (the assumed six months less the actual three months) at the valuation rate.† In

* Note: This assumption is a customary one (reducing the range of calculation within the ordinary algebraical processes), and generally its divergence from the experienced incidence of mortality introduces no appreciable error in the computations. Since the function expressive of the rate of mortality is a continuous one, varying with every small interval of time, the incongruity of the supposition is evident, and may be illustrated by reference to a Table of Mortality itself. If the deaths in the n^{th} year be 100, the hypothesis in question assigns 25 of them to each quarter of the year, independent of the number of deaths in the $(n+1)^{th}$ and following years, but if the deaths in the $(n+1)^{th}$ year be 132 or 68, the quarterly distribution of the n^{th} year would be represented in a more exact manner by the allotment to the several quarters (not of 25 to each, but) of 22, 24, 26, and 28 in the former case, and of 28, 26, 24, and 22 in the second instance, since while these series provide the deaths of the n^{th} year consistently, their continuation for four additional terms would produce precisely also the deaths of the $(n+1)^{th}$ year.

† *Vide* the appendix to this chapter.

other words, the ${}_nV_x$ for each valuation age should be augmented by the addition of three months' interest thereon. Two methods are adopted for effecting this adjustment: the values of the reversions and pure premiums may be modified for the prompter payment; or the policy may be valued as though the sum assured were payable at the end of the year of death and the resulting reserve multiplied by $(1 + i)^{\frac{1}{4}}$ if, for example, the amount be claimable three months after death. If the A_x be thus increased, the π_x is implicitly and properly augmented. In endowment assurances, however, the adjustment will only require to be applied to that portion of the reserve which refers to the involved temporary assurance.

It is to be observed that as a Company advances in age and a consequently increased opportunity is afforded for more extensive legal dealings with its policies, the average actual interval will tend to increase (unless the new business be largely augmented), and thus successively present a constant approach to identity between theory and practice on this point.

Adverting now to the premium revenue, the annuity-value employed for its capitalized estimate (a_x) assumes that the first payment becomes due at the end of each year. It is accordingly inapplicable in this form to the valuation of the premiums in mass, since they are payable at every epoch during the year: hence this factor requires adjustment for coincidence with facts. Nor of course could the expression $1 + a_x$ be adopted in the aggregate (though necessarily employed when, for purely illustrative purposes, and independent of actual conditions, the value of a single policy is presented) since this would imply that the whole premium income became simultaneously due the day after the valuation date.

If the premium revenue were practically distributed uniformly over the various months of the year the total might fairly be considered to be payable in the middle of the year, and the annuity-value consequently might be modified by half a year's purchase, or ·5. For regarding the date of valuation, we might assume that half a year's current income remained in hand, which would be represented by the ·5, and hence should be reserved; or surveying the future, we might give effect to the actual incidence of the revenue by increasing the ordinary annuity-value by the half year's income on the assumption that the aggregate payments would occur six

months thence. But in practice the premium revenue is not disposed with regularity; and the custom of many Companies, immediately prior to valuation, in admitting assurances effected in January or during part of that month with a view to their being dated back to the 31st of December (if that be the close of the valuation period) for the purpose of participating in the profits realized to that epoch, tends to render the income derived from the latter portion of the year in excess, more or less, of that proportion which becomes due in the earlier months. We may properly ascertain the requisite amount of adjustment by the following process: assume that each month's premium revenue in the year preceding the valuation falls due in the middle of that month: then the actual income for January following the valuation should be multiplied by half a month—its average distance, so to speak, from the valuation date: that becoming due in February by $1\frac{1}{2}$ months—the interval of time from the valuation epoch, and so on for the remaining months of the year; the actual "weights" being consequently assigned to the several portions of income. Summing these products, and dividing by the aggregate revenue, we obtain the average period from the date of valuation at which the total income may be assumed to be payable. Let this average term be seven months: then $1 - \frac{7}{12} = \cdot 417$ is the constant to be added to a_x in place of $\cdot 5$; and thus by producing a diminished value of the premiums than the addition of $\cdot 5$ would effect, a higher reserve is obtained. It might be noted that since our object is the exhibition of a typical complete year's result we must, where new policies were effected in the second half of the preceding year at half-yearly premiums, increase the actual income of the prior half by the addition of these half-yearly amounts, and in a similar manner for assurances subject, and completed during the last three-quarters of the year, to quarterly premiums.

The amount of this adjustment may be important. It has been known that the appropriate constant, determined by the real distribution of the premium revenue, amounted only to $\cdot 375$, so that a difference of one-eighth of a year's purchase existed between assumption ($\cdot 5$) and fact: and if the pure premium income were £200,000, the addition to the reserve on this account alone would be $\cdot 125 \times 200{,}000$, or £25,000.

In the case of assurances subject to a limited number of premiums an adjustment is also necessary; for if no provision were introduced into the valuation for expenses of management (commission, of course, will then cease) after the payment of such premiums had terminated, the absence of contribution to the Company's charges from these policies would require to be compensated by the loading of ordinary assurances and by the surplus interest realized upon these particular reserves; in other words, the future loading without this modification in the valuation would be anticipated and distributed erroneously as profit. A corresponding remark applies to the share of the loading appropriated to the creation of bonus. This additional reserve may thus be formed: let ϕ^1 be the annual loading in the actual premiums payable under assurances of this description: let x be the present age and n the term from the valuation to which the premiums are restricted: let ϕ be the loading under an ordinary policy effected at the same age of entry and with premiums continuing throughout life: then if ${}_{n}a_x$ be the annuity value on the life for the unexpired term, and a_x the value of the life annuity, the required additional reserve is obtained from the expression $\phi\,(a_x) - \phi^1\,({}_{n}a_x)$, modified in accordance with the incidence of the premium revenue. If exactitude be required, the element of commission may be first deducted from each ϕ, since no commission will be entailed on the cessation of premiums. Where the premiums upon a policy are fully paid up, either by being effected at a single premium or through subsequent commutation, the reserve of loading will consist of the annual loading (less commission) at the age at entry for an assurance completed at the whole life rate, multiplied by the value of a life annuity for the valuation age.

A reference may be useful with respect to the reserve for policies subject to what are termed "discounted bonus" premiums—that is to say, where a rate of reversionary bonus has been assumed somewhat less than the rate actually declared, and being discounted to date is applied by way of its equivalent annuity-value to diminution of the future premiums. (The premiums are sometimes otherwise abated by a cash percentage of each amount, with interest chargeable upon the balance, but this method obviously introduces no difficulty in the valuation,—the remaining portion of the premium simply constituting a loan upon the security of the policy.) Where a

rate of reversionary bonus is employed in this process, an examination of the arrangements will indicate the mode of valuation. The Company according to this scheme receiving less than the full contract premium, the value of the reduction forms a liability: on the other hand, as compensation for this permitted diminution, the Office has purchased, so to speak, the future bonuses expected to be allotted, and their present value constitutes an asset. Seeing that the amount of Bonus employed in assessing the reduction is inferior to the rate actually realized, the asset side of the account is in excess of the liability side; and hence if it were certain that the scale of bonus declared in the past (where only a portion has been absorbed in the calculated reduction) would be maintained throughout the duration of the policy, the asset side would exceed the liability side, and the total reserve for the policy would be reduced. But regarding the possible fluctuations of the future, the two opposed values should be treated as exactly compensatory; and hence we arrive at the conclusion that the reserve to be created under a "discounted bonus" policy should be precisely identical with that retained for a corresponding assurance of the ordinary type—the liability for the diminution and the value of the bonus being disregarded and the contract premium assumed to be payable. Moreover, it is to be remembered that any surplus beyond the discounted rate will be applied to the benefit of the assured.

The reserve for temporary assurances (policies effected for a definite number of years only) is sometimes assessed at one year's premium income, but frequently that for half a year only, on the assumption, in the latter proceeding, that one-half of the previous premium received remains in hand for the remainder of the unexpired year of risk. This course would not apply naturally to term policies effected, for example, in the last quarter of the former year; but looking to the comparatively insignificant proportion of this description of assurances, no comment requires to be submitted upon the merits of the two modes.

Extra premiums are imposed for the additional rate of mortality consequent upon residence abroad, with climates and conditions less adapted to longevity than those of England, and due also to the risks of hazardous professions and occupations. It is customary to reserve the extra income for

one year or half a year (the future extra premiums being of course preserved intact as a specific fund for the mortality increment). This reserve, in all probability, is sufficient under ordinary circumstances: steadily considered, however, an enlarged provision might properly be introduced. Where the risks are comparatively few, a reserve enhanced beyond the usual course is not practically requisite, but where they are numerous and severe, prudence and theory alike suggest the more stringent procedure. For the additional risk of briefer life is not restricted to the actual period of foreign residence or continuance in the occupation: of those who return home or relinquish the special pursuit, the normal longevity of some (assumed in the ordinary premiums) will be affected adversely by these hazards, and their average duration of life be correspondingly diminished. The exposure previously incurred will leave in a proportion of instances latent elements of future disease, and provision for this probable deviation from the ordinary chances of life should be retained out of the past extra payments received. We recall here again the principle that we necessarily deal with lives in the mass, and hence the validity of this consideration. In former years it was sometimes assumed, in analogy with the relation between the reserve (for assurances free from extra risks) upon the Valuation Tables then generally employed and the amount of premiums paid, that the special fund in connexion with extra premiums of that description should consist of fifty per cent. of the total sum received.

The question of negative values has already been mentioned. These occur, as has been explained, when the value of the premiums surpasses that of the sums assured: but the importance of the subject in modern systems of valuation is practically limited to the case of the winding-up of insolvent companies, when it will be discussed in its proper place. A capital principle in valuations is the absolute exclusion of all results of this kind, and the vigilance of Companies never fails to be concentrated upon the avoidance of this error. A brief theoretical reference, however, to the question may be interposed. The account representing the reserve value of a policy expresses on the debtor side the present value (as a reversion) of the obligation under the sum assured and bonuses, and on the creditor side the present value (as an annuity) of the future pure premiums to be discharged by the policy-holder.

At the origin of the contract we have $A_x - \pi_x\ (1 + a_x) = 0$: at age $x + 1$, $A_{x+1} - \pi_x\,(1 + a_{x+1})$; and since π_x is less than π_{x+1}, it is evident that a present value exists for the liability.

If, however, the contract premium be included (say $(\pi_x + \phi_x)$), we obtain $A_{x+1} - (\pi_x + \phi_x)\ (1 + a_{x+1})$, or, generally, the creditor side exceeds the debtor side, and the liability therefore is exhibited as a negative quantity: in other words, the value of the obligation of the assured in respect of premiums surpasses the value of the Company's liability as regards the sum assured and bonuses. This result is technically termed the negative value of a policy. The company accordingly on this basis would treat this fictitious item as a genuine asset, including it in the Balance-sheet among the Stock Exchange investments and mortgages, although the amount might vanish at any moment by the discontinuance of the policy; and if meantime the assured should die, no provision, but the contrary, would exist for part payment of the claim; that is to say, no accumulated reserve would be available in reduction of the stress upon the general resources of the Company, increased as they would delusively be by a non-existent asset.

A result of this character, the student will notice, would not express a null value, but would possess the meaning, in mathematics and physics, of a negative quantity. The Company would not simply fail to provide a reserve for its liability, but, on the contrary, would take credit for a fictitious sum as a real constituent of its fund. There would not be merely the *absence* of resources to satisfy the liabilities, but (unless the misrepresentation were unintentional) a mendacious overstatement of its assets.

A gross premium valuation is of course under present conditions of assurance administration impossible, but a negative value may occur, for certain ages and durations, where the premium valued is not the pure premium deduced from the data employed by the ordinary process, but the contract premium reduced by a certain percentage.

A negative value may also arise where the premium valued is derived from different data from those selected for determining the annuity and reversion factors. If, for example, the pure premium be calculated at a lower rate of interest or a higher rate of mortality than that which forms the basis of the annuity and reversion, a negative value may be produced. Thus the pure premium

at age 35 on the O^m $2\frac{1}{2}$ per cent. Table is ·02247 per unit, while the values of the reversion and annuity (at the valuation age of 37) on the same table at $3\frac{1}{2}$ per cent. are ·38832 and 17·088. Hence the reserve of ·38832 – ·02247 (18·088) is – ·018.

We turn now to the vital question of the treatment of the loading at a valuation or that portion of the tabular or contract premium which is in excess of the part providing for the risk of death and the further margin included for the formation of profits. The demands of the future for commission and expenses of management are as urgent as those relating to the payment of the claims, and require equally the sedulous attention of the actuary for their adequate provision. Taking age 35, and a participating premium of 2·45, the uniform pure premium for the risk of death is 2·116 : hence an annual margin is preserved of ·334 per cent. on the sum assured.

In a pure premium valuation, where the amount of 2·116 would be valued, the whole of this yearly provision is retained unvalued and unanticipated for the express purpose of discharging the expenses as they occur and the creation of profits : assuming that the Company's actual charges for commission and administrative costs amount to $12\frac{1}{2}$ per cent. upon the tabular premium, we obtain an annual requirement of one-eighth of 2·45 or ·306, or a margin of ·028 in excess of the anticipated needs ; the saving consequently of ·028 is accumulated at compound interest as a contribution to the general profit fund.

Economy of management accordingly and the retention of the future margin unimpaired form a significant index both of stability and prosperity.

Should a premium for valuation be adopted which is not based exclusively upon the selected rates of mortality and interest, but exceeds that deduced result, a portion of the margin, as has been stated, is included in the value of the liabilities for death, and diverted consequently from its proper and intended service as a fund for expenditure, fluctuations and profits. This process is termed the anticipation of loading.

It should be added that it is quite impossible to scrutinize and assess the efficiency of a valuation unless the present value of the loading is ascertained, so that the percentage of its capitalized amount upon the value of the tabular premiums may be compared

with the percentage of the actual annual expenditure upon the premium revenue of the Company, and thus reveal the adequacy of this particular unanticipated reserve. It is necessary to discover the amount of the provision for both the assurances and the attendant expenses and profits: on the one hand accordingly we should express the net value of the liabilities at death and the value of the actual expenditure in the future (based upon the more recent experience of the past): then against the former item we place for comparison the Assurance Fund, and opposite the second the value of the loading itself, which is retained unanticipated; the difference between the two prior items expresses the realized profit; the difference between the two latter amounts will indicate the excess available for contingencies of experience and for prospective profits. We may also discover the sufficiency or insufficiency of the unvalued loading in its relation to the real expenditure which it is required to provide, by comparing the value of the total premium income with that of the aggregate pure premiums contained in the 5th schedule appended to the Act of 1870, excluding the extra premiums; and having calculated the percentage of the difference between these two amounts (which constitutes the capitalized value of the loading) we can compare the ratio which this difference bears to the value of the contract premiums with the experienced proportion of commission and expenditure to the premium revenue during the preceding five years as presented in the consolidated account. The appropriate mode of effecting this calculation will be hereafter described.

A very practical consideration now confronts us.

If a Company value at a very reduced rate of interest, $2\frac{1}{2}$ per cent. for example, the cash reserve is obviously increased, for the value of the liability under the sums assured and bonuses is augmented by the application of a diminished ratio of accumulation: the annuity-value, though also enhanced, is increased in a less proportion, and notwithstanding the fact that the pure premium is advanced in amount, the net combined result is an addition to the reserve—presuming of course that in the compared instances the same Table of Mortality be employed. Assuming the age of 37 at entry, and the valuation age to be 47, the following are the comparative results per unit assured:—

I. $2\frac{1}{2}$ per cent. O^m elements: the reserve produced is ·58498 — ·024 (17·016), or ·1766.

II. 3 per cent. O^m elements : the corresponding reserve is ·53097 — ·02268 (16·104), or ·1657,

or an excess in the former case of nearly 6 per cent. upon the latter amount. But the point to which special attention should be directed is that a $2\frac{1}{2}$ per cent. pure premium trenches more upon the loading for expenses and other demands (including profits) than a premium constructed at 3 per cent. Thus, on a 3 per cent. valuation, and at age 37, the margin of premium retained for these requirements on a participating assurance is (about) 3 — 2·268 or ·732 ; while on a $2\frac{1}{2}$ per cent. reserve the margin is 3 — 2·40, or ·60 ; if the policy were effected on the non-participating scale at a customary premium of 2·6, the margin on the 3 per cent. assumption is ·332, and on the latter ·2 ; while at age 35, with a non-profit premium of about 2·32, where the $2\frac{1}{2}$ per cent. premium is 2·247, the margin has practically vanished. This final example of course affords an extreme illustration, and fails to represent the facts of a collective valuation : a singular specimen, however, renders the signal service of concentrating attention upon the general significance of principles ; but for perception of the aggregate effect we must obviously survey the expanse of the valuation in the mass. It is evident, however, that throughout the valuation the annual provision for expenditure and other exigencies is reduced by a $2\frac{1}{2}$ per cent. basis compared with the adoption of 3 per cent. These future requirements, whose incidence is certain, must be discharged, and hence a portion of the surplus interest at which the reserve and premiums are accumulated (if $3\frac{3}{4}$ per cent. net be realized, a part of the difference of $3\frac{3}{4}$ and $2\frac{1}{2}$, or $1\frac{1}{4}$ per cent.) cannot be regarded as profit, but must necessarily be utilized as a supplemental fund towards defraying inevitable costs and requirements. This difficulty may be surmounted and the integrity of a valuation at a diminished rate of interest preserved by adding to the actual reserve a sum which shall represent approximately the capitalized value of the annual amount of margin thus anticipated and absorbed as compared with the results of a 3 per cent. basis. But evidently this course modifies the character of what is usually recognized as a pure premium system. And thus, where some compensatory element to this effect be not provided, we cannot exalt a $2\frac{1}{2}$ per cent. valuation by stating that the policy-holders possess the benefit (if $3\frac{3}{4}$ per cent., for example, be the general rate of interest obtained)

of $1\frac{1}{4}$ per cent. per annum excess interest for accumulation into bonus, since it has been proved that a proportionate part must be diverted to the necessary costs of management and other permanent demands in the future.

It will further be remembered that a system of valuation described in a previous section, where 3 per cent. pure premiums were capitalized by means of $2\frac{1}{2}$ per cent. annuities, obviates this difficulty, though the valuation is thereby rendered of a composite nature instead of exhibiting a homogeneous structure in every part. For, however expedient for practical and rectifying purposes, it is incongruous at one and the same time to employ premiums formed on the assumption of accumulation at 3 per cent., and annuities (or capitalizing factors)—of the same nature as premiums—based upon the supposition of accumulation at $2\frac{1}{2}$ per cent.

It should be observed as a partial qualification to this statement that, after a lapse of time, dependent upon the premiums charged, the respective pure premiums, and the net rates of interest realized, the Company valuing at the lower basis of interest, will ultimately produce superior results in profits. Let a Company value at 3 per cent., and another at $2\frac{1}{2}$, both securing $3\frac{3}{4}$ upon the funds; and let the respective pure premiums be π^1 and π, and the values of policies V^1 and V. In the second Company the diminution of loading is $(\pi-\pi^1)$ (1·0375)—(the π being the higher of the two, the loading is encroached upon to the extent of the difference of the two premiums),—and the gain from interest will be $(V + \pi)$ (·0375 — ·025) (the V and π being greater than the corresponding elements in the 3 per cent. valuation, produce an augmented amount of interest):— the balance of gain or diminution as it may occur will be $(V + \pi)$ (·0375 — ·025) $- (\pi - \pi^1)$ (1·0375) $= V$ (·0375 — ·025) $- \pi$ (1·025) + π^1 (1·0375). In the first Company, valuing at 3 per cent., no reduction of loading will arise, and the benefit from interest will be $(V^1 + \pi^1)$ (·0375 — ·03). The results exhibited by the two Companies will prove equal when these expressions coincide in value, that is, when V (·0375 — ·025) $- \pi$ (1·025) $+ \pi^1$ (1·0375) $= V^1$ (·0375 — ·03) $+ \pi^1$ (·0375 — ·03) or when

$$V\,(\cdot0375 - \cdot025) - V^1\,(\cdot0375 - \cdot03) = \pi\,(1\cdot025) - \pi^1\,(1\cdot03)$$

By simple trials of interpolation the epoch of equivalence can be obtained from a table of policy values. At the age of 35 at entry and assuming rates of interest of 4 per cent. (realized), and for

valuation 3 and $2\frac{1}{2}$ per cent., this date is found to occur at the end of twelve to thirteen years when the superior excess of interest obtained by the second Company will exceed the reduction of loading produced by the higher π.

It may be suggested that, under existing and probable circumstances of the scope of investments appropriate to Life Offices, and in harmony with genuine business principles, a valuation rate of 3 per cent. is undoubtedly sound and practically reasonable, while an inferior rate does not appear to be consonant with a just appreciation of facts. A proportion of the surplus would be reserved for general precautionary purposes—for example, for possible fluctuations of experience and losses on securities—and could include a sum even to the extent, if desired, of the difference between the 3 per cent. and $2\frac{1}{2}$ per cent. estimates. And it is submitted that this plan of procedure, in place of basing the valuation directly upon $2\frac{1}{2}$ per cent., is more adjusted to commercial conceptions, and expresses a truer recognition of facts.

For the purpose of completing this portion of the subject in an elementary form, the effect of valuing by what we may roughly term an average Table of Mortality when an appreciable proportion of the business consists of new assurances may be briefly considered.

By an average table is intended one which is constructed upon the combined experience of lives recently admitted and of others assured for varying terms of duration, so that the functions at any age apply to fresh entrants at that age and to lives previously accepted at younger ages who have survived. It is an established fact of experience that assured lives during at least the first five years from entry exhibit a rate of mortality inferior to that prevailing among persons of corresponding ages who have been assured for more extended periods. This is the consequence, as has been explained, of their recent selection by the application of medical and other tests for ascertaining their possession of normal vigour and tenacity of constitution. During the earlier years of assurance consequently a larger number will survive than would happen in a general community, and hence a heavier incidence of claims must be provided for in the amount reserved. But a General or Mixed Table, from the nature of its formation, comprises, according to the length of the continuance of their policies, a proportion of deteriorated lives, compared with the vitality of the new and recent

admissions. Hence that table implies an enhanced rate of mortality, a greater number of deaths, than the Select Table appropriate to these fresh members, with the resulting assumption that a reduced extent of liabilities will require provision than that which will actually accrue in consequence of a larger proportion of these recent assured surviving. If then a rate of mortality of this composite character be applied to newly-chosen lives, the effect will be apparent in a reserve diminished below the necessary and appropriate measure. The student's attention is rightly directed to the consideration of this point, although presumably in British Companies the ratio of new business to the existing mass is not sufficiently pronounced to affect substantially the final results. But where a Company has applied at its first valuation (when the entire body of the assured exhibited a select character) a general table in place of a specially adapted measure of mortality which included this select element (or, in default of such a table, has failed to recognize the true condition which exists by creating a reserve for suspended mortality) the subsequent valuation has presented a diminished ratio of profit by reason of the occurrence during the preceding period of the deferred mortality for which adequate provision had not been previously formed. It may be added that the reduced mortality of newly-admitted lives has sometimes been described as "suspended" or deferred mortality, or mortality which, expressive of the force of selection, is held over or postponed for a term and will occur when the period of selective benefit has been practically exhausted.

The effect upon reserves of an alteration in the Table of Mortality, or of the rate of interest, employed, forms an interesting inquiry, and may be briefly examined.

An increase in the rate of interest adopted produces a reduction in the policy values provided a_x, or the annuity-value at any age, decreases with the advance in age. The student will remember that the reserve may be expressed in the form of $1 - \frac{1+a_{x+1}}{1+a_x}$ or $1 - \frac{a_x}{vp_x\ (1+a_x)}$, where p_x is the probability of living a year. Now it is merely an exceptional circumstance that the annuity-value will be augmented with the age. This result only occurs at

infantile ages or at very advanced ages of some imperfectly graduated tables. Hence it may be generally affirmed that for all cases which enter into cognisance in the valuations of Assurance Companies, reserves are decreased with an augmentation in the rate of interest employed, and increased with the diminution of the rate, The preceding deduction naturally assumes that the same Table of Mortality is adopted in the instances compared, for a different scale of mortality might neutralize the effect of the modified rate of interest.

The special case of a_x being inferior to the annuity-value at one or higher ages may be noted. From a practical aspect the question is immaterial, but it will prove of interest to present a demonstration which has been furnished in connexion with two consecutive ages. If a_x be $< a_{x+1}$, π_x (the pure premium at age x) is $> \pi_{x+1}$, for $\pi_x = \frac{1}{1+a_x} - d$, and $\pi_{x+1} = \frac{1}{1+a_{x+1}} - d$, and consequently on the assumption that the annuity factor in the denominator of the second fraction is the greater, π_{x+1} is less than π_x. Now $a_x = vp_x (1 + a_{x+1})$, where p_x is the probability of existing for a year at age x: for the value *now* of an annuity to be entered upon if the person, aged x, survive a year is clearly the present value *then* of the annuity (or $1 + a_{x+1}$) multiplied by the chance of receiving it by survivance and discounted for the intervening period. Hence a_x being supposed to be $< a_{x+1}$, $vp_x (1 + a_{x+1})$, (that is, a_x) must be less than a_{x+1}; and consequently p_x (or the chance of living a year) is less than $\frac{a_{x+1}}{v(1+a_{x+1})}$ or $\left(\text{since } v = \frac{1}{1+i}\right) p_x$ is less than $\frac{(1+i)(a_{x+1})}{1+a_{x+1}}$: q_x or the probability of dying in a year is $1 - p_x$ (for $p_x + q_x = 1$) and therefore there results $1 - q_x$ less than $\frac{(1+i)(a_{x+1})}{1+a_{x+1}}$ or transposing, $1 - \frac{(1+i)(a_{x+1})}{1+a_{x+1}}$ is less than q_x; that is, q_x is greater than $1 - \frac{(1+i)(a_{x+1})}{1+a_{x+1}}$; that is, q_x is greater than $\frac{1 - i\,a_{x+1}}{1+a_{x+1}}$. Now A_{x+1} (or the value of £1 at death) $= 1 - d(1+a_{x+1}) = 1 - \frac{i(1+a_{x+1})}{1+i}$

$= \frac{1 - i\,a_{x+1}}{1+i}$ $\therefore A_{x+1}(1+i) = 1 - i\,a_{x+1}$. Substituting this expression in the numerator of the preceding fraction, we obtain $q_x > \frac{(1+i)A_{x+1}}{1+a_{x+1}}$; multiplying both sides of the inequality by $\frac{1}{1+i}$ or v, $vq_x > \pi_{x+1}$, since $\frac{A_{x+1}}{1+a_{x+1}} = \pi_{x+1}$: vq_x is the premium at age x for an assurance for one year, and hence this expression shows that if the premium for an assurance for a single year at age x be greater than the whole-life annual rate at the next succeeding age, a_x will be less than a_{x+1}.

The effect upon policy values produced by a change in the Table of Mortality selected must be determined by an examination of the annuity-values furnished by the tables compared, and depends upon the incidence of the rate—the age or ages at which the heavier or lighter stress exists. If a_x be the value of an annuity by one table and $a^1{}_x$ that deduced from another, the reserves by the latter table are $> = <$ those resulting from the other according as $\frac{1+a^1{}_x}{1+a_x}$ is $> = < \frac{1+a^1{}_{x+n}}{1+a_{x+n}}$. If then the value of the ratio $\frac{1+a^1{}_x}{1+a_x}$ be calculated from the two tables for all values of x, the results will at once indicate the ages at entry and the durations of assurance for which the reserves by the one table exceed, equal, or prove deficient to, those derived from the other.

It does not follow that a Table of Mortality presenting an increased rate in comparison with another will produce augmented reserves. The comparative effect depends, as has been stated, upon the mode in which the mortality is incident. A very ingenious plan has been devised for exhibiting this conclusion. The value of a policy, ${}_nV_x$, effected at age x, and continuing in existence for n years, is $1 - \frac{1+a_{x+n}}{1+a_x}$. This may be expressed in the equivalent form of $1 - \frac{1+a_{x+n}}{1+a_{x:\overline{n-1}|} + v^n{}_np_x\,(1+a_{x+n})}$, where the total value of the annuity at age x is decomposed into its constituent values for n years and for the entire duration of life subsequent to that date: the value at the deferred age of $x+n$ being

necessarily reduced to the present epoch by discount at interest (v^n) and multiplication by the probability of the life surviving to enter upon possession after the expiration of the prior term. The expression may be again presented in the form of

$$1 - \frac{1}{\frac{1 + a_{x:\overline{n-1}|}}{1 + a_{x+n}} + v^n {}_np_x}.$$

Here the factors $1 + a_{x:\overline{n-1}|}$ and $v^n {}_np_x$ involve the rate of mortality between the ages of x and $x+n$ only; while the element $1 + a_{x+n}$ in the denominator includes the mortality only for ages exceeding $x + n$. If then the rate of mortality prior to the advent of age $x + n$ be increased, and the rate experienced after that age continue unaltered, the quantities $1 + a_{x:\overline{n-1}|}$ and $v^n {}_np_x$ are diminished, and the other, $1 + a_{x+n}$, remains unchanged: consequently the preceding total fraction will be augmented, and therefore the difference between its value and unity, or the policy value, will be reduced. An increase in the rate of mortality over this defined period alone accordingly diminishes the reserve.

If, on the other hand, the rate of mortality between the ages x and $x + n$ be unaltered while that prevailing beyond the latter age be augmented, the value of $1 + a_{x+n}$ will be reduced, while that of the other functions remains unaffected. The fraction therefore of $\frac{1 + a_{x:\overline{n-1}|}}{1 + a_{x+n}}$ will be increased (the value of a_{x+n} being diminished by reason of a higher rate of mortality operating), and consequently the total fraction will be reduced, so that the resulting policy-value will be enhanced. In this mode of comparative incidence of the increased mortality, the reserve accordingly becomes stronger. Hence a rise in the rate of mortality will produce enlarged or decreased policy-values or reserves in accordance with the mode in which it occurs. We thus perceive that if a table exhibit an augmented rate of mortality throughout its entire extent the effect upon reserves will vary according to the direction of the higher measure: if the increase be proportionately greater at the younger ages, the policy-values will be reduced; if the proportionate excess occur at the older ages, the values will be enhanced.

The subject is worthy of extended remark. The rate of

mortality (that is, the chance of death at any age, x) is represented by a fraction whose numerator consists of the number dying between the ages of x and $x + 1$, and its denominator the number living at the former age. Thus, at age 35, the probability of death in a year is, in the Northampton Table, ·0187, or nearly 19 in the 1,000; in the Carlisle ·01026; in the H^m experience, ·00877; and in the O^m Table, ·00738, so that the highest rate at that age is furnished by the Northampton Table, and the lowest by the O^m. At the age of 75, the Northampton exhibits ·09615, and the H^m ·09836, or the relation is reversed and the chance of death according to the latter experience exceeds the rate expressed by the Northampton. A rate of mortality is heavier throughout the entire duration of life than that presented by another table when, out of equal numbers living, the extinction of the whole body under observation occurs the more speedily. If the ages at entry be adopted of 20, 30, 40 and 50, and the durations of the policies be assumed respectively to be 10, 20, 30 and 40 years (that is, in each instance up to a duration expressed by age 60) the employment of the Northampton Table furnishes reduced policy-values compared with a table exhibiting an inferior rate of mortality like the Carlisle; but the general inference hastily accepted from this survey that a table which involves the heavier mortality would yield diminished reserves is confuted by another calculation where the ages at entry are 20 (with one year's duration), 30, with durations of 1, 2, 3, 4 and 5 years; and age 40 with the several durations of 1 to 8 years. In this latter comparison the Northampton values are throughout in excess of those based upon the Carlisle; and the only possible conclusion accordingly is that where the mortality is higher, the resulting values of policies may be greater or less than those deduced from a table expressing a more favourable rate, or an equivalence between them may exist. The student will here observe the imperative need of testing every theoretical or partial conclusion by numerical experiments and width of survey. If the mortality be increased, the practical result is that the A_{x+n} (or the present value of the sum assured at the valuation age) will be augmented; the annuity value a_{x+n} (by which the premiums are valued) will be diminished; but on the other hand the π_x (in the valuation formula), or the pure premium for the age at entry, will also be

enhanced; and the difficulty of interpretation, by mere inspection, lies in the estimate of the relative measure of change of these constituent elements. The effect of a heavier or lighter mortality upon reserves is consequently, it will be noticed, dependent upon the incidence of the stress at the several ages,—a rapid increase in the *rate* of mortality, and not a mere heavy *mortality*, furnishing the higher policy-values. If we assume, as has been already stated, that below a given age, y, one table presents an increased mortality compared with another, but that for age y and higher ages the two tables coincide in rate, then when the ages x and $x + n$ (the ages at admission and valuation) lie on opposite sides of y, the value of the policy will be less under the table which shows the greater mortality. Thus let x be under y, and $x + n$ beyond y, in the formula $A_{x+n} - \pi_x (1 + a_{x+n})$, the values of A_{x+n} and a_{x+n} will be identical on this supposition in the two tables, and accordingly the reserve will be augmented for the table where π_x is the lower. If the value of the ratio $\frac{1 + a^1_x}{1 + a_x}$, which we have already mentioned, be constant throughout the range of the compared tables, equal policy values would be produced for every age at entry and every duration of assurance. Where therefore a comparison is instituted between the values of policies deduced from different tables it must not be generally expected that the results derived from the one are either greater or less throughout the table than those obtained from another; but that for certain intervals the values furnished by one will prove in excess, while at certain other intervals they will happen to be lower. The conclusion has been expressed that if two tables exhibit the same rate of mortality at the young ages and, at higher ages, an increasing difference in the rate, the table which presents the higher rate will afford larger values; if the same rate of mortality be shown by two tables at the higher ages, with an increasing difference in the rate as the tables proceed to younger ages, the table which expresses the lower mortality at the younger ages will require augmented policy values; and if two tables exhibit the same rate of mortality at the middle ages, say about 50, but at younger ages the one gives a higher mortality, and at advanced ages an inferior rate, than those furnished by the other, the former table will involve the lower values. Hence policy values do not depend upon the absolute rate of mortality

represented by a table but upon the progression which the rate involves. The table in which the mortality increases the more rapidly will produce the larger reserves.

The importance is so pervading of the student entertaining distinct notions upon the subject that a brief repetition in another form must be pardoned.

The rate of mortality, we have explained, expresses the chance of dying year by year: this rate increases from age to age* as measured by the ratio of the number of deaths in any year to the number of persons living at the commencement of that year: but the acceleration of this progression (the rate of the rate) may proceed more rapidly at one stage or stages of life than at another or others. If, for example, year by year at the younger ages the rate increases in a higher annual proportion compared with the velocity at the older ages (the first geometrical or quasi-geometrical progression possessing an augmented celerity over the second),—as contrasted with a Table of Mortality presenting a more regular continuous form,—the annuity-values at the former ages become proportionately more reduced (by reason of the larger proportionate diminution of the survivors as recipients of the annuities) than those at the advanced ages. Hence, employing the relevant formula for the policy-value, or $1 - \frac{1+a_{x+n}}{1+a_x}$, the a_{x+n}, on the preceding supposition, will exhibit a proportionately higher value than the a_x (in consequence of the greater proportionate number of successive survivors or payments), so that the fraction is the more increased in value and the Reserve or policy-value accordingly becomes the more diminished relatively to that furnished by a Table displaying a more uniform progression.

A most interesting investigation has been made into the effect upon premiums and reserves of a total absence of interest for the accumulation of the funds. The result proves that the system of life assurance could still be pursued even though no interest were earned. The consequence of this condition upon the premium is the curious one that a uniform addition of £1 per £100 assured to the current rates at all ages (involving interest at 3 per cent.)

* On a former page a divergence has been pointed out at a specific period in respect of male lives.

would produce a premium which, without accumulation at interest, would suffice to provide the sum assured at death. The student will be interested in considering the mode in which the calculations are performed. The value of an annuity at age x consists of the series of $l_{x+1}\ v + l_{x+2}\ v^2 + \ldots$ continued to the close of life and divided by l_x. Now, if money yield no interest, v (or the present value of £1 due at the end of one year) is obviously 1 : ($v = \frac{1}{1+i}$, and making $i = o$, we obtain unity); and the preceding expression consequently reduces to $\frac{l_{x+1} + l_{x+2} + \ldots}{l_x}$, which represents e_x, or the curtate expectation of life at age x. If we adopt as an example the age of 35, the H^m 3 per cent. pure premium is 2·193 per cent.; and to obtain the corresponding premium where interest does not exist we have $\pi_x = \frac{1}{1+a_x} - d$. With null interest (to borrow a term from the department of dynamics) d vanishes (for $d = 1 - v$, and v having been shown to be unity, d becomes o); the a_x factor is reduced to e_x at age 35, which on the H^m Table is 30·52 : hence the annual premium (with accumulation at interest cancelled) is $\frac{1}{31{\cdot}52}$, or 3·173 per cent., differing from the customary pure premium by ·98. And as we have stated, the difference at all other ages is practically constant and amounts to 1 per cent. upon the sum assured.

The effect of the abrogation of interest upon reserves is much more serious. The reserve for age 40 at admission and after 5 years' duration is, according to the H^m 3 per cent. experience, 8·708. To obtain the value of the reserve (when interest is not included) we have $1 - d\ (1 + a_{45}) - \pi_{40}\ (1 + a_{45})$. Now as d vanishes, this reduces to $1 - \pi_{40}\ (1 + e_{45})$; deducing the values of π_{40} and a_{45} as before (where a_{45} assumes the form of the expectation of life at that age), we obtain 100 — 3·584 (24·29)—the sum assured being assumed to be £100—or nearly £13 as compared with the ordinary result of £8·708. It will be observed that the expenses of administering the business would require to be separately contributed by the members.

The relation of the actual premiums received to the necessary reserves may be exemplified in the following specimen for a

specific age at entry: the annual pure premium for age 30 (H^m 0 per cent.) is 2·843 (compared with the H^m 3 per cent. rate of 1·88), and the Reserves at null interest on the termination of 5 years and 30 years respectively would be 10·4 and 59·17; while at these dates the premiums paid (without accumulation) would amount to 14·215 (against the reserve of 10·4) and 85·29 in relation to the required reserve of 59·17—the value of the reserves being determined by the inclusion of mortality only. On the expiry of the preceding terms, the reserves (involving interest at 3 per cent.) would amount to £6·135 and 46·156 respectively, while the premiums of 1·88 received (omitting interest for equality of comparison) would be 9·4 and 56·4. Referring to the longer period, the premiums discharged amount to upwards of 144 per cent. of the reserve (where interest is absent in its construction), while in the ordinary form the ratio is 122 per cent.

In the detailed illustration of the process of valuation which has been furnished, we have exemplified the pure premium method, and its practical operation may be viewed under two aspects,—one termed the Retrospective, because the reserve is erected upon the basis of the actual receipts and payments in the past, with their accumulation at interest to the date of valuation; that is to say, the plan concentrates its survey upon the past; the other is named the Prospective, where the formation of its estimates is founded upon a contemplation of the prospects of the future. Our complete description of the usual method of valuation illustrated the Prospective course. The results of each plan are obviously identical, for the present, as the basis of the future, is constructed from the past. Regard the comparison mathematically: in the Retrospective scheme the accumulated premiums already received diminished by the claims already discharged (omitting the consideration of the loading) leave a certain balance, which constitutes the reserve for the remaining policies; and since the original premiums (which provide both the claims and reserves) have been formed upon the entire future experience of life, their residue on the expiry of any portion of that experience must clearly prove sufficient to meet the requirements of the remainder of the experience if the anticipations which they involve be coincident with the realized facts.

If the assured entered at age x, and the policy at the valuation

have existed for n years, the accumulated amount of the premium π_x (employing the columnar notation) is $\pi_x \frac{(N_{x-1}-N_{x+n-1})}{D_{x+n}}$, while the similarly accumulated sums disbursed in claims are $\frac{M_x - M_{x+n}}{D_{x+n}}$, and the difference of the debtor and creditor sides of the account, or $\frac{\pi_x (N_{x-1}-N_{x+n-1})-(M_x - M_{x+n})}{D_{x+n}}$ represents the fund remaining, or reserve, furnished by the Retrospective method of investigation. Now $\frac{M_x}{N_{x-1}} = \pi_x$, and therefore $M_x = \pi_x \ N_{x-1}$; substituting this value in the numerator, we obtain

$$\frac{M_x - \pi_x (N_{x+n-1}) - M_x + M_{x+n}}{D_{x+n}}$$

$$= \frac{M_{x+n} - \pi_x (N_{x+n-1})}{D_{x+n}}.$$

But $\frac{M_{x+n}}{D_{x+n}} = A_{x+n}$; and $\frac{N_{x+n-1}}{D_{x+n}} = 1 + a_{x+n}$; consequently we arrive at the expression $A_{x+n} - \pi_x (1 + a_{x+n})$, the reserve obtained by the Prospective method, showing that the transformed formula for the reserve produced by the results of past experience is identical with the formula employed in the Prospective system, or that which deducts the estimated value of the future premiums from the present value of the future obligations under the Assurances.

This necessary coincidence of the two plans of deducing the reserves affords so excellent an illustration of the consistency of the principles of assurance, and the accuracy of actuarial methods, that a numerical example will prove serviceable.

Retrospective Method.

Let 88,995 persons be admitted into a Company at the age of 35,—being the number represented as living at that age in the O^m Table of observations of assured lives; the pure annual premium at 35 is £2·116 per £100; and assume that each member is assured for that amount, and that the realized rate of interest is 3 per cent. per annum on which the premium is constructed—

I. At the origin, the Company receives premiums of 88,995 × 2·116, or £188,313 . . . . . .

II. This sum accumulated for one year at 3 per cent. amounts at the end of the year to £193,963

III. The Table of Mortality shows that during the course of this year 657 persons die in accordance with its implicit mode of construction: the claims, therefore, for which the Company is required to provide at the close of the year amount to 657 × £100, or £65,700

Leaving the fund at £128,263

IV. The balance of the fund of £128,263 at the expiry of the first year earns interest at 3 per cent. during this second year to the extent of £3,848

V. Premiums are also received at the beginning of this year from the 88,338 (88,995 − 657) survivors, amounting together to £88,338 × 2·116 or £186,923, and by the end of the year are augmented by interest to £192,531

£324,642

VI. 681 persons die during that year, and consequently the claims discharged at its close absorb £68,100

So that the balance of the fund amounts to £256,542

VII. By the end of the third year the fund of £256,542 secures in interest £7,696

VIII. Premiums are also paid by the surviving members, 88,338 − 681, or 87,657 × 2·116, and their accumulated amount during this year is £191,048

£455,286

IX. 705 deaths occur at the increased age, and the Company accordingly provides the payment of claims to the extent of £70,500

Leaving the fund at the termination of the third year at £384,786

This balance of £384,786 forms the reserve, on this method, required to meet the future liabilities on the assurances of (87,657 − 705 =) 86,952 surviving members.

Prospective Method.

There exist 86,952 remaining assured: the common age of the body is now 38: the Prospective formula is $A_{38} - \pi_{35}\ (1 + a_{38})$, assuming, as is the case, that at the date of valuation a year's revenue is immediately due. Hence we obtain for the requisite reserve (on the O^m 3 per cent. basis) 100 $[A_{38} - \cdot 02116\ (1 + a_{38})]$: substituting the values we get 8,695,200 $[\cdot 44646 - \cdot 02116\ (19 \cdot 004)]$, which gives a result of £385,545 as the reserve, and this would have exactly coincided with that produced on the retrospective plan by an accumulation of the receipts less the accumulated claims which have occurred in the past had we (with the extensive numbers involved) troubled to include precise account of the rejected decimal places.

Although the subject now possesses merely a historical interest, the student should be acquainted with the leading feature of the reassurance or hypothetical method of valuation.

In a pure premium process the factors for multiplying the sums assured and premiums (that is, the values of the reversions and annuities employed) are deducible from the pure premiums themselves; that is to say, the three elements constitute a homogeneous triad consistently based upon the data; thus $\frac{1}{d + \pi_x} - 1 = a_x$; and since $A_x = \pi_x\ (1 + a_x)$, and $\frac{1}{d + \pi_x} = 1 + a_x$, therefore $A_x = \frac{\pi_x}{d + \pi_x}$.

On the hypothetical plan, the values of the reversions and annuities are derived, however, from the contract or loaded premiums actually charged.

The value of the annuity in this mode being $\frac{1}{d + p_x} - 1$ (where p_x is the entire premium), it is evident that since p_x is greater than the pure premium π_x, the fraction is diminished, and hence the annuity-value for multiplying the premiums is reduced. The value of the reversionary factor is $1 - d(1 + a_x)$, and since a_x has been shown to be diminished when derived from the Office premiums, the multiplying factor applicable to the sum assured is enhanced. Thus the premium valued (p_x) exceeds the pure premium; the multiplying annuity is inferior to that deduced by the ordinary process; but on the other hand the value of the reversion is increased. An

examination of the relative financial effects of these differences alone will indicate the comparative amounts of the resulting reserves of the two methods.

This mode of valuation is designated hypothetical in consequence of the valuation factors, in place of a direct derivation from the rates of mortality and interest adopted, being deduced from the contract premiums, so that they possess a hypothetical character dependent (partly) upon the arbitrary (or partially arbitrary) element of loading for expenses and profits; these factors accordingly exhibit the nature of assumptions instead of consisting of facts displayed by the actual mortality and interest selected; the loading, which constitutes a portion of their basis, evidently resting, to an extent, upon the judgment and experience of each actuary.

It is unnecessary to discuss in detail the conditions under which the values of these functions agree with or diverge from those obtained upon the pure premium plan,—the differences of result varying with the mode in which the premiums are loaded for practical use. It will be sufficient to indicate briefly the circumstances under which the reserves by the hypothetical method and the pure premium process coincide. This equivalence will occur if (assuming π_x to be the pure premium) the loading of ϕ_x is $k\ (\pi_x + d)$, that is to say, where, at every age, the pure premium for that age is increased by d (the discount on £1 for a year at the valuation rate) and the loading is a constant percentage of that sum or, dissected, a uniform percentage upon the pure premium and a constant quantity which bears the same ratio to d. In this event the values of policies for every age at entry and every duration of assurance will be identical. Assume that a policy-holder enters the Company at the age of 35; the pure premium (H^m 3 per cent) is ·02193; d is ·02913; $\pi_x + d$ (as the foundation of the loading in question) is consequently ·05106, and if k, the constant percentage, be 20, the loading or $\phi = ·01021$; hence we obtain for the contract premium at age 35 the sum of ·02193 + ·01021, or ·03214. Now the mathematical form of the expression for the reserve on the hypothetical plan is identical with that applicable to the pure premium system; and adopting a convenient equivalent form, we have, as the formula for the Reserve, $\frac{P_{x+n} - P_x}{P_{x+n} + d}$, where P on the hypothetical basis is the premium charged to the assured. Pursuing the preceding method

for obtaining the contract premium at age 40 (assuming that the valuation occurs after five years), we have $\pi_{40} = \cdot 02589$, and adding d, we find the loading of 20 per cent. of the sum to be ·01101 : hence P_{40} is ·0369. Our expression thus becomes $\frac{\cdot 0369 - \cdot 03214}{\cdot 0369 + \cdot 02913}$, or ·072. The pure premium reserve, it will be found on calculation, is precisely coincident.

The term "reinsurance" is also applied to the hypothetical method on the ground that the reserve (founded on the contract premiums charged) consists of the sum at which a transfer or reassurance of the entire liabilities to another Company could be effected whose rates of premium and interest were practically equivalent with those of the original Office. Thus : on the expiry of n years after admission at age x, the reserve, employing the tabular premium, is $A_{x+n} - P_x(1 + a_{x+n})$, where A and a are deduced from the premiums charged, and if this sum be offered in compensation to another Company, whose rates were constructed on similar data, the accepting Office would actually receive the $A_{x+n} - P_x(1 + a_{x+n})$, together with the capitalized value of the future full premiums of $P_x(1 + a_{x+n})$,—the two combined forming A_{x+n}, or the single contract premium for the age which the lives had attained at the date of transfer, including thus both the pure premium for the risk of death and the original loading imposed.

The method of valuation where the whole of the elements included are directly derived from the same data of mortality and interest is usually adopted, and involves the distinctive merit of retaining unanticipated for future demands the entire difference between the premium for the risk of death and the premium charged. Its advocates do not contend for its adaptation to every form of circumstances ; it possesses no magical properties ; it indicates no inherent fact of Nature as disclosed in the probabilities of life ; a somewhat lower pure premium might, for example, be theoretically assumed for the first year of assurance (provided the amount were adequate to cover the current risk) and a permanently higher premium for the remainder of life—the present value of the entire series equalizing with the value of the uniform pure premium amount ; but the systematic adoption of a process based homogeneously upon the pure premium foundation is most admirably and precisely adjusted to the condition of a Company pursuing a steady and prosperous

career, and should constitute the standard ideal of valuation work. The system, again, would be inappropriate in the transfer of a Company's obligations to another Office, since, in that instance, the reserve produced being quite independent of the premiums charged, a transferred Company with a comparatively small margin included in its premiums would be purchased upon the same basis as a Company with actual rates of a superior amount, which would evidently be incongruous. The relative contract premiums, by the comparative extent of their loadings, exhibit a more extensive or restricted annual fund presenting (after discharging expenses) a genuine index to the probable range of future profits. In the case, again, of a reduction of contracts compulsory upon a deficiency of assets compared with the estimate of the liabilities, the pure premium method would be objectionable. The scope of contention, however, against the pure premium system has been extended, and a special modification of the method has been suggested where a Company is securing its business at a considerable proportionate and lavish cost. It is obvious that on the pure premium plan the assumption is implied that the total expenses applicable to the policies are distributed with practical evenness over their entire duration, since the whole of the loading intended to provide for them is retained unvalued as a prospective annual fund of uniform amount. It is undoubtedly the fact that the largest proportionate charge in procuring new business occurs at the completion of the assurance; and for the purpose of giving practical effect to this unequal incidence of the aggregate expenditure attendant upon the duration of the policies, and with a view also to distributing the profits, so far as may prove feasible, in an equalized form throughout the continuance of the assurances, a modification of the reserve upon a homogeneous pure premium basis has been proposed. Without introducing the mathematical processes (which are of an interesting character) by which this special mode of adjustment is obtained, it is sufficient to state that the proposition involves (1) a reserve for policies in the early years of their existence adequate simply to provide for the current unexpired risk of death, and (2) a reserve for all policies of longer duration constructed upon the assumption that they were accepted by the Company at an age in excess of their actual age. Thus, presuming that the costs incurred in the acquisition of new business have amounted to such a sum

that the current claims on that business and its attendant expenses have entirely absorbed the whole of the new premiums for the year of entry, the total suggested reserve should be formed by (1) retaining an adequate amount in hand to provide for the unexpired risk of the year from entry in respect of assurances in their first year, and (2) valuing all the remaining policies at their next higher age—that is, if the age at admission were x, employing the pure premium of π_{x+1}. Similarly, if the preceding demands had proved still more onerous, and required for their payment the total new premiums for the first two years, those policies which had existed for less than two years should be assigned a reserve consisting of the value of the unexpired risk still current, while assurances of a longer duration would be furnished with a reserve which assumed that the pure premium at entry was π_{x+2}. This course naturally cannot continue for a lengthened period, and is only feasible so long as t (the addition to the age at acceptance in obtaining the pure premium of the older assurances for valuation) is so small that $\pi_{x+t} < P_x$, where P_x is the premium charged after diminution by commission. An inspection of a table of the values of π_x will determine the boundary line where the process will terminate.

It is not the purpose of this book to advocate any method in preference to another, unless vital principles are involved, but this scheme can hardly be described without a brief comment. It is admitted—the experience of the soundest Companies attests the fact—that the expenses accompanying the acquisition of new business are very considerably heavier than those attendant upon the conduct of a business already obtained; evidence upon this point is adduced in the present volume hereafter, abstracted from the accounts of some of the stablest and most cautious Companies; it may also be added that the proposition in itself is based upon judicious views and reasonable arguments as well as an ample recognition of facts so far as the question of the incidence of expenditure is concerned and the importance of an equalization of profits; but, however theoretically or practically the scheme be valid, the element of expediency and the righteousness and value of a salutary example should enter, with their efficient weight, into our conceptions. And the sequent considerations appear necessary to be scrutinized and judged in assessing the practicable merit and utility of this suggestion. (1) Many of the policies—and the more

predominantly so as the business assumes a "forced" character which excessive expenditure implies—will, by reason of the manner in which it has been obtained, promptly disappear in lapses, and thus frustrate the hope of regaining a portion of the initial cost by the reduced expenses incident to their continuance; (2) a Company which attempts to secure business at any abnormal cost beyond the full scope of its requirements in the interests of the entire membership should properly suffer the loss produced by its unwise enterprise, and probably therefore be benefited in the future by the detrimental effect of its exorbitant financial procedure; (3) the compensatory reduction of reserves will tend to repel the public, and it may be reasonably anticipated that as those reserves become more weakened, an increased cost (with diminished reputation) will be requisite if the same scale of new business is disastrously pursued; (4) a public example of a high type should be exhibited by Companies, and the difficulty of this proposition, if adopted, might be that inferior Offices, if any should at any time exist, might be tempted to screen their extravagant expenditure (for, unhappily, extensive figures of new business possess an influence with us all and often tend to obscure that vigilant foresight directed to the future conditions and prospects of a Company which the actuary should sedulously exercise) under the power and authority of a scheme propounded by able and skilful minds, under whose administration the plan would be sagaciously guarded, but not when entrusted to feebler capacity; (5) it is asserted in the advocacy of this modification that the *fact* is recognized of the unequal pressure of expenditure; this is true, but on the other hand a more weighty *fact* is misrepresented—the assumption, namely, that the assured entering at the actual age of x are regarded as completing at the fictitious age of $x+t$; and (6) finally it may be urged that a Company which is reduced to this condition of need would act more wisely in the permanent interests of its constituents if it sought a transfer to a more vigorous and stable institution.

These remarks are submitted with a full appreciation of the fact that the pure premium method of valuation is not an integral part of the assurance scheme, which possesses an inherent universality of authority, and with a clear admission also of the facts in practice relating to the distribution of expenditure which suggested the modification. The homogeneous pure premium plan, it may

again be urged, should, in its integrity, form the standard of excellence, in its distinct bearing upon stability and prosperity, at which the actuary should steadfastly aim, and consistently pursue.

The necessary machinery for tabulating the data of an Office as the materials of the valuation need not be particularly described. The author would simply express his preference for the preparation of Classification Sheets afresh at each investigation, where the facts are arranged under the several valuation ages. A continuous debtor and creditor record of the Assurances and their changes is liable to many confusions and errors which the independent compilation of the statistics at every investigation avoids. The Card system again is simply needful when the experience of mortality is required to be deduced.

An interesting reference may here be made in connexion with the law of mortality expressed by the formula $F_x = a + bq^x$; where the total force is represented as composed of a partial force which is constant at all ages, and another partial force which increases with the age in a geometrical progression. If the constant q were supposed to be unity, the force of mortality would become $a + b$, and would thus be the same for all ages (as the second member of the equation is thus independent of x, the age, that is, does not vary with the age). The annual premiums also would be uniform at all ages; and hence no reserve would be required for the Assurance liabilities of a Company beyond any provision which might be deemed necessary for unusual fluctuations in the mortality. Now the series represented by $a + bq^x$ may be made as nearly constant as we please by increasing the other constant, a. Hence it follows that a constant addition to the force of mortality at all ages, while it will necessarily augment the annual premiums, will concurrently reduce the reserve required,—not relatively to the increased premium only, but—absolutely. We thus perceive that it is fallacious to estimate the relative financial positions of Companies by the number of years' premium income in possession. An examination of the expression $a + bq^x$ also explodes the popular delusion that the reserve of a Company is requisite on the ground that death must at some epoch or other occur. If the force of mortality were constant, the certainty of death would be obviously unaffected. Nevertheless, as has been already shown, the assumption of a constant mortality would avoid the necessity of accumulating a

reserve, and hence it is evident that the reserve is required, not on account of the certainty of the event of death, but, by reason of the continual augmentation in the force of mortality from age to age. The expression, it may be added, explains the reason why the rates of premium increase so much more rapidly at the older than the younger ages. At the more youthful ages, the constant, a, is much greater than bq^x, and the force of mortality consequently at that period is nearly constant, while, at the more advanced ages, the term bq^x exceeds the a, and the force of mortality thus increases rapidly. Since no aid to the comprehension of the meaning and service of a Mortality Table can be neglected by the student, it may be useful to present a more detailed explanation. At age x, the single premium for an Assurance for life is $vq_x + v^2p_x \cdot q_{x+1} + v^3p_x \cdot p_{x+1} \cdot q_{x+2} + \ldots$, and the ordinary annuity-value is $vp_x + v^2p_x \cdot p_{x+1} + v^3p_x \cdot p_{x+1} \cdot p_{x+2} + \ldots$ The annual premium accordingly is

$$\frac{vq_x + v^2p_x \cdot q_{x+1} + v^3p_x \cdot p_{x+1} \cdot q_{x+2} + \ldots}{1 + vp_x + v^2p_x \cdot p_{x+1} + v^3p_x \cdot p_{x+1} \cdot p_{x+2} + \ldots}$$

Now assume that the rate of mortality, q, is constant at all ages, or $q_x = q_{x+1} = q_{x+2} = \ldots$ Then the probabilities of living a year are also constant, for $p_x = 1 - q_x$, and is evidently a constant quantity. Let the constants q and p be represented by Q and P: we then obtain for the annual premium

$$\frac{vQ + v^2P.Q + v^3P.P.Q + \ldots}{1 + vP + v^2P.P + v^3P.P.P + \ldots} = \frac{vQ + v^2P.Q. + v^3P^2Q + \ldots}{1 + vP + v^2P^2 + v^3P^3 + \ldots}$$

Here the common ratio in the numerator is vP, and in the denominator also vP. The sum of the geometrical progression exhibited by the numerator is (adopting the ordinary formula)

$\frac{vQ\,[(vP)^{\omega-x} - 1]}{vP - 1}$; and that of the denominator, $\frac{(vP)^{\omega-x} - 1}{vP - 1}$

(ω is the limiting age in the Table which no life attains, and hence the term of the series is accordingly $\omega - x$: in the denominator, the series, of course, stops at one term earlier than that of the numerator, but this deficiency is compensated by the addition of unity to the value of the annuity, and hence the series of the numerator and denominator contain the same number of terms).

The annual premium is, therefore,

$$\frac{vQ\,[(vP)^{\omega-x}-1]}{vP-1}\times\frac{vP-1}{(vP)^{\omega-x}-1}$$

or, cancelling, vQ, which is a constant quantity unaffected by the age.

The value of the policy, or reserve, may be expressed in the form of $1-\frac{P_x+d}{P_{x+n}+d}$: each P being, on the hypothesis, constant, this reduces to $1-\frac{vQ+d}{vQ+d}=1-1=0$. The abrogation of any reserve on this hypothesis is evident when we consider that since the rate of mortality is uniform, the annual pure premium for the year's risk at any age is vq, so that as each premium on this assumption is exactly sufficient for discharge of the claims of its respective year, no need occurs at any duration after entry for any prior accumulation (and, indeed, no source for such a purpose is available) to supplement a deficiency which does not exist. If, on the contrary, the rate of mortality diminished as a function of the age, the policy-values at any date would be negative.

The rate of interest adopted in the valuation is of course less than the net realized return for patent reasons, and in ascertaining the latter amount the actual interest revenue should be brought into comparison with the unproductive assets (for example, balances at the bank and at the agencies and branch offices) as well as with the invested funds, diminished by any income tax and (properly) by permanent depreciations in the values of the several securities unless an independent investment Reserve Fund is maintained.

If 3 per cent. be the rate assumed in the valuation and income tax be 1*s.* in the £1, the actual realized return must at the least prove in excess of £3 3*s.* 2*d.* per cent. per annum in order to yield the requisite clear 3 per cent. No Company naturally would be content with this insignificant and perilous margin, looking, with prudential regard, to the possibility of future variations in the money market, the provision of solid reserves, and a confident assurance of continued prosperity.

It is significant to note that the effective rate of interest (where allowance, that is to say, is introduced for compounding interest where the nominal rate is received more frequently than once a

year) which is actually obtained upon the funds of the principal Life Offices (and deducting income-tax) has fallen from £4 5*s*. 7*d*. per cent. in 1870 to £3 15*s*. 5*d*. per cent. in 1899.

A few observations may usefully be submitted upon the mode of valuing the items on the credit side of the Valuation Account, or the several fluctuating elements which constitute the Life Fund.

It is first to be remembered that at a valuation epoch (which surveys the entire expected future duration of the obligations) we are considering essentially the probable value of the investments during the complete currency of the assurance contracts, so far as trained insight and experience can forecast, and we are not restricted to a contemplation of the values at the present or any particular date. If, neglecting this primary condition, we valued the investments at the time of valuation and adjusted them in the account at those punctual prices we might, in a crisis of intense depression or panic in the money market, find that the values, although the securities possessed the soundest intrinsic character, had become so reduced that the bonus to the policy-holders and shareholders would approximately disappear. In the case of superior investments, so drastic a process would be unreasonable and unjust; for in the period of years which we survey the original values (the cost price, if the investments have been wisely selected, and allowing for any "wasting" securities) may be rightly expected to recur; and the course in question would not merely produce inequity generally but would also deprive those policy-holders who may die prior to a revival of prices of a share in the appreciated amount. If, on the other hand, the investments under the stimulus of exaggerated expectations and wild speculation in the market, advanced far beyond their intrinsic worth, it would clearly be unjustifiable and objectionable to adopt these fictitious values (fictitious, that is to say, in relation to the general course of prices and monetary conditions), and thus produce an unreal surplus with the sequence (having respect again to the future which our valuation contemplates) of a diminished surplus at a subsequent investigation, when values had returned to their normal and average level. If then a prudent Company would not adopt the latter proceeding, is it consistent to pursue to its extreme extent the former plan, instead of employing judgment based upon an extended view?

In a banking institution the diminished range of prices actually exhibited must be adopted or reserved-for, since the very panic which has produced them may entail a run upon the bank, and sales for the purpose of meeting it could only be effected at the ruling rates in the market. But no such consequence or contingency attaches to a Life Office, which can quietly wait, with the retention of all its securities, until more propitious times have appeared.

It may be useful to remove a popular misapprehension. The settlement of a definite day (e.g., the 31st of December) for the valuation does not possess any relation to the date for fixation of the prices of the assets : it is necessarily and simply adopted for the purpose of forming a point of time for the determination of the ages on which the values of the functions employed depend. The day from which the ages are measured in no way involves any implication as to the suitable epoch for assessing the values of the investments. Moreover, it would seem to be evident that the two sides of the valuation balance sheet—related essentially, as they are, to each other,—should be constructed on similar principles : the debtor side, or the value of the liabilities, has express reference to the *future* history of the contracts ; and the creditor side, or the values of the assets (which, with the premiums, form the provision for the satisfaction of those liabilities *as they mature*) should in a corresponding manner be viewed also in relation to their probable *future* products on successive realisation in place of being fixed upon the transient experience of a single day.

Hence, with due regard to possible fluctuations of value (founded upon the experience of the past and a consideration of the probable advent of changing conditions in the market generally or in connexion with any particular class of security) and to the actual decline of value in investments of a " wasting " nature, like stocks purchased at a premium and redeemable at par, leasehold properties, and other terminable securities, the index to the assessed values must be the probable future course of prices based upon the examination of a sufficiently lengthy record of experiences in the past and, as has been stated, upon a sagacious interpretation of any varying conditions which have intervened and may possibly effect some divergence between experience and expectation. A striking instance of a change of value occurred in the history of one of our most ancient Companies between the years 1795 and 1797. In accordance

with the custom of those times, its investments consisted of Government securities; and although between 1795 and 1797 its investments had increased by 19 per cent. in nominal amount, the value of the augmented assets as a whole had in the two years diminished by 15 per cent. A further illustration is furnished in the report of a Select Committee of the House of Commons upon the Savings' Banks' Funds, issued in 1902. In 1897, when Consols stood at 113, a valuation of assets exhibited a surplus of £4,453,000 in the Trustee Savings' Bank Funds, and one of £12,780,000 in the Post Office Savings' Bank Fund. In 1901, when Consols had fallen heavily, these results were converted into deficiencies respectively of £2,680,000 and £5,000,000.

It is the practice of Companies where the market value at the date of valuation exceeds the original cost price to retain the latter value in the balance-sheet, and treat the excess simply as a latent additional reserve, which may or may not be maintained in the future. Theoretically, the value of the investment should be adjusted according to the indications of an adequate experience, having regard to any disturbing influences, or their possibility of occurrence, which appear likely to be permanent, but as a practical and expedient procedure the retention of the cost price in these instances is sound and salutary.

With the object of avoiding the isolating and partial effect of adopting the prices at any definite epoch, it has been suggested that certain values should be ascertained during the period preceding the date of valuation, which should consist of the price paid for the investment, the highest price recorded in the interval, the lowest price similarly obtainable, and the value at the present date: it is then assumed that the average of these several amounts will accurately express the appropriate assessment at that time. This is a cumbrous operation: it does not equally survey the past, but simply selects detached aspects; it fails to take into account the "weights" of the observations; larger and more numerous amounts, for example, might have been sold at the lower price than those at the higher; and it omits, consequently, to notice the continuous history of the fluctuations of value which the security has experienced.

A scientific and adequate process—always bearing, however, in mind any disturbing causes of apparently a permanent or

long-enduring nature which have intervened or may hereafter supervene in respect of any particular securities—would be founded upon a physical deduction which appears to be validly established connecting the periodical coincidence of solar activity (manifested by the intensity of sun-spots) with special physical phenomena occurring on our globe, and their influence upon mental energy and industrial enterprise.

The astrological doctrine enshrined a nucleus of truth, and certainly, in relation to the sun, science has demonstrated the " star raining " influence of the heavens and the interesting fact that solar changes, operating upon the human nervous system, thus mediately affect the feelings and hence, in time, may modify the character itself. It has been ascertained, over a prolonged range of observation, that increased heat is received from the sun at intervals of about 10 to 11 years : that excellent vintage years, cyclones, and droughts recur at similar periods ; and that credit cycles in trade endure for a practically coincident term. The augmented supply of solar heat renders harvests more abundant, and this result acts upon trade and commerce ; a deficient amount of heat operates in the opposite direction, and seems to be causally associated with droughts and meteorological disturbances which tend, through their adverse effects upon the materials of commerce, to derange commercial affairs and above all to produce nervous influences which in the form of mental hope or depression constitute the prime factors in industrial vigour and enterprise. In this form of survey we obtain a connected and continuous period which so far as it is reflected in trade comprises about three years of adverse commercial conditions ; the same subsequent stage of normal trade ; two years of speculative extension and exaggerated values naturally ensue, with the resulting " bubble " of insecure trade, which endures for about a year ; and then the advent of the inevitable comprehensive collapse which terminates the cycle of 10 or 11 years, with a gradual reascent into a recurrent and repeated stage. Hence a reasonable and validly-grounded index to the general value of securities would be furnished if the average prices of the preceding 11 years were adopted, seeing that this term of observation comprises the several fluctuating phases we have mentioned which periodically and regularly appertain to the course of values. Until some scheme of this nature is conceived to be feasible, Companies

should rightly pursue the following course : retaining investments at cost price which present an increase of value ; definitely writing down those which appear to be permanently affected adversely ; for example, the debentures in a sound trading Company may be thus reduced in price if a change of fashion occur in the manufactured articles which it was founded and equipped to produce and supply, and where adjustment to the altered conditions cannot be effected without serious cost ; reducing the value of securities purchased at a premium with redemption at par, either by writing off the amount of depreciation due to effluxion of time or creating a sinking-fund for that purpose ; and proportionately diminishing the book-value of terminating securities. Where, however, an intrinsically sound investment has suffered temporary depression, and definite expectations exist of improvement in value, the sum required to place it upon a reasonable (not necessarily, as has already been remarked, the transient market) level should not be permanently written-off as a loss but carried to a Suspense Account, whence, as the security advanced, portions would be transferred to the Revenue Account with the object of reinstating its diminished amount.

The present place is probably as appropriate as any other for the introduction of another important function relative to the assessment of mortality, with the principle of which the student should be familiar, and to which a reference has been made on a prior page. The ordinary rate of mortality applies to an entire year's experience but the new function involves a more precise mode of measurement adapted to indefinitely small periods of time during which deaths occur, and furnishes the annual rate of mortality as computed at the *exact* point of any age, x, or as assessed upon the basis of the rate of death during an infinitesimal interval succeeding the given age. The serviceableness of this function can only, at the student's present stage of knowledge, be indicated in general terms.

Regarding mankind in the mass (for, as we have insisted generally, our reasonings upon Assurance problems must involve the conception of magnitudes), death presents itself as a continuously acting force, analogous to the forces which produce the assemblage of effects observable in physical Nature : its destructive operation, thus viewed, proceeds, not by starts or with intervals of breaks, but, with incessant and uninterrupted regularity. We cannot, of course,

apply this notion of continuous decrease to a single life or a small group of lives ; but in relation to an ample mass, we can evidently consider the number of deaths (or the decrements in the living) produced in, for example, every moment of time, and thus form our measures of the probabilities of surviving an infinitely short period or dying within its compass. And since, in our calculations, the associated element of interest possesses a similarly continuous character, when investments in the aggregate are surveyed, we are thus enabled, in this form of conception, to range our problems in Assurance, and particularly those of a complicated nature, within the processes of the higher calculus with a necessarily preciser method of treatment. To the function in question has been assigned the title of the force of mortality which—varying, in dependence upon the age, at every fractional period, however minute be the fraction, intervening between any two consecutive ages—expresses the rapidity with which at any stated *instant* of time a body of persons of a given age and of sufficient magnitude, is diminishing by death. Now death itself forming a force, this descriptive name, when expanded, becomes the force of the force of mortality, or, more distinctively phrased, the rate of the force of mortality. The compound expression is legitimate, but the more appropriate term would be the instantaneous rate of mortality, since this possesses the advantage of analogy with the designation of the customary " rate of mortality," while it is differentiated by the qualifying adjective. The symbol allotted to the function is μ, with the subscript of the age to which it appertains in any particular investigation. Take the fraction, $\frac{l_x - l_{x+1}}{1}$: this expresses the relation between the number of deaths produced in the period of 1 year and the time (1 year) during which these effects have been occasioned ; and when divided by l_x (the number existing at the commencement of the term in question), the quotient represents the rate at which, by the end of the year, the original aggregate of lives has been diminished by death, or $\frac{l_x - l_{x+1}}{l_x}$. If we adopt a briefer interval between the ages, say $\triangle x$ (where $\triangle x$ is a fraction of unity just employed), the ratio between the number of deaths which occur and the time $\triangle x$ in which they happen becomes

$\frac{l_x - l_{x+\Delta x}}{\Delta x}$, and the quotient of $\frac{l_x - l_{x+\Delta x}}{l_x \backsim x}$ represents the annual rate of mortality based upon the rate for the short period contemplated. This may be presented in the form of $-\frac{1}{l_x} \cdot \frac{\Delta l_x}{\Delta x}$, where the Δl_x represents the former numerator, and Δx symbolises the space of time surveyed, regarded as a proportion of the unit-interval of 1 year. (In the Calculus of Finite Differences, the difference of u_x or Δu_x is $u_{x+1} - u_x$: in mortality computations, we therefore obtain $\Delta l_x = l_{x+\Delta x} - l_x$, but l_x is $> l_{x+\Delta x}$, so that the expression is negative (as indicating a diminution of quantity) and hence, to exhibit it in the usual form, we get $-\Delta l_x = l_x - l_{x+\Delta x}$). It will be observed that the ordinary function furnishing the annual rate of mortality possesses an identical form by substituting unity (or 1 year of time) for the abbreviated period of age involved in the Δx. Assume once more that the time contemplated in the range of survey (and the increment or addition to the age) is represented by dx, a considerably shorter interval of time—that is, substitute dx for Δx (the dx implying that the Δx has thus been greatly reduced in extent)—and we obtain an approximation for the force of mortality or the annual rate of mortality founded on the assessment of its value at an age near to x and assuming a very minute difference between that age and its successor: this is $-\frac{1}{l_x} \cdot \frac{dl_x}{dx}$, presenting the same form as that of the prior expressions. Now successively diminishing the increment dx (and the difference in the numerator corresponding to it by bringing l_{x+dx} closer and closer to l_x) we finally arrive, in the limit as it is termed, at a value of the expression which furnishes the exact rate at the moment when age x is attained, or the instantaneous measure of mortality existing at the precise instant of x between the number of deaths which happen in an infinitely short interval of age (less than any interval we may choose to assign*) and the infinitely brief space of time during which those deaths are occasioned.

The fundamental principle in the doctrine of Kinetics (the science which treats of forces in relation to motion) is analogous to or

* That is to say, if any interval be selected the interval in question is briefer; if a minuter interval be chosen, still the interval in question is less in extent, and so on continuously. And see Appendix II. to this Chapter.

rather identical with, the conception involved in the force of mortality, for death being comprised within the category of the physical forces, its operations are subject to corresponding uniformities of action. Where the velocity (rate of motion or rate of change of position) of a moving point (or body) is variable (that is, where it does not describe equal spaces in equal times—the definition of uniform velocity), the velocity *at any instant* is measured by the space which would be described (or traversed) in a unit of time *if the point moved uniformly during that unit with the velocity which it possessed at the instant contemplated.* (A railway train, to adopt a concrete illustration, on starting, gradually and continuously increases its speed so that longer and longer spaces are traversed in equal times : but we can imagine that, at a particular *instant* during the motion, the steam-power might be so adjusted as to keep the train travelling from that instant for a prescribed time at a perfectly uniform velocity or rate of speed—the same as that acquired at the instant regarded. This uniform velocity then would express the velocity which the train possessed at the instant in question—such instantaneous velocity being thus measured by the subsequent relation between the uniform spaces traversed in the same periods of time.) Now the principle involved in this mode of measurement of an instantaneous velocity is necessarily implied in all accurate measures ever devised for the purpose of estimating the rate at which *change of any kind is going on.* Hence it must be applicable to the consideration of problems of this type occurring in mortality assessments. Reverting to the moving point, we can therefore substitute the increasing force of death for the point itself ; the term of 1 year for the unit of time ; the instantaneous rate of mortality at any exact age for the velocity at any definite instant ; and the subsequent uniform progress of the point (i.e., the effect it produces in the form of the spaces traversed) which measures its instantaneous velocity will be paralleled by the force of death preserving after the age just stated a uniformity of the strength which it exhibited at that moment of age, i.e., producing identical effects (in the form of the number of deaths occasioned among a fixed number of lives at risk) in every sequent equal interval of time. Now to effect this result—i.e., the production of the same extent of mortality-effects—we must obviously assume that the abstractions by death which successively occur in the mass of lives originally

subject to the power of death are restored from time to time during the year by the immediate substitution of fresh lives of the fractional ages attained by those who die ; for only in this way do we obtain a uniformity of effect, identical in amount with the initial effect during the instant considered, i.e., l_x is treated as constant throughout the year. Hence we may, with strict scientific accuracy define the instantaneous rate of mortality as the ratio possessing as denominator the number of persons living at a given age, and as numerator the number of deaths which would be produced in a year were the force of death prevailing at that instant of age to be maintained uniform for the remainder of the year, and were the numbers originally exposed to risk concurrently preserved constant by the addition, at each minute age of life between the ages x and $x + 1$, of new lives equal in number and similar in age to those successively removed by death. It will be observed that as μ increases in intensity with the advance in age, the number of the living acted upon by the force of death at successive minute intervals of time is simultaneously reduced, but the resulting number also of the deaths as affected by the diminishing numbers exposed to risk is wholly or partially compensated by the (commonly in practice, though not necessarily,) continuous augmentation in power of the force which operates from stage to stage. The preceding exposition is somewhat difficult, but we shall furnish a concrete illustration hereafter : meantime, although the subject has been implicitly anticipated, we proceed to explain the reason which renders the customary annual rate unsuitable as a representative of the instantaneous rate. The O^m Table of Mortality shows that the numbers living (l_x) at the following several ages were 96,453 at age 20, 96,063 at 21, 95,251 at 23, 93,933 at 26, and 90,828 at 32. The rate of mortality for age 20 is $\frac{l_{20} - l_{21}}{l_{20}} = \cdot 00404$, and expresses the rate of reduction during 1 year of age. Now instead of employing an interval of 1 year, let us notice the extent to which we should obtain this rate of ·00404 if we selected a period of 12 years, and adopted the *average* annual deaths during that term for division by the l_{20}. The formula becomes $\frac{1}{l_{20}} \cdot \frac{l_{20} - l_{32}}{12}$, and calculation produces the ratio of ·00486. This furnishes the average annual rate of mortality during the

period in question as related to the number living at the commencement of the series, and on account of the length of time selected exhibits a result which does not coincide with the ordinary rate for the first year of the term, or ·00404. The deaths between 20 and 32 are 5,625 in the aggregate, and the average adopted is 468·75 per annum, while the actual series consists of 390 in the first year, 400 in the second, then 412, and so on to 567 between the ages of 31 and 32; and the assumption, therefore, of the average value of an increasing progression of deaths furnishes too great a numerator for the determination of the fraction for the first interval of the period. (This remark applies, of course, to the major part of the Table, since at advanced ages, the annual number of deaths decreases.) It is evident then that on account of employing the arithmetical average of an increasing progression in the number of deaths, we fail to represent with accuracy the rate at the earliest age of the series. Apply now the formula to an interval of 6 years, and we obtain $\frac{1}{l_{20}} \cdot \frac{l_{20} - l_{25}}{6}$, or ·00435, a closer approximation to the initial ratio: the adoption of a term of 3 years would yield the nearer approximation of ·00415; and the correspondence becomes more exact as we descend to the original interval of 1 year. The shorter then be the interval selected (beyond 1 year) between age 20 (and of course the observation applies to any other age) and a succeeding age, during which interval the deaths which occur are adopted as the antecedent of the ratio, the more precisely we shall arrive at the ordinary rate of mortality at that youngest age of the group. Now the relation (as a gradually approaching, yet still inefficient, measure) here shown between the true yearly rate at any age and the average rate which involves a term exceeding 1 year, may be applied to the instantaneous rate of mortality in comparison with the customary annual rate. At age 20, the value of q is ·00404, while μ_{20} (or the instantaneous rate) possesses the value of ·00399, and hence the former does not afford a measure, for the reasons already adduced, of the annual rate when reckoned on the basis of the precise instant from age 20; for q includes in its estimate the deaths of an entire year, while the instantaneous rate involves the deaths during an infinitely minute space of time following that age. Now just as we ascertained that the *average* rate over a term of years more and more closely expressed the customary rate for 1

year as we reduced the interval of time surveyed from 12 years to 6 and 3 (until the calculations were merged in the initial year), so will the year's rate at age 20 (since the force of mortality is incessantly varying between 20 and 21) more approximately represent the instantaneous rate at that age if we adopt 6 months instead of 12 months $\left(\frac{l_{20}-l_{20\frac{1}{2}}}{l_{20}}\right)$; more closely still if we employ 3 months or $\frac{l_{20}-l_{20\frac{1}{4}}}{l_{20}}$, and in the limit (just as we proceeded from 12 years to the 1 year), when the interval of time observed and the increment to the age are both made infinitely small, we shall exactly obtain the instantaneous rate at age 20; or, if dx be this infinitely minute space of time, $-\frac{1}{l_x}\cdot\frac{dl_x}{dx}$, or $\frac{1}{l_x}\cdot\frac{(l_x-l_{x+dx})}{dx}$

It has sometimes been stated that q_x represents the *average* value of the varying values of μ_x at the several moments of time in proceeding, through all intermediate ages, from x to $x+1$. But, on the contrary, it does not appear that this mathematical relation exists between these two functions. At ages 40 and 41, for example, on the O^m Table, μ has the respective values of ·009 and ·0094, while q_{40} is ·00915, or q_{40} is $> \mu_{40}$; but at age 74 and upwards q is $< \mu$ at each age, while at age 90, for instance, q_{90} is $< \mu_{89}$. These results would seem clearly to show (in supplement to our preceding investigation) that no such arithmetical relation connects the two functions. An able friend has pointed out to me, however, that q_x might correctly be termed the mean* value of μ_x, when the latter is "weighted"* in proportion to the numbers successively left under observation by the operation of the force of mortality. For $q_x = \frac{d_x}{l_x}$ = the integral of $\mu_{x+t}\cdot\frac{l_{x+t}}{l_x}\cdot dt$, between the limits of 0 and 1 (i.e., between the present instant and the end of one year), where $\frac{l_{x+t}}{l_x}$ (as continuously diminishing) is the "weight" affixed to the μ_{x+t} (increasing). The following approximate relation has been exhibited between q and μ : $q_x = \frac{l_x-l_{x+1}}{l_x}$: μ_x (approximately) $= \frac{l_{x-1}-l_{x+1}}{2l_x}$

* See Chapter I.

q_x then is $> < \mu_x$ as $2l_x - 2l_{x+1}$ is $> < l_{x-1} - l_{x+1}$

(Adding $2l_{x+1}$ to each member of this inequality):

as $2l_x$ is $> < l_{x-1} + l_{x+1}$

(Deducting l_x from each member, and adding to each, $-l_{x+1}$):

as $l_x - l_{x+1}$ is $> < l_{x-1} - l_x$

as d_x is $> < d_{x-1}$

That is to say, the instantaneous rate is greater than the probability of dying in a year when the number dying in a year at age $x-1$ is greater than the number dying in the same interval at age x. It may similarly be shown that when d_x (or the deaths occurring during a year) increases, q_x is $> \mu_x$, while if d_x be a constant quantity, $q_x = \mu_x$.

A fair approximation to the value of μ_x is $\frac{l_{x-1} - l_{x+1}}{2l_x}$ or its equivalent, $\frac{d_{x-1} + d_x}{2l_x}$. In the O^m Table, $\mu_{40} = \cdot 009$, and $\frac{d_{39} + d_{40}}{2l_{40}} = \cdot 009$.

The central death-rate at x, or m_x, is $\frac{d_x}{l_{x+\frac{1}{2}}}$, and forms an approximate value of the instantaneous rate at an age half a year older. Thus, approximately, $m_x = \mu_{x+\frac{1}{2}}$. At age 40, according to the O^m Table, $\frac{d_{40}}{l_{40+\frac{1}{2}}} = \cdot 0092$: $\mu_{40} = \cdot 009$; $\mu_{41} = \cdot 0094$, and $\mu_{40+\frac{1}{2}}$, consequently, is approximately .0092 as above. In other words, μ is approximately equal to the ratio which the number of deaths in a year bears to the number living in the middle of that year or the average number living during that year. (But the student, in regarding this calculation of $\mu_{40\frac{1}{2}}$, will bear its inappropriateness in mind and will remember that the true method for obtaining μ is based upon infinitesimal quantities of time). It may be proved that q_x is always $< \mu_{x+\frac{1}{2}}$, i.e., the *rate* of mortality for a year is always $<$ the mean *force* of mortality for that year. Thus: q_x is $> = < \mu_{x+\frac{1}{2}}$

as $\frac{d_x}{l_x}$ is $> = < \frac{d_x}{l_{x+\frac{1}{2}}}$

as $l_{x+\frac{1}{2}}$ is $> = < l_x$

But $l_{x+\frac{1}{2}}$ is $< l_x$

$\therefore q_x$ is $< \mu_{x+\frac{1}{2}}$

This result is evident, since in each instance, the deaths for the

year are considered, but in the one case they are compared with l_x, and in the other with $l_{x+\frac{1}{2}}$.

A rough concrete illustration has been cited. A person walks one mile in 15 minutes with increasing speed. He may start slowly and gradually augment his rate of progress continuously during the time; or at different intervals he may adopt different accelerations of the previous rates of speed. Whatever be the modes of progress, however, which he may pursue, he accomplishes the distance within the prescribed time; and his average rate of walking would be described as 4 miles per hour. This average rate per hour may be regarded as corresponding to the customary rate of mortality at any age x taken as the point of start: the rate at which he *commenced* to walk is analogous to the instantaneous rate of mortality at the precise age x; and the rate at which he was walking at any specified instant during the 15 minutes would correspond to the instantaneous rate of mortality at some definite fractional age between x and $x + 1$. Further illustrations upon the subject—in relation to the interpretation of the terms, "infinitely small" and "limit"—are furnished in Appendix II to this Chapter.

Appendix I.

In the example of adjustment of the value of the reversion for the payment of claims prior to the date implied in the Table of Mortality employed, we have added three months' interest at the valuation rate. This is a rough but convenient course, and suggests the question of the proper mode of deducing the present value of £1 due at the end of a fractional portion of a year, and the corresponding amount of £1 at the expiry of that fraction. In one mode of viewing the problem, the present value would be stated (if the fractional interval be 6 months) at $\frac{1}{1+\frac{i}{2}}$, and in the other mode $\frac{1}{\sqrt{1+i}}$, presuming i to be the interest per unit for a year. The interest for the half-year on the one supposition would be $\frac{i}{2}$, and on the other $\sqrt{1+i}-1$: so that the amount of £1 at the end of six months would be represented on the one method by $1+\frac{i}{2}$ and on the second

by $\sqrt{1+i}$. Hence, according to the view adopted, the value of the reserve ${}_nV_x$, obtained in the ordinary manner, would, if the former hypothesis be entertained, amount at the assumed instant of death to ${}_nV_x\left(1+\frac{i}{2}\right)$, while, if the latter be accurate, the value would be ${}_nV_x\,(\sqrt{1+i})$. (At 3 per cent. the one multiplier would be $\left(1+\frac{\cdot 03}{2}\right)$ or 1·015, and the other $\sqrt{1\cdot 03}$ or 1·0149. In our illustration the interval selected is three months, and hence the present value is ${}_4\sqrt{\frac{1}{1+i}}$, or the addition to be made to the customary reserve is ${}^4\sqrt{1+i}-1$ or ·0074 per unit instead of ·0075.

The factor of $\sqrt{1+i}$ (and its reciprocal) is undoubtedly correct. The function expressive of the rate of interest is a continuous one, and exhibits a uniform progression not simply during a given finite period, but also during the course of every smaller interval of time of which that period may be composed. Let x be the undetermined interest on £1 for half a year: the amount of £1 at the end of that interval will be $1+x$: in the second half year this sum of $1+x$ will accumulate to $(1+x)^2$, and should then be equivalent to the stipulated $1+i$, if the interest to be received at the close of the year be i per unit.

Hence, $(1+x)^2 = 1+i$, $\therefore\ 1+x = \sqrt{1+i}$.

Appendix II

The section upon the instantaneous rate of mortality may well be illustrated, so prominent is its importance, by an explanation of the terms employed. A complete grasp of the conception which those terms embody can, however, only be attained when the student has mastered the elementary principles of the Calculus; we do not mean the analytical mechanism of that branch of mathematics, but the clear apprehension of the fundamental notion which it involves—the nature of a limit. Concepts (or the ideas existing in the mind answering to general names) are divided into Imaged and Unimaged: in the former, they are competent of being accompanied by a mental picture of the object to which the concept refers: in the latter, the power of imagining fails. We can picture a house of which we think after the actual perception has ceased;

we cannot form the faintest image of infinity, and its correlative, the infinitesimal. It is thus impossible to conceive, in the former sense of the term, of anything infinitely small or infinitely great, and in employing these phrases we must apply a process of interpretation. It must be remembered, too, that these expressions are not absolute but *relative* terms : if a magnitude be regarded as being infinitely small in comparison with a finite magnitude, it is termed infinitely small, and similarly of infinitely great. A microscopic particle of sand can only be said to be infinitely small when viewed in relation to the extent of the universe of which it forms a part. We shall best exhibit the use of the leading terms by means of examples ; and, for the purpose of comparative completeness, the definition of a limit should first be furnished. When A (a fixed magnitude) is described as the limit of B (a variable magnitude) it is implied (1) that B never becomes equal to A, and (2) that B must be capable of being made as nearly equal to A, as we please. (If a quantity increase or decrease *without* limit, then there exists no limit to which it can never attain, or rather we should say that, in the former case, its limit—in the sense just explained—is infinity : in the latter, zero.)

(i) A finite quantity is divided by an infinitely small quantity and the quotient is infinity or ∞ ; i.e., the division by an infinitely small quantity results in an infinitely great one. If 1 be divided by a small magnitude, ·00001, the quotient will be very great ; if the divisor be reduced to ·0000001, the quotient will prove considerably larger ; and no assignable number is sufficiently great to satisfy the conditions of the case, yet still no divisor, however minute, can render the quotient absolutely equal to infinity. Hence we must consider the language of the preceding proposition (i) as equivalent to, or an abbreviated form of, the statement that the quotient can be made as great as we please (greater than any magnitude we like to assign) if concurrently we can take the divisor as small as we please (smaller than any quantity we may choose to name). Now translate this proposition into the language of limits thus : when the denominator of a fraction diminishes without limit, the numerator remaining unchanged, the fraction increases without limit, i.e., it continually approaches to ∞, which, however, it can never reach since no quantity divided by another can equal infinity.

(ii) Consider, again, the proposition that every circle is a regular polygon formed by an infinitely great number of sides, each of which is infinitely small or evanescent. Now a circle can never become a polygon or a polygon a circle: and the interpretation we place upon this abbreviated language is this: that an inscribed polygon of a considerable number of minute sides nearly coincides with the circle: if the sides be increased in number and diminished in length, the coincidence becomes closer; so that we may bring the polygon and circle into a nearer and nearer approximation to identity if the sides may be made as large in number as we please, and consequently the length of each side as small as we please, though no polygon, however great be the number of its sides and however minute their length, can absolutely coincide with the enclosing circle. In the language of limits, this proposition would be thus expressed; if the number of the sides be increased without limit, the polygon will approximate without limit to the circle, or the circle forms the limit of all regular polygons which can be inscribed. But the polygon never reaches the limit or becomes lidentical in contour with the circle. The student will afterwards earn the mode of reasoning upon propositions relating to limits without actually supposing the quantities to have attained their limits. This course, he will find, is defined by the demonstration of the theorem, which seems almost self-evident, but yet does not form a perfect axiom, namely, that if, for example, A and B be two variable quantities so related that A is always equal to B (or, say, double of B), and if A have the limit, a, while b constitutes the limit of B, to which respectively they continually approach, then $a = b$ in the one case, and $a = 2b$ in the other. Let us apply the preceding considerations to certain functions employed in our calculations. v^t is the present value of 1 due at the end of the time t; v^{dt} expresses the present value when the period to the date of payment is dt, or an infinitely small part of t. Now the limit of the index, dt, is obviously 0; and the limit of the function itself, v^t, is clearly unity: the former corresponding, as it were, to the infinitely small side of the inscribed polygon, and the latter to the infinitely close approximation to the circle. Now if we were actually to adopt the limit itself (the 0) in the index, we should at the same time be brought to the limit (or unity) of the function, for $v^0 = 1$, or the present value of a sum instantly due is obviously the sum itself.

But in pursuing this course we clearly should lose every vestige of the sense of "present value," for that term, by its definition, necessarily implies an interval (however minute) between the moment *now* and the moment *then* when the payment becomes due. Hence we employ the dt (not as though the limiting value had been attained, but) in its continual approach *to* the limit of 0, and thus preserve the conception and nature of a "present value." The probability of surviving the infinitely short period dt is $\frac{l_{x+dt}}{l_x}$. If, again, we adopt the limit of 0 for dt, we obtain $\frac{l_x}{l_x} = 1$, or the chance of survival is certainty. We thus should altogether abolish the notion of survivance which necessarily implies an interval of time, infinitely minute though it be, between the present age contemplated, and the succeeding fractional age. Hence we use the l_{x+dt} in the numerator as it infinitely closely verges *on* its limit. So, too, with respect to the probability of dying in the infinitely brief period of time, dt, or $\frac{l_x - l_{x+dt}}{l_x}$. If the actual limit of dt or 0 be introduced, the fraction reduces to $\frac{0}{l_x} = 0$, that is to say, all probability of dying has vanished, and we thus negative the conception of probability here which must involve the present instant and an interval separating it from the sequent instant. Hence we here consider dt in its gradual approach *to* its limit of 0, or at the ultimate point (so to speak) instantaneously prior to the attainment of the limit. Now the differential calculus, in general terms, enables us to ascertain the relations between these differential parts of any quantity (i.e., the infinitely small differences between the successive values of that quantity), while the integral calculus determines the whole sum (or integral magnitude) of any quantity of which the differential parts are given, within whatever limits of age, period, or range we may desire; and hence, by the introduction of the notion of limits into our calculations we are able to utilise the calculus as a mode of precise measurement. It may be worth while pointing out the interpretation of the expression, $\frac{l_{x+t}}{l_x} \cdot \mu_{x+t} \cdot dt$. This furnishes, for the moment of time, t, the probability that a person, aged x, shall survive to that moment and then instantaneously die.

CHAPTER VI

The Sources of Profit and its Distribution

Precision of thought and its exactness of verbal expression being at the foundation of all accurate observation and deduction, a brief space may be usefully devoted to a preliminary consideration of the two terms, "surplus" and "profit."

We are constantly the slaves, and forfeit the mastership, of words, and through looseness or indefiniteness in conception and phrase, are frequently beguiled into error which a steady concentration of mental apprehension would have avoided.

The word "surplus" is obviously a compound (and a pleonasm or iteration of terms) of the Latin *super*, above, and *plus*, more, and signifies an excess beyond what is required.

"Profit" is formed from the Latin *profectus*, a progress, advance, gain, or benefit, which again directs us to *proficio* (*pro*, before, and *facio*, to make), to make progress, to advance, to improve.

All our terms, it may be added, in every department of science or practice, and even in philosophy, whose scope embraces principles or ultimate grounds of thought and reasoning, are based upon words expressive of sensible objects or actions, and their relations; the conception in the region of thought suggests an analogon in the world of sensation, and adopts the verbal expression which depicts the material experience; thus, to adduce one illustration only, the process of *pondering* over a subject implies the Latin *pondus*, a sensible weight, and presents the picture in the metaphysical domain of comparing one subject with another by means of a mental pair of scales.

In popular usage surplus and profit are generally employed (in error) synonymously, and the excess or surplus of the income in a revenue account beyond the expenditure, or the resulting balance, is not infrequently termed profit erroneously. Hence a discrimination is needed; in the example just furnished the appropriate term is "surplus"—that which remains over the total charge, which may, or may not, consist of profit according to the prior satisfaction of all requirements—special reserves, for example—which the nature and exigencies of the business may entail, just as the term "deficiency" (*deficio*, to fail, or be wanting; *de*, from;

and *facio*) expresses the "falling-off" of the income from the expenditure.

Applying this distinction to the subject of the present chapter, we should define surplus as the balance remaining after making the requisite provision for all prospective obligations under contracts by means of adequate reserves, both in respect of the amounts assured and of a sufficient margin for future expenses, fluctuations, and profits; while the profit would consist of that portion of the surplus which expediency and a consideration of all the circumstances of probable future experience suggest may be safely distributed as bonus, having adequate regard to the interests of the existing policy-holders, the maintenance of those interests, and the benefits of future entrants—in a word, the solidity and continued prosperity of the corporation as a permanent body. The former condition involves the customary valuation elements adjusted to the circumstances of the business; the latter may require the application of special judgment and foresight to any particular views or contemplated objects relating to future administration. For example, the surplus having been ascertained, it may be deemed advisable, in the permanent interests of the Company, to retain a portion (1) as a precautionary provision for possible contingencies, or the business-like practice of always maintaining a balance in possession; (2) for the purpose of approximately equalizing or rendering uniform a progression in the rate of bonus allotments in the future; (3) with the object of gradually accumulating funds in order that, at a subsequent date, the valuation may be based upon a reduced rate of interest or an increased rate of mortality, without producing abrupt discrepancies between the successive apportionments of profits, and thus again preserving a comparatively uniform or advancing series of bonuses; or (4) the future substitution of a different method of division of profits and the facilitation of the change by a retention of surplus, so that grave or inconvenient divergencies of rates may be avoided between the results of that system and individual or class results appearing in the past.

The primal aim to be constantly retained in view should be, as far as possible, with a clear regard to the advantages of all members, the preservation of harmony and consistency of administration throughout the entire duration of the contracts.

Three main sources of surplus exist in life assurance; and, for

clearness of statement these will be separately treated. They consist of (1) a higher net return of interest being realized than that at which, in the valuation, the reserve and future premiums were expected to be accumulated; the net return in question will be adjusted for income tax, for losses and depreciations in the investments if an independent fund for this purpose does not exist, and for the absence of interest upon unproductive amounts, such as balances at the bank, agencies, and branch offices; (2) economy of administration, resulting in the saving of as substantial a proportion of the loading as possible after expenses and fluctuations have been provided; and (3) the experience of a rate of mortality inferior to that involved in the valuation estimates. With reference to any possible surplus yielded by the lapse and surrender of policies, the observations contained in Chapter IV would appear conclusively to show that this source is either insignificant or entirely unproductive. Even if a minute fragment did exist in any case, it would merely consist in the former instance of the reserve (reduced by a protective provision for the effect of withdrawals upon the rates of expenditure and mortality), and in the latter event of the difference between the surrender value granted and the reserve, diminished for the purposes just mentioned. Looking to the comparatively early dates after admission upon assurance at which surrenders, and especially lapses, generally occur, it may justly be affirmed that no surplus is derived from these sources.

With regard to the sums invested in the purchase of reversions —selecting this item as a specimen of the surplus obtained from the investments generally—the amount consists of the excess of the rate of interest (accumulated) which the reversions were brought to yield beyond the valuation rate. But in the event of the early deaths of life-tenants, the difference between the fund or *corpus* falling into possession and the accumulated amount of the purchase money will not constitute an absolute surplus for division as profit, for here again the conception of aggregate results, or results in the mass, must be introduced, and a portion of the accrued surplus (apart from the difference between the valuation rate and the return which the purchases were effected to produce) should be retained in compensation for the deficiencies which must occur, over the entire range of these transactions, in consequence of the prolonged existence of other life-tenants, so that the financial position of the

future may not be depleted for the superior benefit of the present. The subject will be more fully discussed in Chapter VII.

Interest

This source constitutes at present the principal contributor to profits.

The rate of interest at which the valuation reserves are created is always lower than the net return realized upon the investments and securities, and consequently an annual surplus is produced for accumulation at compound interest into profits. If 3 per cent. be the adopted rate, and £3 15*s.* per cent. be obtained after the requisite deductions, there remains a yearly excess during the quinquennial period of 15*s.* per cent. upon both the assurance fund and the premiums received during that term, which, with a substantial amount of assets and a considerable revenue, will naturally constitute a most important factor in the aggregate sum to be divided.

No special comment appears to be necessary upon this obvious fact; and the student should submit the accounts of Companies to scrutiny, when he will appreciate the signal value of this contributory element, and the imperativeness of devotion, with assiduous vigilance and energy, to the extension of a sound and profitable range of the area of investments of an Office. Hence the constant need of discovering fresh and promising outlets of stability for the employment of capital which are not so open and adapted to the private or trust investor, and where, in consequence of the demand for money exceeding the supply, an enhancement of rate is necessarily sequent. Indeed, it may be stated as an administrative axiom that the actuary discovers his highest and most serviceable vocation in gradually training himself specifically as a sagacious and capable financial adviser. It is an easy employment for the administrator to select without much difficulty a few first-class Stock Exchange securities as funds accumulate, and simply furnish instructions to the broker to purchase; but this facile and futile course of management is not assurance administration; it possesses the grave defects (1) of amassing a large proportion of investments which yield a minimum rate of interest and thus tend to the diminution of surplus and the prevention of ample reserves; (2) of accumulating securities which from their nature are the first to depreciate in value when a speculative spirit arises among the public, and when a period of stress occurs consequent upon national

trouble or misadventure; and (3) the administrator who pursues this course has failed to master the rudiments of the responsibility with which he has been entrusted, and the resourceful and competent execution of which demands energy in place of supineness, vigilant yet cautious enterprise instead of inactive ease.

With the rate of mortality customarily adopted by Assurance Companies essentially permanent and constant (for the experience recorded in the O^m Table confirms with punctual exactitude the H^m Table which was previously employed), the two domains of remunerative enterprise now peculiarly open to the industry and survey of the actuary or administrator, on which the profitable issue of his labours eminently depends, consist of (1) the unceasing and capable supervision of finance in the mode we have indicated, and (2) the management of the Company upon a scale of economy consistent with effective work. The comparative proportion of the surplus derived from excess interest in relation to the remaining sources will be observed in the experience of a particular company furnished on a later page.

Loading

In proceeding to consider the question of economy of management as a fertile source of surplus we first refer to the phrase just employed of the conjunction of saving with efficiency. The expression itself is essentially valueless as a guide; its terms are sufficiently vague and puerile to admit of almost any scope of interpretation; wherever we are confronted with this statement, and with similarly sonorous but usually void expressions, without the attachment of a reasonable precision, we may confidently consider them as belonging to that class of innumerable platitudes with which people deceive themselves into believing that they are adopting theorems and principles of vast practical worth. To employ philosophical or scientific terms is not necessarily to think and speak as a philosopher or scientist. The width of such statements resides merely in the indefiniteness of their meaning; and a sounding phrase frequently serves as an imposing mode of evading the intellectual energy and strain which precision of thought and expression essentially involves. A single star for concentrated brilliancy surpasses the diffused brightness of a nebulous mass. What we ourselves intend by this expression is the definite principle that all expenditure should be determined and defined by the aim of solidity and profit; that no

expenditure should be incurred for the sake of eclipsing a competitor in extent of new business or in displaying the superficial appearance of remarkable numerical results, or casting a fictitious halo of enterprise over operations which may really prove a deepening detriment to the permanent interests and prosperity of a Company; for it is a lamentable fact of experience that heavy expenditure once originated cannot readily or promptly be restrained in expansion without, at all events, involving a grave reflection upon the wisdom and judgment of those upon whom the responsibility of initiation rests. In no commercial undertaking is the principle of a commensurate profit forming the criterion of the scale of expenditure so vital and far-reaching as in life assurance by reason of the lengthened duration of its contracts, nor are the consequences, in any sphere of industry, of an ill-advised procedure in this respect more deeply fraught with important and, it may prove, permanent influence upon future stability and success than in the administration of these special business trusts. A poorly equipped actuary of impoverished capacity can readily command a plethora of business if he is prepared (neglectful of the effect upon profits) to subordinate the future to the present. It demands a man of strength of character as well as competence of financial skill and training so to forecast the future and weigh the tendency of actions of this kind beyond the boundary of the immediate and proximate period as to restrain judiciously and severely the lavish hand. For our natural desires, even apart from personal reputation, are in favour universally of extensive augmentations and surprising results, and, as we have just observed, it requires fortitude of character equally with clear and steadfast survey over the distant future to remain contented with acquisitions of minor extent though involving a more profoundly auspicious influence upon the prospects of solidity and profit. In this restraint and skill reside the stable and permanent conditions of continued success. An extensive new business constitutes in no degree in itself an index to wisdom of administration or prosperity of result. It may prove illustrative if we indicate, in connexion with the conduct of life business, a specially significant consideration from this aspect. In Companies trading in other descriptions of insurance the results of expenditure are related to a *fixed* amount of capital—that of the proprietors—so that if a minute margin only appear upon the transaction

(assuming that it also contains a sufficient provision for unexpired risks) the consequence forms a definite gain; but in life assurance we are essentially concerned, so far as the policy-holders are interested, with the results or profits in relation to an *increasing* capital—the capital, that is to say, which is represented by the augmented number of the assured among whom the balance of benefit is to be distributed. The large proportion of assurance business consists of policies on the participating scale of premium, and obviously the wider the extent of new business obtained (beyond the necessary requirements) the more expanded is the area over which the acquired profits are to be allotted; and unless the business be restrained within profitable limits, the share of each participant must be diminished in amount below the result which would otherwise have been attained had a more rational ambition and foresight prevailed. It is well known that the introductory charges for the completion of new business—the cost of branch offices and agencies, advertising, inspectors, and other means of extension—practically absorb (when the value of the first year's risk is included in the calculation) the whole of the year's premiums derived from fresh entrants; and hence the more enlarged be the business secured, accompanied by this inevitable burden, the greater must be the extent to which the benefits to existing policy-holders must be trenched upon in order to yield any surplus at all to the additional incomers. New business must be procured of adequate amount to supply the successive cancelments of income produced by death and withdrawal, and to provide an increasing revenue if the Company is to be maintained as a progressive and enduring corporation; but clearly, in the interests of membership in the long run and in the mass, the scale of competition in excess of these normal demands must be restricted within reasonable and remunerative limits. The members as a body implicitly assent to this expensive effort after business for the reasons and objects already expressed, but they rightly expect that the practical accomplishment of this purpose shall be conducted upon business lines, that is to say, advantageous principles and methods. The members are not concerned with *numbers* (beyond the needs of protection against appreciable fluctuations of experience); their principal and legitimate regard is the substantial condition of their Office as a financial institution and, subsequent to

this, the acquisition of the largest benefit in the form of periodical bonus returns.

It may be added incidentally that the introduction and definite insistence of the element of bonus has hardly proved to be an intrinsically propitious feature of assurance generally. The primary principle should have been preserved from the outset of a minimum scale of premiums consistent with safety, and the allotment of any periodical excess in the single form of an equivalent addition to the sums assured (thus maintaining the conception of a family provision) or in the sole mode of reducing, or rather extinguishing by commutation, the future premiums. And the method of allotment, whichever form might constitute the regulation of the Company, should have been indiscriminately applied to every contract. But the notion of bonus is deeply founded in the public mind in consonance with the spirit of speculation which attends all human enterprise, and our observations accordingly simply relate to the production of the highest returns, based upon and determined by solidity of reserves and judicious economy in management. These observations, again, must not be misconceived as tending to foster an enfeebled or inactive administration : every faculty and form of energy should be sedulously exerted in the interests of the corporation towards efficient development and widespread power of usefulness, but governed strictly throughout, in every range of work, by the considerations already expressed.

It is, of course, impracticable to indicate any definite proposition as the controlling criterion of the appropriate area of business ; but if the actuary accepts the general principles now propounded he will, if he survey the future with vigilance and thought equal to the contemplation which he bestows upon the present, speedily discover the permissible standard by which his administrative zeal should be inspired and restrained.

It is usually affirmed that in the most vigorously and cautiously administered Companies the initial expenses (including commission) of procuring new business (if, say, 7 per cent. be allowed for the conduct of the renewal revenue alone should the Company cease to compete) absorb, with the value of the unexpired current risk of death, the whole of the first year's premiums derived from the fresh members.

We have, entirely at random, examined the latest accounts of five

prominent Companies whose administration has always been marked by judgment and soundness of enterprise : no bias of selection was exercised (except that of restricting the inquiry to Offices transacting a uniformly steady and moderate new business, with successful results), and the Companies whose figures were adopted as the basis of the following tabular statement were absolutely those whose reports came haphazard to hand.

The summation of their figures shows—

New premiums for the year in question	Renewal premiums exclusive of the new business	Commission and expenses
£	£	£
85,348	1,100,172	153,345

If then the renewal income alone be capable of administration at the rate of 7½ per cent., the annual charge would be £82,513 ; deducting this amount from the total commission and expenses, we obtain the balance of expenditure of £70,832, which, in relation to the new premiums of the year, amounts to 83 per cent. This reference suggests a consideration of the proposition frequently submitted that the aggregate expenditure of a Company should be dissected into the special charges attendant upon the acquisition of business and the remaining cost of ordinary administration, with a view to furnishing a test of comparative economy of management. The analysis here advised is important and valuable, but it does not appear, from the policy-holder's point of view, to be of practical moment. We are dealing with the total effect upon profits in relation to the *entire* and *actual* burden to the assured, and we are compelled to affirm that the concern of the policy-holders does not rest in any separation in the elements of charge, however excellently and precisely effected, but essentially and solely in the bearing of the aggregate expenditure upon the aggregate premium income and the consequent result produced upon the prospects of profit and enhanced stability. If two Companies transact business at 12 per cent. and 15 per cent. respectively, the fact that the latter is spending 3 per cent. more is not touched, in relation to the benefit to policy-holders, by the circumstance that the proportion allotted to the new business in the latter percentage is inferior to that included in the former.

The preceding statements may be summarized :—

1. It is imperative that new business should be acquired in order to maintain and reasonably extend the natural growth of the corporation.

2. New business, whether restricted in extent or not, presses severely upon the premiums contributed to the common resources by reason of the necessarily expensive effort which its acquisition entails. Companies, speaking generally, whether the business be comparatively small or enlarged, must endure this considerable cost of maintenance.

3. New business therefore should be strictly and vigilantly viewed in relation to the permanent interests and prosperity of membership, and adjusted in extent as carefully as possible to adequacy of profitable return. In other words, the business (and hence its judicious limitation) should be closely scrutinised from the aspect, not of amount, but of probable permanent continuance; so that, by the receipt of future renewal premiums at a reduced expense, opportunity may be afforded for recouping the whole or the greater portion of the excess of the introductory charges beyond the regular administrative cost. The table already furnished, derived from the experience of the soundest type of Companies, supplies a significant comment upon this subject ; and

4. The amount of new business beyond the imperative requirements explained in section 1 will thus be governed, not by chance or mere persistency of effort without foresight, but under the selective principle of quality and permanence in place of quantity.

Where a Company, by want of vigour and judgment of management, has fallen into a retrograde and enfeebled condition, and where its prolonged continuance appears on valid grounds to be preferable and more hopeful (under more competent administration) than its transfer to a more powerful rival, additional expenditure must at first be necessarily and justly incurred in retrieving its impoverished fortunes and restarting it upon a career of steady advance. Here, again, the principle of guidance will be, not the mere acquisition of business as business only, or the display of large figures in effective contrast to a decadent past, but such an amount as will conduce to the recovery from languid prosperity and the declension of financial strength and provide a reconstructed foundation for future vigour.

Having regard to the grave influence upon profits attendant on the acquirement of new business, suggestive adjustments have been proposed. It has been stated that the benefit of selection (that is, the temporarily reduced rate of mortality among selected lives compared with the general experience) might be utilized as a counterpoise to the initial charges. But this proposal fails to mark the essential difference between these two elements: the expenses are an actual and immediate reduction of the fund: the advantage of selection assumes more of a deferred character, and may be largely defeated if heavy lapses occur, especially when the business involves to an appreciable extent a "forced" nature. Moreover, a portion of the financial benefit from selection must be retained in reserve in order to provide the augmented mortality which will ensue when the selective influence has naturally disappeared, independent of the adverse effect of lapses already noted. It has also been proposed that, in the allotment of bonus, the first year's premium should not be reckoned as part of the basis of participation. This appears to be unjust: the weight of expenditure is not due to the voluntary action of the fresh entrants, but to the efforts of the Company acting in the presumed interests of its existent members, and if those members consider the business to be worth acquiring upon the terms that prevail, they should share fully in the general burden of the cost.

With these remarks we proceed to a more detailed examination of this source of profit, which consists of the excess of the loading reserved in the valuation beyond the rate of expenditure actually incident: the closer the economy exercised, the more prominent obviously becomes this factor of profit, and the securer the basis for increasing achievement. If the loading reserved at the valuation be 20 per cent. upon the contract premiums, and the ratio of expenses incurred should be 13 per cent., an annual margin of 7 per cent. upon the entire net premium revenue is thus provided for accumulation at compound interest into profits. In this domain resides the practical organizing capacity of the actuary in so adjusting the actual expenditure to adequate results as to liberate as large a proportion of the unvalued loading as is practicable for the advantage of the members generally. In a preceding section we, from inveterate and loose habit of speech, repeated the worn and decrepit aphorism that the scale of expenditure should be

consistent with effectiveness of toil and progress; but we have atoned for this weakness by expounding the considerations which should define the phrase and render it practically intelligible and efficient.

It has been stated, at least on one public occasion, by the manager of an Assurance Company (which passed out of separate existence in consequence of the vigour with which in practice he carried out his doctrine) that, regarding life assurance as a social duty, a Company was justified from this aspect in expending profusely with a view to promoting this national form of prudence to the widest extent. The double folly of this proposition hardly requires exposure. Habits of thrift in the people must be the spontaneous result of individual improvement of character and foresight, produced by the stress and difficulties of life and the unhappy experience of imprudent practice, and obviously forfeits its nature as a voluntary and inherent development of mind if it be attempted by compulsory enforcement. Let sound channels and facilities for the exercise of thrift be available, but the tendency to these channels must be a deliberate choice; and, on the other hand, a Life Office exists for the sole benefit, in the shape of solvency and profits, of its particular constituents.

From the Board of Trade returns we have calculated that the rate of commission and expenses of the whole of the Life Offices (excluding Industrial Assurance Companies) was 13·79 per cent. in 1883, and 13·82 in 1900, while, during the interval, a substantial increase was apparent for many years, with a descent in recent times. The difference of ·03 per cent. appears insignificant in itself, but assessed upon the combined premium revenue it amounts to nearly £64,000 a year.

In the acquisition of new business (especially near the close of a valuation period, when interest has possessed meagre power of effective accumulation) it is to be observed that the demands upon the premiums recently obtained comprise the discharge of the abnormally heavy expenditure in securing them; the provision of a fund for the current risk of death; the proportionate compensation towards the enhanced mortality which the speedy lapse of many of these policies will produce upon the common fund; the creation of their requisite reserves, which presses with marked severity when the valuation rate of interest is very low; and the provision of a

contribution to profits. Hence, as the first year's premiums prove insufficient for these purposes, the general resources of the Company must be utilized in supplementing the deficiency, and with a new business extended considerably beyond the imperative needs of the Office the scale of bonus to existing members must suffer diminution, or, in a more favourable issue, its increase will be stayed.

In illustration of the relation between the ratio of expenditure (and consequently the rate of profit) to the extent of the new business transacted, a very interesting and instructive statement was published many years ago, based upon the returns to the Board of Trade for 1874, in which the Companies were classified according to the amount of new business effected in the course of a year. The results are here presented—

Amount of new business completed in the previous year	Number of Companies in each class	Average premium revenue	Average percentage of expenses to the premium revenue in individual Offices
£ Over 500,000	8	£ 323,000	14·5
Between 400,000 and 500,000	6	162,000	14·0
,, 300,000 ,, 400,000	18	156,000	13·4
,, 200,000 ,, 300,000	14	103,000	13·1
,, 100,000 ,, 200,000	9	72,000	15·6
	55		

It was ingeniously pointed out that regarding the column of expenditure there appeared to be two forces concurrently operating upon the ratio of expenses to the premium income. The one attains its maximum intensity (15·6) as the Company diminishes in size. This result expresses the effect upon a comparatively reduced business of the expenses invariably attendant upon a "going" concern. As the new business is augmented, this force decreases in power (owing to the extended area of the amount of business) and reaches its minimum in the largest Companies. But simultaneously with its decay another force advances in intensity; for the ratio which has been successively diminishing reascends

until it attains another maximum at the point where the former influence practically disappears—that is, where the business is of the widest extent. The framer of this analysis suggested, and no doubt with validity, that the explanation of this second change in the ratio indicated the fact that as the new business was developed the cost of securing it also advanced: a certain magnitude of assurance appears to be practicable at a moderate expenditure, but any attempt to exceed this boundary, indefinite though it be, is accompanied by more than a proportionately augmented cost. It will be observed, therefore, that a stage may be reached where an increased new business (beyond the necessary demands for maintaining the Company as a continuing corporation) produces no additional profit to the policy-holders. The same effects, in almost an identical degree, were observed when the returns were analyzed for three years and the average obtained. The figures were subsequently re-examined with a confirmation of the preceding results. It has since been stated that the Board of Trade figures for 1872, 1877, and 1882 support the view that Companies transacting a new business ranging between £300,000 and £400,000 in sums assured are conducted at a lower charge than that of Offices whose new business either surpasses or descends below that grade, while the statistics for the single year 1887 show that the most favourable ratio of expenditure (and correlatively of profit so far as this element is concerned) appertains to Companies whose new business varies from £400,000 to £500,000 in sums assured. It is at all events conclusively evident from these actual observations—looking to the rate of cost as the index of saving or profit—that the extension of new business is not coincident with expansion of profitable results to the policy-holders. A definite service of a most practical character in its relation to soundness and effectiveness of administration would be conferred if some student would examine the most recent returns and base his research upon the results of a series of five and ten years.

We have recently analysed the Returns for 1903, and find that the ratio is 16·4 for the lowest amount of new business (£200,000 to £300,000), while the most favourable percentage appertains to a business ranging between £500,000 and £600,000.

Every problem in life assurance administration—the scope of investments, the ratio of expenditure, and the amount of new

business which will probably produce a favourable or disadvantageous effect upon profits—possesses an actuarial aspect of definite significance and demands the application of professional knowledge and experience. For all these elements affect the reserves and consequently the profits realized. It may even occur, as has already been mentioned, that the effect of a considerable augmentation of new business may entail diverse consequences to an extent upon the individual ratios of bonus according to the method of distribution adopted.

A practical suggestion may here be submitted upon the assessment of expenses which pertain to the administration of the assurance business. The conduct of an annuity business (where these contracts are comprehended in the general Assurance Fund) involves commission and expenses specifically attached to these risks : and hence a truer index would be furnished to the incidence of costs entailed by the life business, and the position of the Company in this respect would be exhibited in a fairer light, if the expenses were dissected, where the Annuity Department is of any extent, and a statement presented of the portion which applies to the life premium income and to the annuity section respectively. The necessary charges for administering the Annuity Department could be reserved in the valuation and gradually appropriated to actual needs. And following upon this reasonable suggestion, it may further be counselled—and the consideration assumes a greater importance as the funds of a Company increase—that the approximate costs of conducting the finances of the Office, which can be ascertained with adequate accuracy, should also be separately mentioned and excluded from the charges applicable to the premium revenue. These specific expenses should properly be deducted from the interest earned as constituting a portion, according to the ancient doctrine, of the aggregate return required to be realized in the business of investing. It is true that, as regards the charge upon interest, the Company as a whole is not benefited by the decomposition of the total sum, since the gain in the reduced ratio of cost is balanced by the deduction from the interest-rate, but it is evident that on such a plan, if it could be made feasible, we should more exactly perceive the real pressure of the expenditure incurred in the administration of the Assurance Department of the Office.

Mortality.

The third source of surplus consists of a more favourable experience of mortality than that anticipated in the construction of the premiums and reserves.

The subject is a difficult one: a variety of opinions may properly exist both as to the reality (on the whole) and the extent of this source (treated, that is to say, as actual profit), and enlarged consideration of the question, beyond the limited bounds of this volume, appears to be required.

If a Company exclusively, or mainly, accepted assurances upon lives belonging to a class of persons which possessed as a body a superior duration of life, and charged premiums based upon a more general experience and therefore exhibiting a heavier rate of mortality, the difference between the actual and expected results would constitute a legitimate and realized surplus which, reserving a provision for periodical and accidental fluctuations, in order to secure a uniformity of experience, might rightly be regarded as profit. But the consideration and import of the position are necessarily modified by the current practice of Companies, where the anticipated mortality is measured by premiums constructed upon the observations of lives of a condition and circumstances closely corresponding in nature with those of the persons who will constitute the membership—where, for example, a Company estimates its premiums and valuations upon the recorded experience of assured lives themselves. In such a case the actual experience will in the long run, subject to intervals of deviation from time to time in opposite directions, coincide, with intimate approximation, if the risks be sufficiently numerous, with the results of the Table of Mortality employed. If for a period a favourable effect should be prevalent—that is, the expected claims should prove to be in excess of those which occur—it is clear that, since the results of the table adopted—appropriate as it has been assumed to be to the class of lives admitted—will be realized in the mass and over the whole extent of the Company's existence, the beneficial events will be followed, sooner or later, by an experience of a contrary character in order to maintain the entire uniformity of the table. This alternate sequence is obvious too from consideration of the fact that the disappearance of inferior or enfeebled lives will leave the remaining mass, by abstraction of

its weakly elements, more competent on the whole to resist death, and likely therefore to exhibit for an ensuing stage a diminished range of mortality. The reduction of claims in this subsequent interval cannot be regarded as surplus in its aspect of profit, for a portion of the balance consists of what is frequently termed suspended mortality and must consequently be retained for the purpose of compensating the increased death-rate which will occur as the ages of the assured community advance, and as fluctuations in a divergent form arise. The object of administration in this department, as in others, should be the preservation of regularity of experience, so that one stage of the Company's history should not be accidentally benefited to the disadvantage of another era. It may be reasonably assumed, regarding the present constitution of Assurance Companies' premiums and Reserves, that, on the whole, and in the long run, no very appreciable surplus (or loss) can be anticipated from this source.

This exposition is furnished in general terms only, for the consideration of details in the distribution of the amount assured and of the relative ages must enter into the survey.

We proceed in our remarks upon the assumption that the Company has founded its expectation of the death-rate upon a Table of Mortality congruent with the general character of the lives existing upon the Registers—either the H^m or the O^m Table.

If then in any year (and obviously the experience of a single year affords too meagre an area for any definite conclusion) the claims anticipated according to the Table of Mortality employed in the formation of the premiums and valuation should exceed those which have actually occurred—say, five deaths have happened instead of eight—does the difference between the two sets of sums assured, that is, the amount saved, constitute surplus in a realized sense? Evidently not, since the remaining three lives are still in existence and the Company's liability for claim at their subsequent death is not discharged. The occurrence of death is simply postponed. If each of the persons be assured for £100, the balance of £300 continues to form a liability upon the Company's resources. This incidence of the obligation is merely suspended for an indefinite time, and these additional claims may either become payable promptly or more probably, and especially where the divergence between the anticipated

and actual number is considerable, they may be distributed over a certain period, and may either coincide with the normal rate of mortality for the ages attained or again present fluctuations of occurrence. If the balance thus temporarily saved (or rather deferred) were divided as profit, the reserves of the policy-holders generally would require to be more largely depleted in atonement for the improvidence of the act. If these suspended claims arise speedily, the position will be that the remaining members are benefited by (1) receiving the actually realized rate of interest upon the sums assured for the deferred period, and (2) obtaining an increased number of premiums upon the policies, and thus being able to create an enhanced reserve for payment when the claims arrive. If the suspended claims ensue gradually after the date of calculation of the expected and real mortality, the advantage just mentioned will clearly be increased, and the surplus would consist of the difference between the anticipated and actual amounts diminished by the reserves required to be retained against the postponed demands. And again the time will occur when the benefit described will be succeeded by opposite effects, where the premiums received will be reduced below the calculated number and the interest on the sums assured be not merely lost but payable out of the general funds. From the nature of the case moreover this mode of consideration possesses too general and theoretical a character to admit of numerical estimate. A preciser and more adequate method of assessment has been devised, which we proceed to explain. This method endeavours to ascertain the " strain " (as it has been termed) or the pressure of the claims upon the resources of a Company. Every policy after completion possessing a reserve value increasing annually in amount, the extent of risk consists of the difference between the sum assured and bonuses (if any) and the existent reserve : the cost therefore of the claim is expressed by this balance, which accordingly provides the measure of the strain of mortality experienced. The claims which occur are discharged partly by the reserves retained against these specific assurances, and partly by the current premium income ; and so far as the difference between the amounts payable and the accumulated reserves diminishes, as the duration of the policies (which become claims) is prolonged, to that extent the strain upon the general resources of the Company is reduced. An

apparently favourable mortality experience (on a mere comparison of the actual number and amount of the claims with those anticipated) will really prove the reverse if the deaths have occurred in connexion with policies of brief continuance, where the reserves are necessarily small; and again a total amount of claims in excess of that expected may display a fortunate issue if the assurances thus becoming due possessed enhanced reserves in consequence of extended duration. In calculating from the Table of Mortality adopted the sum expected to become payable by death during the course of a year the aggregate amount assured by the Company is viewed in relation to that proportion of it which the estimate shows likely to be demanded, and in the same manner should be set aside from the value of the liability under the whole of the policies that portion of such value which may be regarded as the reserve for the expected amount. The difference between these two aggregates furnishes the standard by which the strain by death can be assessed, and a surplus or deficiency* be attributed to the mortality experienced according as the balance between the sum assured and the reserves for the claims actually occurring is less or greater than the difference above ascertained. Let S represent the total sum assured, E the amount of expected claims; V the reserve value for S, and V^1 the expected reserve for E: then $E-V^1$ forms the criterion of the anticipated strain. If the actual claims amount to A with a reserve of V^{11}, the difference between $(E-V^1)$ and $(A-V^{11})$ expresses the surplus or deficiency upon the mortality experience according to the result whether $(E-V^1)$ exceed $(A-V^{11})$ when a surplus remains, or prove less than the $(A-V^{11})$ when a deficiency appears. Thus if $E-V^1$ be $200-30=170$, and $A-V^{11}$ be $150-60=90$, a surplus of 80 is evident; while if the figures be reversed a deficiency of 80 is the consequence. In the former event the sum of 170 was expected to be derived from the general assets, while the real demand upon the assets has only proved to be 90, and so reversely in the succeeding event.

Or, more generally, let ΣS represent the total sum assured (where Σ is the symbol of summation), and ΣV the aggregate reserve: if q be the probability of death in the year, $q\ (\Sigma S - \Sigma V)$

* It will be observed that the terms "surplus" and "deficiency" are restricted to the relation ascertained to exist between the anticipated and the experienced condition of the result.

measures the anticipated strain: if the actual claims amount to $(q + y)\ \Sigma S$ and possess a reserve value of $(q + z)\ \Sigma V$—where y and z may be positive or negative*—then the real death-strain upon the resources generally is expressed by $(q + y)\ \Sigma S - (q + z)\ \Sigma V$, or $q\ (\Sigma S - \Sigma V) + y\ \Sigma S - z\ \Sigma V$: so that the quantity $y\ \Sigma S - z\ \Sigma V$ shows a surplus or deficiency from favourable or adverse mortality according to its negative or positive character as a whole. If the accrued claims prove to be less, but their reserve value greater, than the amount expected—that is, if y be negative and z positive—the quantity $y\ \Sigma S - z\ \Sigma V$ will be negative, and will thus signify surplus. If y be positive and z negative, the total quantity will be positive and will thus exhibit a deficiency; or the actual claims may be excessive in amount compared with those expected, and their reserve value may also be higher than that anticipated, in which case y and z will be positive: or both the sum and reserve may be inferior to the amounts expected, when y and z will be negative; and in both these circumstances the final sign of the quantity will depend upon the comparative magnitudes of the parts of which it is composed. Briefly, the expected aggregate strain is $q\ (\Sigma S - \Sigma V)$: the actual strain, $(q+y)\ \Sigma S - (q+z)\Sigma V$, or $q\ (\Sigma S - \Sigma V) + y\ \Sigma S - z\Sigma V$—where, as has been shown, $q\ (\Sigma S - \Sigma V)$ represents the measure of the anticipated strain. Hence if $(y\ \Sigma S - z\Sigma V)$ be positive as a whole, the $q\ (\Sigma S - \Sigma V)$ is increased or the actual strain surpasses the expected, with an unfavourable result; while if the expression $y\Sigma S - z\ \Sigma V$ be negative as a whole, the $q\ (\Sigma S - \Sigma V)$ is diminished or the experienced strain is reduced and a fortunate result created.

The comparison has been expressed in the following form: the actual strain is less than, equal to, or greater than the expected strain—that is, there results a surplus, an equality, or a deficiency,

* If y, for example, be negative, and therefore $q - y$ be less than the expected q, the actual amount of claims is inferior to that expected: if z be negative, and accordingly $q - z$ be less than the anticipated q, the reserve retained against the actual claims is less than their expected reserve. On the contrary, if y be positive, then $q + y$ being greater than q, the actual claims exceed those anticipated: if z be positive, and therefore $q + z$ is $> q$, the reserve actually in possession against the claims that occur is superior to the relevant reserve which it was expected would exist. If y and z be each equal to zero, the indication is that the actual claims and their actual reserves are equivalent to the amounts respectively anticipated. If y and z be equal to $-q$ no claims occur: the supposition of y and z being less than $-q$ would be unmeaning.

according as $q\ (\Sigma S - \Sigma V) + y\ \Sigma S - z\ \Sigma V$ is $< = > q\ (\Sigma S - \Sigma V)$, or, cancelling and transposing, as $y\ \Sigma S$ is $< = > z\ \Sigma V$—that is, according as the variation in the amount of the sum assured which has become demanded by death is less than, equal to, or greater than the variation in the reserve value appertaining to that amount, and having regard to the signs of the different sides of the inequality. As an example : let $y\ \Sigma S - z\ \Sigma\ V$ be negative, or, what is the same thing, $y\ \Sigma S$ is less than $z\ \Sigma V$, then the left-hand side of the inequality (by the deduction of the quantity) is diminished below the right-hand side—that is, the actual strain is inferior to that expected and a favourable result is produced. Similarly, if $y\ \Sigma S$ be $> z\ \Sigma V$, a loss is shown, while if $y\ \Sigma S$ be equal to $z\ \Sigma V$ (or the expected condition be precisely fulfilled) the expression becomes $q\ (\Sigma S - \Sigma V) = q\ (\Sigma S - \Sigma V)$ or the actual strain is identical with the strain anticipated. It will, of course, be borne in mind that, in the preceding investigation, q is used of a *status*, so that probably the expressions ΣqS and ΣqV would be preferable, q naturally varying with the age.

The following concrete illustration which has been published affords a significant comment upon the futility of statements that the number of deaths has proved less than the number expected, or the amount of claims inferior to the anticipated payments, as though these bare facts, without an analysis in the mode just described, furnished any index whatever to the favourable or adverse incidence of the experienced mortality. Let a Company possess assurances of £208,000 in force at the age of 70, comprising policies effected for various amounts at the quinquennial ages of 15 to 65, both inclusive, distributed in relation to the ages employed in this illustration as follows : £4,000 at age 15, £8,000 at age 20, £8,000 at age 60, and £4,000 at age 65. The total pure premiums (including those for the 4 selected ages) will amount to £5,578 ; the entire reserve on the H^m 4 per cent. Table will be £108,991 ; so that the amount actually at risk is £208,000 – £108,991, or £99,009. According to that table a sum of £11,926 is payable in respect of the claims for the past year (q_{69} = ·05734, and ·05734 × £208,000 = £11,926). The total reserve value at age 70 will, as shown above, amount to £108,991 ; and assuming that the mortality is uniformly distributed over the entire assurances of all ages at entry, so that every section of the business

is affected by death, we obtain for the estimated reserve value of the expected claims of £11,926 the sum of £108,991 × ·05734, or £6,249; hence £6,249 is provided from reserves, and the balance of claims, or £11,926 – £6,249, or £5,677, must be obtained from the general resources, and represents the strain upon the funds.

Thus it will be noticed that q (£208,000 — £108,991), or £5,677, is the q ($\Sigma S - \Sigma V$) above described, or the measure of the strain by death. But let it happen that the actual deaths occur entirely under the oldest policies, namely, those effected at ages 15 and 20 (say, the whole of the amount completed at age 15, or £4,000, and £7,926 out of the £8,000 accepted at age 20), the total amount is identical with the sum formerly assumed, or £11,926; but the corresponding calculation shows (on the incidence of mortality now supposed) a reserve value at age 70 of £7,554 in respect of these claims, leaving therefore a balance of liability upon the Company of £11,926 — £7,554, or £4,372, as expressive of the actual strain; the total expected strain as already exhibited being £5,677, there results the favourable balance of £5,677 – £4,372, or £1,305; in this mode of direction, consequently, of the mortality, the strain is 77 per cent. of the expected (4,372 : 5,677 :: x : 100). Assume, on the other hand, that the lives recently admitted alone have died (involving, say, £7,926 out of £8,000 effected at age 60 and the entire £4,000 completed at age 65)—the actual aggregate claims thus continuing to be constant, or £11,926: the reserve value of these assurances will be £3,109, so that the real strain exists of £11,926 — £3,109, or £8,817; and the measure of the total strain being as before £5,677, there results a deficiency of £3,140 (or £8,817 – £5,677). The experienced strain of £8,817 bears the proportion to the estimated strain of £5,677 of 155 per cent. Hence with a precisely identical amount of actual claims in the year there may exist a divergence between the real and the expected events ranging from 77 to 155 per cent.—that is to say, a surplus of £1,305 or a deficiency of £3,140, or, in dependence upon the distribution of the mortality, any other intervening percentage of surplus and deficiency between these limits of the illustration.

A summary may be presented, adopting for illustration the appearance of a deficiency :—

I. The total expected claims are $q\ \Sigma S$ = £208,000 × ·05734 = £11,926;

II. The expected reserve applicable to these claims is $q\ \Sigma V$, or £108,991 × ·05734 = £6,249 ;

III. The expected strain is consequently £11,926 — £6,249 = £5,677 ;

IV. The amount of the actual claims where the deaths have occurred exclusively in connexion with the most recent policies is £11,926 ; the reserve retained for their provision is £3,109 ;

V. The experienced strain is accordingly £8,817.

VI. The difference between the anticipated and real strains, or £3,140 (that is, £8,817 — £5,677), constitutes the measure of the deficiency.

These detailed figures conclusively prove, as a general consideration of the problem would indicate with equal validity, that the mere statement of a comparison between the expected and actual number of deaths, or between the amount demanded for the satisfaction of claims and the sum anticipated, is meaningless and furnishes no indication of the beneficial or unfavourable character of the mortality which has been experienced. It is here, again, to be noted that the resulting surplus cannot be construed as profit in the customary acceptation of the term.

An admirable and serviceable process has been suggested for ascertaining the surplus or deficiency resulting from the mortality experience, not in respect of a single year but, during the entire currency of a valuation period. This method possesses the additional merit of acting as an effective check upon the yearly analysis. Applied to an individual year's experience, the reserve at the commencement of the year is increased by a year's interest (which of course it earns), and to this accumulated amount the difference is added between the pure premiums and the claims of the year augmented by half a year's interest (on the usual assumption of a fairly uniform distribution of receipts and payments, this balance may be supposed to occur in the middle of the year and hence to be invested for half a year) ; the reserve created at the close of the year is then deducted, when the result furnishes the surplus or deficiency derived from the mortality experienced during the year. The excellent suggestion in question extends the process to the quinquennial period by accumulating the total reserves at the beginning of that term for five years ; adding the accumulated differences between the pure premiums and claims (for the

respective years), and deducting from the resulting sum the reserves formed at the expiry of the period. Expressed symbolically, the method becomes

$$V_0(1+i)^5 + \left\{ \begin{array}{l} (\pi_1 - d_1)(1+i)^4 \\ +(\pi_2 - d_2)(1+i)^3 \\ +(\pi_3 - d_3)(1+i)^2 \\ +(\pi_4 - d_4)(1+i) \\ +(\pi_5 - d_5) \end{array} \right\} \times \left(1 + \frac{i}{2}\right) - V_5$$

Here the factor $\left(1 + \frac{i}{2}\right)$ provides the additional half year's accumulation upon the sum of the balances between pure premiums and claims; thus, selecting the third year, the balance is to be compounded for three and a half years, and so on; while V_0 and V_5 represent the aggregate reserves at the commencement and end of the quinquennial period, and π, d, etc., express the total pure premiums and claims appertaining to the year indicated by the subscript.

It has, we think, been generally experienced that the rate of mortality in a Company, in respect of its advantageous or unfavourable nature, is, to a large extent, a function of the amount assured upon the individual lives where the policies are effected for ordinary prudential purposes—the benefit increasing with the augmentation of the sum. The explanation appears to be evident. The largest policies are effected by persons occupying a higher social and financial position, with the control of all the appliances for the preservation of health which wealth can command. It might also be stated that this beneficial consequence is a function of the rate of premium per unit assured, for it is well known, for example, that policies for a temporary period present the least satisfactory results. This again, without suggesting other considerations, is apparently due in a large degree to the fact that if the object for which the brief protection was required should terminate prior to the expiry of the assurance, those policies where the assured had meantime succumbed to deterioration of health or disease would tend to be maintained as a speculation, looking to the comparatively insignificant cost involved in the premium charged.

An interesting tabular statement was presented some years ago, which may here be reproduced in a somewhat different form from

the original, exhibiting in actual experience the varied proportions in which the surpluses of four quinquennial periods were distributed between the three sources we have described, and confirming the observation that, under existing conditions of assurance business, the primary fount of profit consists of the excess of interest realized beyond the valuation rate upon the aggregate funds.

The entire divisible profit included some minor items, which we omit.

Of the profit declared at	The following were the respective percentages of contribution from the sources of		
	Surplus interest	Mortality experience	Loading saved
1. One division	91·95	(—) 32·86*	40·91
2. The succeeding division .	61·44	4·42	34·14
3. The next division . . .	43·85	24·14	32·01
4. The fourth division . .	41·36	25·41	33·23
5. The average for the four periods	54·08	11·76	34·16

We now proceed to the explanation of some principal modes in which profits are distributed among the assured, with a few comments for the purpose of expounding them more clearly by indicating their results. It is first desirable to express the general principles which should govern the system of division which may be adopted.

I. The interests of existing members who have built up the solidity and prosperity of the business should be carefully regarded consistently with general attention to the common interests (and therefore also to their own).

II. Equity of treatment should be observed between members of different ages, durations of membership, and descriptions of assurance. A natural corollary is, that if a system is ascertained by experience to press unjustly, for example, upon longer or shorter terms of membership, a complete change should be effected,

* The mortality results in this quinquennial period were adverse.

notwithstanding the dislocation which may ensue between previous results and those produced by the more adequate substituted scheme. Changing conditions and lengthened experience will generally demand at intervals a revision of any method, especially of those organized in less enlightened times of actuarial knowledge ; and if, under the cancelled plan, the older policy-holders (that is, the assurances of more extended duration) have benefited to an excessive extent, the depression of their bonuses under a modified system has been compensated by the abnormal profits they have received in the past. All alterations of method, however effectively constructed, must necessarily entail some anomalies in practice ; and when, after an adequate trial of an existing scheme has been experienced, a change has become imperative in the permanent interests of all members, it appears to be the expedient and juster course to adopt the substituted method universally and summarily without introducing the invidiousness and appearance of partiality and preference for older policy-holders, which might be suggested by treating them as a separate section subject to their simultaneous retention of the method which is rejected for future entrants. This practice does not contravene the first principle, since a new plan which possesses an equitable character, fair to all, will tend to induce assurances to the Company (and probably, in consequence of its attractiveness, at a diminished expense), and will thus aid, by the profit on a wider and less costly business, in compensating the temporary reduction which the present policy-holders must sustain. But independent of all special considerations, the dominant claims of uniform justice demand the decisive abrogation of any system, whatever be the partial consequences, which does not, with equal scrupulousness and approximate precision, conserve the several interests of the entire body of members. Circumstances, however, may exist where a limited series of sections of Assurance may be reasonably adopted and different methods of allotment be employed, provided general equity be not impaired by appropriating to any series proportions of surplus in excess of their estimated contributions. As the conditions of Companies may widely vary in this aspect, and accordingly involve diverse considerations, it is only feasible to express upon the subject a general judgment, with the addition of the remark that primary attention should be assigned to the fact, that an Assurance Office

should, as closely as possible, be essentially regarded as a harmonious whole of interests in place of an aggregation of different (and frequently divergent) classes.

III. Care should be exercised that the mode adopted should not, by exceptional or selective treatment of any class (unintentional, of course), tend to repel fresh entrants, and thus produce stagnation or retrogression.

IV. Profits should, as far as possible, be recognized as a common fund produced by the members as a whole ; a reasonable and general appropriation might be attempted in proportion to the several contributions ; but this course, if deemed wise, should essentially affect the profit derived from surplus interest alone. And, as will be perceived hereafter, even this limited principle, when embodied in practice, may demand some modification for equitable working.

The assurances of extended continuance undoubtedly contribute more largely than those of more recent admission by reason of the enhancing reserves, invested at the realized rate of interest, yielding an increasing return beyond the valuation rate ; and this definite circumstance, it may be urged, should be recognized in the apportionment of shares.

With reference to the sources involved in the loading and the mortality experience, the distinction is not so clear. The new business, if secured in moderate and needful amount, is obtained presumably for the benefit of all ; the mortality results lie beyond the determination of the policy-holders, and rest primarily upon judgment in the original selection and upon natural vicissitudes incident to every enterprise of this nature. Minute subdivision of profits, based upon an intricate analysis, may assume an arbitrary character, and the general and sound conclusion would appear to be that if any discrimination of specific contribution be attempted, the analysis should not extend beyond the element of surplus interest. Even here, however, as will be shown at a later stage, the operation of this principle in a literal form may produce (with the duration of policies, and especially of endowment assurances) effects of a disadvantageous nature in relation to the interests of the entire body ; and the inherent difficulties of every suggested mode of analysis present so formidable an aspect, that a general and simple system, like the compound reversionary bonus plan (which involves, without specific appropriation, the consideration of the

additional interest earned), appears to exhibit supremacy of practical adaptability and comprehensive fitness of service. It may be repeated that, in general, different series of policy-holders as a part of the system of distribution should as far as practicable be avoided. The obstacle to an equitable apportionment of the general expenditure, for example, is serious; the trouble and cost of separate accounts are involved; assessments, arbitrary to some extent, cannot therefore be evaded; and the paramount aspect of a Company should be that of a commonwealth of assurance. The prediction may be hazarded that, in some future time, the voluntary contributions by the policy-holders to profits (as to risk), expressed by the contract premiums themselves, will constitute the primary element in the allotment of surplus: excess interest realized, an advantageous incidence of mortality, and the margin of loading, forming merely (so to speak) involuntary and indirect additions to the common stock. The justification of the premiums alone as the principal factor of distribution is based upon their constituting the exclusive source of interest, reserves, payment of expenditure, discharge of claims, and consequent creation of profit.

V. A primary consideration in the choice of a method of division is its elasticity and pliability. Many systems have signally failed on account of the rigidity of their construction, so that even a moderate change of popular fashion in its main selected form of assurance, or a comparatively slight alteration in the conditions of business, has rendered the method inequitable and impracticable. A framework, therefore, of adaptable character, competent of prompt and facile adjustment to varying states of experience without the production of severe and abrupt divergence between recorded and future results, constitutes a primal condition of method. In all departments of commercial enterprise altered circumstances demand amended modes; and that system testifies to the soundest judgment and most cultured forecast which involves, as these variations occur, the least possible departure from its original form, or, more properly expressed, is capable of the most ready and continuous adjustment to the conditions of acquired experience that may arise. No method can be devised against which theoretical and practical objections cannot be validly urged, for the difficulty resides in the happy conjunction of expediency with equity and sagacious foresight.

VI. The aim of any plan should, so far as is accordant with equity and consistency, be the provision of a reversionary bonus increasing with the duration of the policy. It is true that profits are not produced in the form of reversionary sums, but of cash amounts, and with the advance of age a stated cash allotment necessarily yields a diminishing reversionary equivalent. But a method of this description—that, namely, of a cash percentage apportionment—which fails to include in its application the bonuses also which remain attached neglects improperly a cognizance of the fact of the annually augmenting reserves, and the contribution which they accordingly render to the principal source of profit—the surplus interest. The public consequently appears to exercise a sound judgment when it concentrates its chief regard upon the reversionary amount as the index of a profitable membership.

VII. A capital feature should be the simplicity, and consequently the ready intelligibility to the public, of the plan pursued, and its congruity with the general notions of profit-sharing which are derived from the experience and practice of ordinary business affairs. The introduction of theoretical and minute refinements, which are both perplexing and repugnant to common-sense perception, should be avoided. This promptness and approval of apprehension are not limited to the verbal enunciation of the system, but apply also to the clear and commercial acceptance of its operation and effects. The public can recognize and appreciate a scheme which involves the commercial principle of allotting profits—without nice and complex distinctions, which are sometimes dependent upon the judgment of the individual actuary—in proportion to the respective contributions to their formation.

Curious notions were occasionally current upon the true measure of apportionment in the earlier history of actuarial science. The relation between the amount of bonus assigned and the extent of a policy's contribution to the profit fund—which constitutes, as regards surplus interest, the measure in the judgment of many actuaries of the benefit it should receive—rarely entered into consideration, or only in the crudest form.

One method of division consisted of the exclusive allotment of the whole of the profits realized in any year among the representatives of the assured who happened to die during the currency of that year. This extraordinary arrangement was defended on the plea

that co-partnership terminated with death, and that the retiring partners were entitled to the profits existing at the date of their withdrawal. Not a thought was bestowed upon the continuing partners who had equally contributed to the creation of the surplus, and the necessity of retaining a proportion for ultimate apportionment when their retirement from partnership had also occurred by death.

Another system distributed the profits at a uniform percentage per annum upon the sum assured, reckoned at each division from the date of original entry into the Company. Thus, at any valuation when the policy had endured for n years, the allotment was granted for that period; at the succeeding distribution, when the assurance had continued for $2n$ years, a share of profits was assigned in respect of $2n$ years, and similarly at each investigation which occurred during the policy's entire duration. The increasing injustice which this plan inflicted upon the recent entrants is evident, and the effect would naturally be that the influx of new business would be thwarted, or could only be sustained against the repellent force of this adverse treatment by augmenting expenditure, which again would produce results in the same unhappy direction by reduction of the profit fund through the prior mortgage upon its accumulation in favour of the assurances of extended standing. The principle of this method roughly expressed no doubt the consideration that the early policy-holders, who had incurred the risk of establishing a stable and solid business, and had thus enabled subsequent entrants to apply with confidence, were entitled to some compensation for the hazard they had originally sustained: the plan further involved the important element (though without any express recognition) that as a policy became more valuable, by reason of its expanding reserve, the expectation of an ampler share in the successive profits was justified. But the soundness of the implicit principle was marred in its practical execution, since the course pursued failed to appreciate the fact that each valuation formed a closed account, and that consequently a new era originated with the ensuing valuation period to which the fresh allotments, from the nature of the case, are restricted in respect of their range.

In accordance with another plan, adopted in earlier times, the profits were divided in the proportion indicated by the difference

between the contract premiums accumulated at compound interest and the reserve values of the assurances. This scheme received the sanction of able actuaries at that date, but is now merely mentioned for the sake of directing the student's attention to the neat mathematical analysis which was employed to expose its invalidity. The proposition in question, expressed symbolically, is $\Sigma p - \Sigma V$, where Σ indicates the process of accumulation, p is the contract premium received, and V the reserve. Now $\Sigma V = \Sigma\pi - \Sigma c$, where π is the pure premium, and c represents the claims. The first expression becomes by substitution $\Sigma p - (\Sigma\pi - \Sigma c)$ or—decomposing p into its constituents of π and the loading ϕ—$\Sigma\pi + \Sigma\phi - \Sigma\pi + \Sigma c$ or $\Sigma\phi + \Sigma c$. Hence the method allotted the profits, not simply in the proportion in which the policies had contributed to produce them, but in the strange ratio of their combined contribution to the surplus and to the payment of the claims.*

A few examples of other methods may be adduced.

I. The division may be based upon the proportions exhibited by the contract premiums received since the preceding distribution and accumulated at interest. If the premium be formed from the pure premium by the addition of a percentage, some reason may be alleged in favour of this plan. But the proper mode of loading consists of a constant quantity and a percentage—the percentage representing the commission and margin for profit, and the constant being the common contribution to the fixed expenses of administration; for it is obvious that, apart from commission, each policy entails the same amount of financial burden in respect of the conduct of the renewal business; hence with the assumption of the mode of loading which would tend to support the justice of this scheme of division, the scheme would again be condemned by the unfairness of the construction of the loading for general expenditure. At age 35, the pure premium (O^m 3 per cent.) is ·2·116 per cent.; at age 55, 4·641 per cent., and a

* Even at the present date—though very rarely—a somewhat similar method is pursued. The surplus is distributed in proportion to the difference between the accumulated pure premiums, received during the valuation period, and the reserves at its close. The ratio of participation is consequently $\Sigma\pi - \Sigma V$. But $\Sigma V = \Sigma\pi - \Sigma c$ (where c represents the claims which have occurred): by substitution, accordingly, we obtain as the ratio $\Sigma\pi - (\Sigma\pi - \Sigma c) = \Sigma c$, that is to say, the extraordinary ratio of the contribution to the payment of claims.

loading for current charges of management of (say) $2\frac{1}{2}$ per cent. would be 1*s.* 1*d.* at the former age, and 2*s.* 4*d.* at the latter; or the policy-holder of the older age at entry would contribute more than twice the amount subscribed by the younger assured towards the permanent demands in place of the fairer assessment of a constant sum. A practical difficulty also exists—the fact, namely, that since the premiums received during a quinquennial period form a constant quantity, the ratio of participation expressed in cash will continue identical or become reduced according as the total surplus shows, or fails to show, a proportionate increase; hence as the ages at valuation advance the reversionary bonus will gradually diminish with the duration of the policy, and thus defeat the reasonable expectations of the assured in respect of maintained or enhancing bonuses. The plan neglects the important constituent in the contribution to the Profit Fund (and consequently the extent of participation), consisting of the surplus interest upon the augmenting amount of reserve.

II. The bonus has been allotted in proportion to the values of the policies.

If the rates of premium in a Company exhibit a diversity—where, for example, other Offices have been absorbed—the assurances whose premiums include a larger loading will receive the same bonus as those where the loading is inferior (the ages of the policy-holders and the dates of their policies being the same), since the reserves are based upon a pure premium standard, whence the loadings are excluded. If the participating premiums were constructed upon a uniform scale (composed of a percentage and a constant for margin), and founded upon a true basis representative of the Company's actual experience of mortality or the collective experience of Assurance Offices generally, the method would provide compensation to new policies at future valuations, since the values of the policies annually increase. But the important difficulty enters in practice that, since the profits contributed by the more recent policies are chiefly diverted to the assurances of prolonged duration, these depleted policies can only anticipate a recompense for their sacrifice out of the contributions of future members, which may never arrive. For the tendency of a method of this description is to prevent the acquisition of new business, and hence to frustrate irretrievably

or largely the intended compensation by the repellent power of a plan of division partial in its practical execution.

III. The profits may be distributed among those policies only upon which the premiums accumulated at a defined rate of interest equal the sum assured—the origin of participation occurring when this equilibrium has been attained. This plan has been described as opposed to the principles and intention of the system of life assurance. This criticism appears to be somewhat extreme, and omits consideration of the different views which the public may entertain and appreciate as determining their decision when they assure. If the results are clearly explained, no valid objection, however unscientific the method may be, can be urged; each person must individually select the mode which appears to satisfy his particular requirements. The plan involves the benefit of reduced premiums and the disadvantage of the absence of surplus during the course of perhaps twenty to forty years. The scheme, of course, is irrelevant to the system of life assurance in itself where the premature, or, more precisely, the early deaths require essentially as an integral part of the contract to be provided by the contributions from policies which longer endure, but it does not seem to follow necessarily that arrangements relating to the allotment of surplus should exactly embody the principle on which assurance itself is based.

IV. The plan may be adopted of allotting to each assurance a uniform reversionary addition to the sum assured and existing bonuses payable on the maturity of the policy, and calculated from the close of the preceding valuation. This is termed the "compound reversionary bonus scheme." Although unscientific in formation, the plan possesses real and popular merits. It is readily intelligible to the public; it rightly, from a practical and genuine point of view, includes approximately the proportionate distribution of the surplus interest realized upon the augmenting reserves, so that the reversionary bonuses (where previous bonuses remain attached) will exhibit a series of gradual increments in accordance with popular expectation; and its practical execution is exceedingly simple. It has been proved that the results produced are fair if the net rate of interest realized exceeds the valuation rate by from $\frac{3}{4}$ to 1 per cent. per annum; and any theoretical defect is probably neutralized by the consideration

of the benefit to the Company (that is to say, to the whole of the policy-holders) of the enhanced profit derived from excess interest upon long-continued assurances, and the provision of a permanent source for payment of expenses in consequence of the attraction of maintained membership due to the prospect of increasing returns.

The following illustration may be of service, although in life assurance problems the mass of the contracts at the several ages should be included in the survey.

A policy for £100 is effected at age 35 at the pure premium of 2·193 per cent., based upon the H^m Table at 3 per cent., and let the net rate realized be $3\frac{3}{4}$ per cent. The profit derived from interest for the first 10 years or in respect of two valuations (resulting from the additional $\frac{3}{4}$ per cent.) is obtained by the formula—

$$\frac{\pi_{35}\,(N^1_{34} - N^1_{44}) - M^1_{35} + M^1_{45}}{D^1_{45}} - {}_{10}V_{35}$$

where π_{35} is formed at 3 per cent. and also ${}_{10}V_{35}$, and where the functions indicated by the accented symbols on the columnar notation involve a calculation at $3\frac{3}{4}$ per cent.,—the element of the death-rate thus remaining, of course, unchanged.

The corresponding profit for the first five years is deduced by insertion of the appropriate elements in a similar manner. Hence the profit for the second five years is—

$$\left(\begin{array}{c}\text{the profit for}\\ \text{ten years com-}\\ \text{puted as}\\ \text{above}\end{array}\right) - \left(\begin{array}{c}\text{the profit}\\ \text{for the}\\ \text{first}\\ \text{five years}\end{array}\right)\left(\frac{D^1_{40}}{D^1_{45}}\right)$$

(for $\frac{D^1_{45}}{D^1_{40}}$ is the present value at the age at the first distribution of £1 payable five years thence ; hence—

$$\frac{D^1_{45}}{D^1_{40}} : 1 :: \left(\begin{array}{c}\text{the profit}\\ \text{for the}\\ \text{first five}\\ \text{years}\end{array}\right) : x \left(\begin{array}{c}\text{the accumulated}\\ \text{amount of such}\\ \text{profit at the}\\ \text{expiry of the}\\ \text{second five years}\end{array}\right),$$

$$\text{or } x = \left(\begin{array}{c}\text{the profit}\\ \text{for the}\\ \text{first five}\\ \text{years}\end{array}\right)\left(\frac{D^1_{40}}{D^1_{45}}\right)$$

By calculation according to the first portion of the formula the result will be found to be 7·399 at the close of the first five years, and deducting the reserve at 3 per cent., or 7·203, the cash surplus produced by the additional $\frac{3}{4}$ per cent. of interest amounts to ·196 per cent. assured. At the age of 40 then attained the sum required to provide £1 at death (A_{40} at 3 per cent.) is ·4706; hence the ·196 will furnish at death the reversionary bonus of ·4165, which for the past quinquennium affords a reversionary bonus at the rate of ·0833 per cent. per annum. Will the corresponding surplus on the expiration of the second five years prove sufficient to yield this rate of bonus for that quinquennium upon both the original sum assured and the accrued bonus of ·4165? The application of the formula to the calculation for the ten years gives a result of ·75, and deducting the accumulated profit existing at the close of the first five years $\left(\text{involving } \frac{D^1_{40}}{D^1_{45}}\right)$, we obtain, as the cash surplus produced by excess interest during the second quinquennium, the sum of ·501. At age 45, £1 at death will be furnished by the immediate payment of ·51669; hence the sum of ·501 will provide ·97 at death, or distributed over the preceding five years the return of ·194 per cent. per annum upon the original amount assured, or, assessed upon both that amount and the attached bonus of ·4165 together, a compound annual rate of ·193 per cent., being in excess of the maintenance of the previous rate of appropriation. It is true that the pressure involved in the continuance of the original return increases as the duration of the assurance extends, but, on the other hand, a higher amount of surplus interest is derived from the investment of the augmenting reserves.

It may be useful to exhibit the process of ascertaining the uniform compound addition which a stated surplus is sufficient to provide. Let the surplus be £50; let one policy for £500 (effected at age 35) have endured for three years at the date of valuation; another for £1,000 (completed at age 37) for two years; and a third for £700 (effected at age 33), for one year. Assuming as a datum for the required rate of bonus the amount of £1 per cent. per annum, then the bonus addition to be attached on this basis to the first policy is $5 \times 3 = 15$; to the second, $10 \times 2 = 20$; and to the third, $7 \times 1 = 7$. The present values per unit on the H^m 3 per cent. Table at the

valuation ages of 38, 39, and 34 are ·45, ·46, and ·42 ; the values consequently of the supposed bonuses amount to 6·8 (*i.e.* 15 × ·45), 9, and 3, so that the sum of 18·8 will produce a uniform addition of £1 per cent. per annum for the valuation period. Hence the realized uniform increment is obtained from the proportion, 18·8 : 1 : : 50 : x, or 2·66 per cent. per annum. The uniform rate to be ascertained might, of course (and more systematically), be generally designated x : then in the preceding example we have—

$$500 \times 3 \times \frac{x}{100} + 1{,}000 \times 2 \times \frac{x}{100} + 700 \times 1 \times \frac{x}{100},$$

or the aggregate reversionary bonuses to be provided amount to x (15 + 20 + 7). To obtain their present values and equate their sum to the amount of divisible surplus we have $15x$ (·45) + $20x$ (·46) + $7x$ (·42) = 50, whence $x = \frac{50}{18{\cdot}89}$ or the former result, allowing for the abbreviated decimals.

In actual practice the existing bonus additions would be added to the sums assured as the basis of computation.

The mode known as the "simple reversionary bonus plan" declares the uniform rate of bonus addition upon the original sum assured alone. It thus neglects the factor of remaining bonuses allotted at previous distributions, and hence so far deviates from the principle of proportionate assessment by omitting consideration of the contribution made to profits from the surplus interest realized upon the successively increasing reserves for the bonuses already attached and retained.

V. A further method endeavours to analyse the sources of profit, and the several contributions of the assured to the creation of the divisible profit fund. At a valuation a certain reserve is provided on the basis of a specified rate of interest (that is, the valuation rate), say, 3 per cent. At the ensuing valuation this reserve has been accumulated at the realized rate of £3 15*s*. per cent. ; hence it has earned an annual surplus consisting of the difference of its accumulation at £3 15*s*. and 3 per cent. That portion of the total profits declared at the investigation which represents this accretion from excess interest is applied to the policies by which it has been formed, and the remaining balance of profit is then divided among the whole of the policies in

proportion to the premiums received during the quinquennium, or to the loading involved. In a Company steadily progressing and free from extensive fluctuations of experience, this method apparently produces no unjust diversities of apportionment. It will be noted that the primary lien upon the aggregate surplus is the allotment of the increments of excess interest to those assurances possessing a reserve at the prior valuation; hence all fluctuations of mortality and expenses press chiefly upon the portion assigned to the more recent entrants. If during a prevalent form of disease which mainly affected older people a severe rate of mortality should be experienced among the lives of higher ages,* while the younger* assured exhibited a favourable result, then, since the absence or deficiency of any surplus from mortality is cast upon the general profits as essentially occurs under this system, the bonus apportioned to the newer entrants would be greatly reduced, while, of course, the surviving policies of longer duration would be secure in their primary charge upon the Profit Fund. If, on the other hand, the mortality of the more recent members proved to be excessive, the older assurances would still retain their first lien, and the remaining profits would tend to diminish. This consideration is the more significant in consequence of the fact that the principal source of profit consists of the surplus interest realized upon the funds, so that a minor benefit exists for the later policies, which must depend, for their enhanced share, in compensation, upon the results derived from future assurances whose entrance may be repelled by the comparison presented between the excessive bonuses conferred upon the senior policies and the reduced sums secured by those of more recent introduction. Hence a barrier tends to be raised against the acquisition of new business except at an enhanced cost. If, again, a diminished valuation rate of interest be deemed expedient for the purpose of creating stronger reserves, the amount required would be derived from the common profits, and both in respect of the increased basis upon which the surplus interest would be realized and that surplus accretion itself (due to the wider difference between the actual return and the reduced valuation rate), the policies of longer duration would thenceforth receive a double benefit. It will readily

* The student should understand by these expressions the more extended or shorter continuance of the Policies.

be perceived how this diversion of profit to the older policies (and the resulting effect upon the influx of new business and its cost) is accelerated where an appreciable proportion of the business consists of endowment assurances (and particularly where those policies possess comparatively brief terms), with their heavy reserves.

In every system of distribution the possibilities of changes in the character and incidence of the business must, so far as practicable, be vigilantly regarded and forecast, emphasizing the contention that the method pursued should possess an elastic nature and thus prove capable of adaptation to altered conditions without abrupt and sharp divergencies or the relinquishment of its fundamental principle. And this condition, it must be admitted, is one of grave difficulty. No one, for example, many years ago could possibly have predicted the profound diversion in popular selection which has resulted in the existing disproportionate amount of endowment assurances compared with the slender ratio between them and ordinary whole-life policies which previously prevailed—an extension which appears to have been largely accelerated (independent of the general desire for the inclusion of an investment element in the scheme of life assurance) by the widespread agitation relating to old-age pensions. For it will be observed that an endowment assurance provides, in addition to protection at earlier death, and the cessation of premiums after a definite period, a pension for old age in the form of (1) a negative pension secured by the termination of the payment of premiums, and (2) the actual pension provided by the investment of the sum assured on its discharge by the Company during lifetime. Now, the prospect of surplus from a diminished mortality upon endowment assurances is insignificant compared with the result under a similar condition upon policies for the whole term of life. Besides the selection originally exercised by the Company, there appears reason to surmise that an effective power of self-selection is employed by the applicants for endowment assurances on the whole in view of the chance of a considerable benefit to themselves if they survive the specified period and thus receive personally the sum assured: this combined selective process accordingly produces the effect that the great majority of these assurances tend, beyond the expectation implied in a general Table of Mortality, to leave a proportionately

increased number of claimants. Since an endowment assurance is a conjunction of a temporary policy (payable if the assured die within the adopted term) and a pure endowment (claimable only if the deferred age be attained) a lighter mortality acts in different modes upon these component elements. In the temporary assurance a surplus accrues if fewer die than the number expected; in the pure endowment no surplus, or an insignificant amount, is yielded on account of the augmented number who survive: and the favourable result of the term assurance is required to compensate the deficient balance exhibited by the remaining portion of the combined policy. The calculated premiums assumed that a certain number would die whose forfeited contributions would assist in payment of the policies of those who remained and whose own contributions, separately regarded, would not suffice in themselves to amount to the sum assured: if then fewer should die during the interval, an inadequate sum would be obtained from their cancelled premiums to supplement the demands of the augmented number who claim payment of their policies. Hence in this description of policy the element of mortality possesses an insignificant share in the production of surplus, if indeed it prove thus effective in any degree whatever. On the other hand, the loading as a percentage exceeds the amount contributed by a whole-life policy: and since the reserves and pure premiums are proportionately greater, the surplus interest derived from their investment is enhanced; and it has been generally shown that, practically in the mass and when the valuation rate of interest is reasonably low, the allowance of the same rate of reversionary bonus to this class of policy as that accorded to whole-life assurances of the same ages at entry is justified. It may be soundly conjectured, however, that this doctrine would require modification if the observations comprised an appreciable proportion of endowment assurances for comparatively brief terms—of, say, ten or fifteen years. For in contracts of these reduced durations insufficient time is afforded for the accumulative power of interest to produce a prominent effect, and it is not unreasonable to suggest, on the basis of the considerations already adduced, that endowment assurances for these short periods should receive a diminished rate of bonus compared with the bonus assigned to policies for the entire duration of life, completed at the same age, and showing the

same term of existence. If in the fifth mode of division, which we have described, a sensible proportion of the aggregate business consisted of this kind of policy (and particularly where the terms were brief) the wide extent is evident to which these assurances would deplete the general surplus to the detriment of the more recent members, with the attendant difficulty, through the deterrent force of adverse treatment, of attracting new business without increased expense.

The experience derived from the recent investigations by the Institute of Actuaries and the Faculty of Actuaries into the mortality appertaining to assurances of this special class is worth noting :—

Age	Annual rate of mortality according to the	
	O^m (or Whole-Life) Table.	E^m (or Endowment Assurance) Table
25	·00481	·00375
35	·00738	·00500
45	·01153	·00873
50	·01504	·01155
55	·02045	·01674
60	·02887	·02638

Regarding ages 25 and 35, it will be noticed that the rates of mortality experienced under endowment assurances are respectively 22 per cent. and 32 per cent. inferior to those exhibited by policies for the entire duration of life. By mere inspection it will be observed that the rate at age 50 on the Endowment Assurance Table is almost precisely identical with the rate for the younger age of 45 on the whole-life observations. Our remarks are thus confirmed with reference to the improbability of any appreciable benefit being experienced under endowment assurances in the form of surplus from mortality in consequence of the decisive tendency to a proportionate enhancement of survivances.

A reference may here be suitably introduced parenthetically to the distinct change in popular preference displayed in the preponderant proportion of endowment assurances. The vast extent of this divergence of public selection may on many grounds be gravely deprecated, and in the twofold aspect of the Companies and the people. The ancient prudential character of life assurance as a family provision tends to be obliterated, and the sums received during the lifetime of the assured may be diminished by insecure investment, and thus prevent the maintenance unimpaired

of the proposed fund. It is true that the entire amount will be received by the family if death occur during the active period of life and that the payment at a deferred age may generally imply that the needs of the family have been satisfied or provided, so that no important object would be secured by postponing the receipt of the policy-monies to the later date of death. This consideration, however, would not affect endowment assurances completed for brief terms, such as 10 or 15 years: yet here it might be well retorted that, having regard to the substantial premiums required, none but the comparatively rich would resort to assurances of these diminished periods. It may also be urged, and justly urged, that in the case of the young and unmarried, with no specific object to be attained by assurance, no policy probably would be effected unless it were accompanied, as an investment or saving of money, by a settlement at a specified age. And again, the assured generally desires that when the period of practical retirement from commercial toil has arrived, his (probably diminished) income should be relieved from the burden of premiums. This object, however, could be equally attained, and the prudential nature of assurance preserved in its former shape, by completing the policy for the whole duration of life and limiting the premiums (by a moderate increase) to the term intervening to the age of 60 or 65. The policy would then ultimately become an unburdened asset, and if no necessity existed for its continuance, it could be commuted for a substantial surrender value. But the subject may be considered from a wider point of view. On the part of the public, the change of choice may be regarded by many as indicating an alteration in the national character which is not of the highest type; but apart from this general aspect, the extensive substitution of this class of policy for the customary form involves important considerations to the Companies and their members. The premium income will be depleted successively, and with increasing force, by the periodical maturity of these policies; and if the present conditions of competition continue, these grave defections of revenue can only be compensated by augmented expenditure in securing fresh business, and must be consequently attended by a diminished profit. Companies must obviously adapt their methods and service to public demands; the alterations and vicissitudes of popular selection lie beyond their control; but Offices would act wisely, in

their own permanent interests and the genuine interests of the assuring public, if they sagaciously and tactfully endeavoured, by judicious advice and explanations, to arrest the current of this predominant choice and divert it into the more beneficial channel of a substantial preference for the ordinary type of assurance protection.

In Mutual Societies the bonus is usually distributed in the form of a reduction of the premium for the ensuing year ; that is to say, a cash bonus in the shape of a percentage upon the premium is annually allotted. Unlike the diminution of premium granted by Companies which possess a proprietary capital as a guarantee of its permanence, and where the reduction consequently constitutes a specific contract for life (or for a term of years, if the assured should so select), the abatements in a Mutual Society are simply annual, and are dependent year by year upon the successive valuation results—the diminished premium being susceptible of increase (up to the level of the original amount), or of additional reduction, in the event of the future operations of the Society failing to provide the necessary surplus on the one hand or disclosing an augmented surplus on the other. Our object is to express no preference for the one form of Company to be selected, but it is just to add on this point that the history of Mutual Societies (like that of Mixed Companies) has ever proved an honoured and stable one, without retrogression of the abated rates—a history worthy of the highest traditions of British commercial enterprise.

In connexion with the conversion of a cash bonus into its reversionary equivalent a practical consideration necessarily enters —the power of selection capable of exercise by the assured in choosing either form. As in life assurance problems generally, the mass, in discussing the question, must be substituted for the unit, and it will be found that on the whole the mode of adoption follows the general course exhibited in the experience of lapses and surrenders, namely, that in the mass those will prefer the reversion whose prospect of life is not so satisfactory as that of those who demand a payment in cash. In the published experience of a particular Life Office, the interesting result was disclosed that, among the members who selected the reversionary form of bonus, the actual deaths were 998, while the expected number, according to the general experience of the Office, was 939, or an excess of over six per

cent.; in the case of those who preferred an immediate cash commutation the deaths were 1,163 and the anticipated number, 1,212. The option accordingly, so far as the records of this Company are concerned, accorded to the assured to select the form of bonus, does exercise an influence upon the future rate of mortality prevailing in the various classes of participating policies which may be created, and confirms the conclusion deducible from general considerations —though in life assurance, more importantly than in other departments of practical work, these *à priori* assumptions imperatively demand the experimental test. It was further shown by the entire results that the value of this option, measured by its effect upon the mortality, was the more pronounced as the period at which the option was adopted was deferred. At the younger ages the prescience of probable lifetime is less definite, while at the older ages the acquired experience of personal health has produced a distinct power of anticipation. Hence, for the protection of the general interests, some compensatory modification must be introduced in the basis of commutation. An eminent actuary once proposed that, in determining the reversionary equivalent of a cash bonus, the age of the applicant should be obtained (and the value of a reversion to £1 founded upon that adjusted age) by deducting a percentage from the expectation of life, varying in amount with the actual age: the deduction would thus produce an increased age, and consequently a present sum of £1 would yield a diminished amount at death. At the present time, the compensating provision is formed either by converting the cash into reversion by a Table of Mortality expressing a heavier rate of mortality (the $H^{m\,(5)}$ or $O^{m\,(5)}$) than that furnished by the table employed in the valuation, or by conversion of the cash at a lower rate of interest than that on which the valuation is based; in either event, it will be observed that the reversion suffers reduction.

Some concluding observations of a more general character may now be submitted.

A method of division of profits after ample trial of its working is discovered to produce inequality of treatment between, for example, the policies of prolonged and brief duration, where the former obtain an excessive share measured by their comparative contributions: the importance of a change is finally justified in the permanent interests and prosperity of the entire body of the

assured and of future entrants. In any modification of the existing system of distribution—and the more extensively so in proportion to the period during which the method has been in operation—abrupt divergencies between past and future bonuses will probably be disclosed. This result is incidental to all changes, and the general aim of equity of apportionment must obviously override any individual diminutions of benefit. Moreover, as has been pointed out, the older policies which will sustain the larger effect have already enjoyed an advantage in the past in excess of their specific contributions to surplus, and it may well occur that the modification will ultimately produce an enhanced result by securing the advantage of the lighter mortality of fresh and vigorous lives in greater proportion as the public perceive that their contributions are not diverted to existing members, while from the exhibition of an attractive and equitable system the new business may be expected to be acquired at a diminished cost with a consequent increment to the Aggregate Profit Fund. Two courses, however, as we have mentioned, may be pursued: (1) the maintenance of the older policies thus affected as a closed series to which the existing method of distribution would continue to be applied,—the new entrants after a fixed date being subject to the revised plan; or (2) the substituted method might be applied immediately and universally to the whole of the assurances, and the differences of resulting bonuses disregarded. The former course involves the creation of two accounts and funds, with an analysed allotment to each of its premiums, claims, realised interest, and expenditure. The assessment between the two series of premiums, claims, and interest is not difficult; but considerable perplexity will arise in connexion with the division of expenditure. The charges apportioned to the terminated series must obviously approximate to that implied in the conduct of a purely renewal business; and as regards the fresh series, it might even occur that, if the claims entailed upon it should prove excessive during the first valuation period following the change of method, an insignificant, or practically an absence of, bonus might be the result upon their assurances, looking to the additional burden of the necessarily increased expenditure accompanying the accession of that business. Should this event ensue, or even should the consequences be less oppressive, the Company would be confronted with the problem of a diminished

expectation of membership except at a costly price; and indeed adverse reflection might thereby, however unjustly, be cast upon the wisdom and expediency of the scheme of allotment itself. We are undoubtedly presenting an extreme illustration, but, as we have already observed, instances of a "limiting" character frequently supply, from their impressiveness, serviceable suggestions for guidance.

So far as is practicable, a Company should be regarded and administered as a unity of risks and benefits; a commonwealth of interests, and alien from the intrusion of any principles of management which may, however unreasonably, be considered as introducing partiality and discrimination of treatment. Hence as a rule, when a plan of distribution has been ascertained by adequate experience of its operation to produce important inequality of action and effect upon the several classes of the members, the wise and sagacious course would appear to suggest the application of the substituted principle immediately to every assurance in force.

Although we are treating in this chapter of Assurance Companies of the ordinary type, and of their expenditure in relation to profit, it will not be unfitting to introduce a few remarks upon the apparently excessive charges incurred by Industrial Institutions. The object of the completion of policies in these Companies is virtually the provision of a sum for discharging the funeral expenses of the members; the average amount assured is £10 or less, and the average weekly premium about 1¾*d.* to 2*d.* Undoubtedly the expenditure in some Offices is enormous, and beyond the needs of administration exemplified in the superior Industrial Institutions; but dealing exclusively with Companies of the higher type, the ratio of expenses compared with that of Life Offices of the general form is reasonably justified under existing conditions of public practice. The premiums are all paid weekly, while in ordinary Companies the shortest term is usually quarterly; and, what is more significant, the assured in ordinary Offices themselves pay their premiums direct (to the agent, for example), while in Industrial Companies a representative is compelled to call upon each member week by week for the purpose of receiving the individual pence, so that the cost of collection is egregiously increased. Moreover, the extent of bookkeeping, the number of clerks necessary to be employed, and the amount and cost of correspondence, are

clearly almost incalculably in excess of the requirements in these respects of Companies of the customary type. When these considerations are fairly weighed, it must be admitted that, in the superior Industrial Offices, the extent of annual expenditure is not unduly high. To these considerations should be added the vast social benefits which this class of institution is competent to confer by supplying a solvent mode of saving small sums as a provision for the future, and thus indirectly aiding in the cultivation of the habit of thrift by providing a suitable and substantial channel for its effective exercise. A further remark may be made. Industrial policies are effected by less intelligent people on the whole than the constituents of ordinary Life Offices. No disrespect or reflection is intended to be conveyed in this observation; but obviously the persons to whom the Industrial Companies appeal are less versed in business and commercial ways; explanations consequently and interviews detailing the advantages of assurance, and the time expended in securing the proper completion of forms, are vastly more extensive than the requirements of assurance work generally; and when the voluminous clerical labour is added, we discover the need of a discrimination of judgment in surveying and assessing this important accessory of industrial work. It is well established as a general fact of assurance experience that the cost of administration is a function of the individual amount per policy, increasing as that amount is reduced, for reasons which a slight examination and reflection will supply.

NOTE :—We have compiled the following interesting and instructive table, from the Board of Trade Returns for 1903, and other authentic sources, in connexion with the relation existing between the rate of expenditure and the average amount of *New* Policy acquired :—

(Industrial Companies are excluded.)

Average amount per new policy	Number of companies	Average amount of new business transacted in the year	Average net Premium Revenue	Average ratio of expenditure to Premium Revenue
		£	£	
Over £900	4	754,268	266,267	14·20
Between £700 and £900	3	1,056,799	335,507	13·18
,, £500 ,, £700	19	578,431	317,275	14·18
,, £300 ,, £500	17	699,313	355,881	15·94
Under £300	5	344,079	200,702	18·13
	48			

CHAPTER VII

The Investments of a Life Office

The financial administration of a Life Assurance Company is governed by the conception of a trust which regards with equal solicitude the interests of the existing policy-holders and those of the public who may hereafter participate in its benefits and obligations. But its range of action is not limited by the comparatively narrow boundaries within which the execution of an ordinary trust is circumscribed, since by the incorporation of the element of bonus it is implicitly accredited with powers of investment of a wider and more remunerative scope.

We proceed to present the general and specific principles by which this department of work should be controlled and guided, with a practical statement, for the student's study, of the mode in which some typical forms of security should be scrutinized and conducted.

Capacity in finance and economy of management constitute, as has been stated, the essential factors of successful assurance administration ; and the student who aspires to prove a competent actuary, adequately equipped at all points, should from the earliest stages of his career make this mode of education equally prominent with his mathematical learning and his more purely professional accomplishments. Practical knowledge and power of discrimination and foresight in finance indicate by their nature the source of these acquisitions. Books will teach the meaning of terms and the formal modes of commercial relations with more or less completeness and width of view, but the judicial command over the processes, and a direct and consequently effective acquaintance with business affairs, can only be secured by leaving the privacy of the study and becoming immersed in the real activities and intercourse of commercial life. This suggestion does not convey that the student should gain his comprehension of financial business by first entering a commercial office as part of his practical education, or engaging personally in industrial enterprise, but comprises two stages adapted to his professional advance. In his days of training, prior to obtaining an appointment of authority and responsibility, he should acquire from books (or, as a more excellent

form of learning, from friends concerned in commercial and financial pursuits) a clear businesslike knowledge of the terms and modes of work actually employed, with the definite meaning and import attached to them in the schemes of trading and finance to which they apply; he should concurrently secure for himself a *concrete* knowledge (as distinguished from a theoretical understanding) of commercial undertakings and processes by dissecting actual revenue accounts and balance-sheets of joint-stock companies, railways, banks, and assurance institutions; he should deduce from the newspapers, by a comparative examination and analysis, the influences which act and react upon Stock Exchange values and their relation to the changing value of money, with the modes in which and the limits within which they usually operate. (The Author recalls with gratitude the practical benefit which he derived, in his earlier professional days, from placing himself under the tuition of an intelligent stockbroker). He should examine the records of sales in the public mart of life interests and reversions; and, assuming tentative values for the constituents of the *corpus* and for the approximate legal and other costs, determine by a reverse process the data of mortality and interest on which purchases are arranged; and above all he should sedulously study, analyse, and compare the Board of Trade returns of typical companies, not simply for any particular year but for a series of years, in order, for example, to discover the forms of connexion between rates of interest and ratios of expenditure, and the variations in the resulting profits, with the diverse manner in which different descriptions of assurance policies effected at various ages and exhibiting varied durations are affected by the mode of valuation and distribution. These returns are too frequently regarded as simply an armoury in competition; we direct special attention to them as a sound foundation of professional education. In the latter inquiry just mentioned, he will gain a precision of view concerning the relations existing between the loading of premiums, the bases of valuation, and the method of distribution of profits, and will further discern the manner in which, for example, the system of division of profits may be related, as to actual effects, to the extent of the new business secured. If two Companies transact the same increasing ratio of new business, and the one adopts the simple or compound reversionary mode while the other allots its profits as a uniform cash

percentage on the premiums paid within the valuation period the student may observe that, although the fresh acquisitions (let us assume them to be extravagant and needless in amount) press equally upon the resources of both Offices in consequence of the considerable cost and necessary reserves, the strain nevertheless upon the individual shares of bonus in the former Company may prove less severe than that which will be experienced in the latter, since the one method of division fails to include cognizance of the amount of the premiums, while this element, in the second scheme, forms the predominant factor. This suggested course of study, dependent upon the perception and measurement of *real* facts, constitutes the main foundation of ultimate professional aptitude and competent grasp. And, as a general mental principle, the *provision* of knowledge, alertly gained, thoroughly systematized, and nicely applied, passes unconsciously into the power of *prevision*. As the student, thus equipped, succeeds in securing a position of responsible trust, his assiduous care should be exercised in a similar direction : in the direct personal knowledge of men in business, and an acquaintance with the ways in which they deal with commercial and financial problems. The laborious acquisition of this discerning analysis and knowledge of human nature in its business aspect will lead him to the cultivation of tact and judgment ; of genuine humility combined with self-respect ; of genial manners associated with conciliatory though firm and vigorous action ; of discreet silence where words are idle ; of prompt initiative where delay would be futile or dangerous ; of insight into character, with its teaching of a discriminative treatment of men, not in the mass, but adapted to individual nature ; and of sagacious forecast born of the mastery of experienced or observed facts and their results. These capacities can obviously be gained by strenuous and laborious toil and self-control alone, and are utterly impracticable as a steadfast habit to the actuary who confines himself chiefly to the seclusion of his study or office room.

The primal principles on which life assurance investments should be based comprise—

I. The permanent integrity of the capital, with the possible occurrence of the least violent and frequent fluctuations of value.

II. Subordinate only to this principle is the acquisition of securities which yield the most remunerative return.

A word may here be interposed upon the subject of Consols as a type. In former days of assurance business (when the relation of the realized rate of interest to the valuation rate was not so distinctly recognized in its bearing upon profits, and in consequence also of restricted notions of assurance finance) this form of investment was universally popular with Companies, and the preference, again, was no doubt partially due to the prestige and public confidence which were supposed to be conferred upon a Corporation by an extensive holding of the finest Government security. Looking to the decline in the rate obtainable from Consols, and especially to the fluctuations of value in more recent years,—the security within the past five years only having reached 111½ and descended (January, 1905) to 87½,—this mode of investment has justly, in the interest of the policy-holders, ceased to retain the supreme favour of assurance administrators which it formerly possessed.

III. The larger proportion of the Assurance Fund should be invested in securities which are not readily realizable, and therefore command a superior rate. Mortgages for fixed terms of years, Corporation loans, land improvement advances, redeemable over a considerable period, and reversions to well-constituted funds, are comprised within this class. Some of them by reason of the extended duration of their currency are not adapted to individual lenders and private trustees, and hence the supply of capital being limited in relation to the demand, advances of this character exhibit a superior rate of interest.

The assurance commitments of Life Offices are somewhat similar to bills drawn at long-deferred dates: their incidence can as a whole be closely determined; and accordingly, with an increasing or constant revenue from premiums and interest to discharge current claims and expenses, together with the practical absence of recurrent and widespread epidemics, no possibility of danger—corresponding to a sudden drain upon a bank—need be apprehended and these securities with deferred redemption may be freely entertained.

Mortgages are occasionally limited by old-fashioned Companies by a repayment clause on six months' notice from either side: this is a mistake: if soundly selected, they should be constituted for a definite term of years; for where this clause exists and the value of money falls, the mortgage will certainly be discharged and

the Company be thus deprived of one of its more remunerative investments. The only instance where such a notice may be requisite is one in which it appears to be expedient that the Company should possess the right of repayment in the event of the borrower becoming bankrupt, though the course, even in such instances, would probably in most cases be needless.

(It would be well if some consistent usage were adopted in our financial nomenclature : the term " mortgage " should be restricted to transactions where the lender merely advances money by way of security ; while the term "investment" should be employed where the investor purchases both the income and the capital which yields it ; the term " interest " should apply where the annual return is at a fixed rate, and the term " dividend " (*dividendum*, that which is to be divided) where the return or share depends upon the amount of profit out of which it is discharged and of which it forms a proportionate part).

IV. An important principle upon which considerable stress should be placed is not infrequently omitted from adequate recognition, and if neglected may produce unsatisfactory or grave consequences. The principle in question is of universal application, both in the private and public choice of investments, and may be thus expressed : There should always exist a due and reasonable proportion between the different *classes* of securities selected. Experience naturally can alone indicate the practical adaptation of this rule. Assume that a fund possesses a large proportion of the ordinary stocks of railways : a period of financial difficulty in railway administration occurs—the adoption, for example, of electricity in continuous locomotive form ; the undue extension of loop lines ; the " watering " of capitals ; the pressure generally of expenditure discovered to be unproductive ; the increasing keenness of competitive effort—in this event the entire amount of this particular class of holding might be adversely affected ; and the depreciation naturally might become serious if the suspicion or alarm of the public merged into panic with its thoughtless sales. The difficulty of such a possible situation lies in the fact that if an undue proportion of the fund had been invested in this description of securities the whole or a large portion of the total Stock Exchange values may possibly exhibit a diminution. But if these investments generally had been distributed in a judicious apportionment, an appreciation of other

classes of investments would tend to neutralize the result of any financial disturbance in the set under consideration, and restore equilibrium by means of the balance in value ; as the public disposed of the one class of securities with a corresponding diminution in the value of the Company's holding, the remaining investments of the Office would be augmented in value by the superior price to which they would rise in consequence of the public applying the proceeds of their sales to the purchase of stocks which more fully commanded their confidence : the enhanced value thus conferred upon the Company's investments in these latter selected stocks would tend to compensate the fall in the specific class we have mentioned. A reasonable view appears to be that the aggregate investments of a Company should be examined at the close of each year with the object of ascertaining that, in pursuance of the teachings of experience and of technical knowledge, they are ranged in appropriate proportion of amount in every description of holding. Since the issue of the first edition of this book, a practical confirmation of our observations has been, unhappily, afforded by the insolvency of a private bank, of long standing, in consequence of its neglect of a judicious distribution of its assets by locking up capital too exclusively in bills representing one particular class of industry.

V. The investments should so far as practicable be helpful to the increase of the life business, so that a possible twofold profit may be realized—a profit in the rate of interest, and a benefit in the reduction of the ratio of administrative charges. Hence mortgages upon the security of life estates and reversions amply secured, and the purchase of these interests, should be diligently cultivated. Loans upon personal security, judiciously and carefully selected and restricted within reasonable limits, are worthy of the thought and trouble they involve. For in addition to the higher rate of interest which they return, through their practical confinement to Life Offices, a policy on the borrower's life invariably accompanies the transaction.

A few subsidiary observations naturally follow the exposition of these principles.

Securities should as a rule be avoided, however favourable be their present aspect, where some special feature exists a change in which might not improbably affect in an adverse manner the

safety of the capital advanced. The stocks, for example, of a commercial or industrial Company sometimes largely and specifically depend for their sustained value upon the continuance in office of the present managers whose proved capacity and energy have been chiefly instrumental in producing their success; the passage of supervision and control into less competent and experienced administration might be fraught in time with diminished fortunes and depreciated stability. A similar suggestion applies where the productiveness and prosperity of an undertaking of this nature depend in an important degree upon the continued demand for a special description of manufacture: a change in popular fashion or preference may produce a deterioration of position and prospects, or at least may compel an enlarged expenditure in different forms of machinery or processes, and an alteration in the mode and range of business, with the resulting difficulties which attend any grave interruption to an organized and established industry.

It has sometimes been contended that a portion of the funds of a Life Office might be judiciously placed upon deposit with banking or discount institutions. No implication of disparagement of these admirable joint-stock undertakings is intended when it is urged that deposits do not constitute a form of permanent investment for Assurance Companies. The rightful and business-like function of deposits in assurance finance is simply that of a temporary and productive employment of capital until some of the more regular and recognized modes of mortgage or investment have been secured for the utilization of these surplus accumulations. The money so deposited cannot be obtained at a period of panic, however stable may be the position of the accepting institution, and an Assurance Company should contemplate all reasonable possibilities in its investing practice; and, moreover, no specific security exists for the deposited amounts.

Another caution suggests that securities, however excellent intrinsically, should be as a rule avoided where an uncalled liability is attached to the obligations purchased or accepted in mortgage.

Advances upon mortgage on real estate in the most stable and long-established cities in the Colonies, in America, and in foreign countries, are deservedly advancing in assurance recognition and adoption. Undoubtedly special caution and knowledge are requisite, but the superior rate of interest to be secured justifies

the expenditure of particular and continued supervision. The principal considerations which demand satisfactory inquiry and solution comprise (1) the form and extent of taxation, which in some localities is leviable not simply upon the land and buildings mortgaged, but also upon the mortgage debt itself, with the frequent legal enactment that the burden of taxation shall not be transferable from the lender to the mortgagor. Arrangements are sometimes effected (in the form, for example, of covenanting for a higher reserved rate of interest) for the purpose of realizing the stipulated return free from fluctuating taxes, but the possible trouble is involved that the Government may at some time regard this practice as a punishable evasion of statute; (2) questions affecting the validity of title, and the form (and priorities) of legal registration require scrutiny; and especially (3) the capacity of legal ownership by aliens. While the lender receives his interest and other demands this difficulty will not arise; but should default occur and an attempt be made to dispose of the property, any legal restriction relating to alien holding may prevent the recovery of the capital, since, in selling the property, the mortgagee stands in the position of owner and must be able to convey a sound title of possession.

The question has been frequently agitated whether a Company acts wisely and beneficially to its policy-holders in the purchase of its own shares where a proprietary capital exists. It would appear primâ facie that the investment in securities appertaining to the Corporation of which the Life Fund constitutes an integral part naturally presents the aspect of sound and appropriate administration. Many years ago a Life Office with a proprietary capital gradually in this manner acquired the whole of the shares, and thus transformed itself into a Mutual Society. But this operation, possible at that date in consequence of the limited demand by the public and Exchange speculators for assurance shares, and the sequent possibility of purchase on reasonable terms which a Company thus obviously possessed, is impracticable under existing conditions. In response to the rapid and enormous accumulations of capital, and the advancing requirement of remunerative outlets, a definite market is now created for the shares of Life Offices (and those also of Companies transacting other descriptions of insurance business) and this demand is augmenting in extent by reason of the prosperous and competent manner in which these institutions have been

managed. Hence the price tends to advance. Now a Company purchasing its own shares becomes an increasingly important factor in the market : its holding is known : its selection of this class of investment naturally tends to enhance the value on the ground that no holder can be more intimately acquainted with the intrinsic and potential prospects on which the worth of the shares depends than the Company itself : the Company accordingly, so to speak, is continually forming the market against its own operations : and any additional purchases after a time must occur either at prices maintained largely by its own action in the market as an investor, or its purchases must cease with the attainment of the value at which it becomes inexpedient to buy. But the difficulty is intensified by this consideration : if the prosperity of the Company as exhibited in the declared bonuses diminishes in the Life Department—assuming for the purpose of illustration the case of a Company whose transactions are confined to life business,—on which the value of the shares, apart from the separate investment of the capital at interest, depends, the price will fall. This result obviously reduces the value of all the purchased shares as assets in the balance-sheet ; an amount must accordingly be written off for adjustment to the market estimate. An adverse effect is thereby again produced upon the prospects of life profits to the extent of the sum thus absorbed, with its attendant disquiet in the public mind ; this result will react upon the market position of the shares, and thus an inter-dependent influence incessantly tends to prevail. We are presenting an extreme instance possibly, but extreme instances, if confined within practicable and probable limits, afford the clearest evidence and ground of principles.

For the special information of the student—though the application of the rules will only occur at later stages of his career, when, by training and reflection, he will be prepared for their appreciation—it is proposed, with a view to furnishing a thoroughly practical aspect to this part of financial administration, to indicate the manner in which some typical securities should be scrutinized.

I. An advance upon a leasehold property is desired.

The market value of such a property depends on the magnitude of the annual difference for the unexpired term of the lease between the ground-rent payable to the freeholder (who owns the fee simple of the property) and the full or rack rent which may be readily

receivable on a repairing lease from a solvent undertenant, and after deducting an allowance for possible losses of rent through temporary unletting, rates and taxes, bad debts due to non-payment of rent, and all charges connected with the property which fall upon the lessee under the ground-landlord. (The rates and taxes above mentioned only consist of that portion which the original lessee—the applicant for the loan—would pay under any sub-letting.) The value will depend also upon the condition,—stable, fashionable, or otherwise,—of the neighbourhood, its well-established character, and the consequent demand by traders or the public for local accommodation. The market price, which the valuer will assess, will include the usual Sinking Fund for redemption of the value on the determination of the leasehold term, and a provision for leaving the premises in a substantial condition of repair at that date. It is a wise counsel to remark that, except for the exercise of a general supervision and judgment on the valuer's statement, rendered observant and practical by experience, the administrator should not interfere with the details of these external professional reports. A further comment may be interposed. A pure architect is not suited to express the market value of properties, whether freehold or leasehold: his function more expressly lies in the expenditure of money in the erection of buildings, and not the appraisement of the price likely to be realized by a sale: a similar observation applies to a fire surveyor or assessor, whose experience is mainly limited to the character of the structure of buildings or the value of damage caused. The work of valuation should be properly entrusted to a general surveyor and auctioneer, who, from the nature of his professional duties, must be conversant with the varying advantages and defects of properties as mortgage securities, and especially possessed of continual and actual experience of the demands of the mart and the prices at which premises can be disposed of at a public sale.

The margin between the market value and the appropriate amount of loan will be determined by the circumstances of each case, and should be decided by the Office and not by the valuer, in the light of the Company's experience in similar transactions. And the proper mode of fixing and expressing this margin is by way of the percentage of the difference between the value and the

advance assessed upon the former figure. Thus, a market value of £1,000 as the basis of an advance of £660 implies a margin of about 33 per cent.

The values of leasehold properties will obviously differ by reason of the relation between the ground-rent and the rack rental. In the City of London, for example, the rack rent will frequently amount only to twice the ground-rent where the premises are let as offices; but if let to one tenant, the ground-rent, instead of being one-half of the rack-rental, will probably exhibit 10 per cent. to 20 per cent. more in excess of this proportion. In suburbs, on the other hand, a margin of four or five times the ground-rent may prevail—the changed proportion being due to the difference of locality, and the consequent extent of the pressure of demand for occupation. For this relation involves the other determining elements of value.

It need not be added that where suburban leaseholds are accepted they should be chosen in well-established neighbourhoods with convenient railway facilities.

The actuary should carefully study the terms of the lease from the ground-landlord in order to observe and weigh any restrictive or onerous covenants which may exist and which may hinder or depreciate a sale should default occur.

The unexpired length of the lease forms another important consideration: in the City of London, short leases may usually be safely accepted, while in the suburbs or other localities it may be prudent, as a general rule, to decline the security of properties where the remainder of the term is less than fifty or seventy years.

Seeing that the value of a leasehold diminishes naturally by effluxion of time, an estimate—if the lease be comparatively brief, and assuming that a loan is to continue for a term of years—should be obtained of the approximate selling value on the expiry of that latter period. This estimate may suggest that the advance should be reduced by annual instalments of a sufficient amount, in order, at the termination of the period of the loan, that the same or a similar proportion may then exist between the value at that date and the diminished advance as that which was required, on the completion of the transaction, between the market value at the time and the original amount secured upon mortgage.

The actuary should himself prepare the necessary instructions to the valuer, and indicate (in addition to the points which an experienced auctioneer would present) any special questions on which an opinion was deemed desirable.

It is a salutary practical rule in connexion with properties of every description (land and buildings) to arrange a *general* inspection each year (or at other longer reasonable intervals) of (at least) the external appearance of the premises with a view to observing that they are maintained in sound structural repair and that a possible cancelment of the lease may thus be avoided, or a substantial diminution in market value. Instances have occurred where interest has been punctually discharged, but where the buildings, without the knowledge of the mortgagee, have been permitted to deteriorate, so that the value, independent of the lapse of time in the leasehold term, has been seriously diminished. Hence the necessity of the counsel just furnished; with the adjoined advice that every mortgage deed relating to freehold or leasehold premises should contain an ample power to the lender for full periodical inspection by his agents, and at reasonable times, of the condition of the structure, with the right to call in the advance if grave defects be discovered, and the obligation of the mortgagor to effect any necessary repairs with promptitude where the extreme course just mentioned does not appear to be imperative.

It is hardly necessary to add that in all advances, whatever be the nature of the security and its excellence, the character and position of the borrower are of the first importance: an admirable security with a borrower whose character will not sustain the strictest test of enquiry is usually one to be declined: the lender in such an instance can never feel that safety of position which should be involved in every advance.

In obtaining responsible references to the borrower's character and solvency, no doubt of their competency and good faith is expressed, but a sound general precaution merely suggested, in the recommendation that enquiries should also be instituted, independently of the borrower and references, through the medium of friends and business connexions of the lender. This rule applies as a result of experience to loans of every description. High character lies at the foundation of all satisfactory transactions. For in all advances the object of the lender should be that, so far

as knowledge and foresight (derived from adequate enquiry) can extend, the security, viewed from all aspects, should continue for the entire duration of the loan without trouble, anxiety, or the probability of a sale.

II. An advance is proposed upon security of an absolute reversion to a specified fund, expectant on the death of a life-tenant of a certain age, or its purchase is submitted.

Here the primary requisite is the market-value of the reversion at the date of the loan. The age of the life-tenant must be proved by sufficient evidence; and the constitution of the fund—the investments and securities of which it is composed—demands all the knowledge and acquired skill of the Actuary. For the values at which the elements of the fund or corpus are to be assessed are obviously not their values at the present date, but those at the probable epoch of the life-tenant's death. A leasehold property of a comparatively brief term will at that date be worthless or seriously diminished, and must either be excluded from the estimate as valueless or inserted at a large reduction. Furniture, of course, would be omitted as worthless: stocks at a premium and redeemable at par will require to be written down to their approximate value at the deferred date in question; and all terminable securities must be suitably revised as the basis of the calculation of the market-value. Definite rules cannot be furnished for the estimate of these particular cases: each must be the subject of consideration by skilled and trained sagacity. Take a leasehold, for example, with a future currency of 20 years, and a life tenant of 60. It is readily observed that, on the average, this item of the fund may possess no value at the probable period of death: if we adopt the term expressed by the even chance of death, we obtain, on the O^m Table, a period of between 13 and 14 years as against the unexpired leasehold currency at that date of 6 years, but although such a security might properly be considered to be valueless in the case of a suburban locality, it might justly be assigned a value if the premises were situated in the centre of the City of London.

Having then adjusted the ultimate value of the fund, we can compute the market-value upon suitable data of mortality and interest. With regard to the former factor it may be at once assumed as a general rule that reversionary interests are only

submitted to public sale or private purchase where the life-tenant is considered to possess an excellent prospect of longevity. If the contrary be the case, the relatives or reversioner will be careful to retain the property in the family in anticipation of a speedy realization, or an advance, if temporary needs be urgent, will be obtained upon its security in order to preserve the corpus from alienation Hence it may justly be affirmed that the appropriate Table of mortality for valuation should be one which expresses a light mortality at the older ages (at which ages these transactions occur), such, for example, as the experience of the National Debt Office, male or female, as the case may be. Besides the general authority which the Carlisle Table had acquired, its former use in the valuation of reversions (before an accurate discrimination had been recognized and rendered practicable in the distinction in the mortality of the sexes) was no doubt fairly satisfactory on the whole, in consequence of the large proportion of females (55 per cent.) which the observations included. But its employment now would be erroneous.

The rate of interest will depend on many circumstances, among which the age of the life-tenant is prominent: reversions payable on the termination of old lives naturally secure higher terms, that is to say, involve a lower rate of interest in the valuation, seeing that the time is shortened for the entry of disturbing events of the nature hereafter specified, and this result is aided by the fact that where the ages of the life-tenants are comparatively young, private investors do not usually care to purchase, since they desire to see a probability that the reversions will fall into possession during their own lifetimes; hence in the latter instance the competition being reduced, and in the former increased, the value in the case of advanced ages is augmented (that is, the rate of interest is lower), while the rate of interest is enhanced or the price diminished where younger lives are involved.

The points to be noted by the Actuary comprise the nature and range of the investments prescribed by the deed under which the reversionary interest is created; the more akin they be to ordinary Trustee securities the more is the probability reduced of losses or depreciations: where the area of investing powers is extensive special knowledge and financial sagacity are required in the Trustees, and the existing administrators possessing these

capacities may in course of time be replaced by men of inferior ability : the provisions of the deed with respect to any limitations which may affect the saleability of the reversion, and especially any power of granting advances or allotting portions of the corpus to beneficiaries ; the possibility of the appointment of Trustees, as the present administrators die or resign, who may not be persons of sound financial skill ; the probability of changes being effected in the investments and the consequent costs (which should be provided for in the price) involved in the removal and reimposition of distringases (or notices of lien to the Companies whose securities have been purchased) or of stop-orders (or similar notices where the funds remain under the supervision and administration of the Court) ; the expenses to be incurred in the substitution of Trustees ; the costs entailed at the life-tenant's death in the release of Trustees, the sale of the securities, and the distribution of the fund (these estimated charges, with the amount of estate-duty, will require to be deducted from the corpus prior to valuation) ; and the legal and other expenditure appertaining to completion of the transaction which form a deduction from the price arrived at. The practice here again applies of independent and careful inquiry respecting the character, ability, and solvency of the Trustees, and of the solicitors to the Trust.

The Actuary will consider also the certainty, if a purchase be effected, of promptly and readily learning and proving the death of the life-tenant ; and a condition which is essential in every instance consists of an inquiry of the Trustees and solicitors to the Trust whether any notices of charge have been received, and of the despatch of notice to the Trustees (with an acknowledgment returned) when the purchase has been completed or the charge created. A remarkable example may be furnished. An investor purchased an absolute reversion ; the solicitor who introduced the vendor and the vendor himself were ascertained to be persons of high character and responsibility : the buyer was desired by the seller to refrain from serving notice of the transaction upon the Trustees upon very plausible grounds, which were urged, of family quarrels and dissension. Looking to the position and character of the persons concerned, the purchaser waived his practice, and no notice was given. In a few years the reversion fell in, when it was discovered that, prior to the purchase, the reversion had

been already sold, so that the investor lost the whole of the purchase-money with accumulated interest.

The student will appreciate the special difficulty of these transactions when he remembers that the Title deeds, whose possession would afford a primâ facie evidence of full ownership, remain with the Trustees of the fund, and do not, like the deeds relating to premises held in fee-simple or on leasehold tenure, pass into the custody of the buyer or lender.

The circumstances and conditions thus enumerated may appear to the student to be very formidable, and indeed on some points practically insuperable: the estimate, in a numerical form, for example, of the possibility of depreciation in the value of the securities during the lifetime of the life-tenant; the chance of detrimental consequences through improper or unwise acts of Trustees; the probability of sound investments being exchanged for securities of an inferior nature for the purpose of realizing an increased income to the life-tenant; but intricate and difficult though these elements sometimes appear, experience and judgment, based upon practical use, readily surmount them; and the higher return which these securities yield,—particularly adapted as they are to the constitution of a Life fund—compared with the rate upon investments more accessible to the general public, forms an ample compensation for the thought and anxiety which they involve, and the losses which must occasionally occur. Moreover, we must revert to the conception of masses of transactions, where depreciations in some instances are counterbalanced by surpluses which result in others, so that on the whole the anticipated rate of remuneration on which the purchases were arranged becomes competent of realization.

The preceding exposition relating to reversions chiefly contemplates a purchase, but the student will readily perceive the form of application to a mortgage upon an interest of this description.

The extent of the margin between the value and the amount of the advance on absolute reversions is not so important, of course, as in the case of leaseholds and freeholds, since reversions by mere effluxion of time are annually enhancing in value at the rate generally of 2 to 3 per cent. per annum. In order to trace the death of the life-tenant with promptitude, where a reversion

is purchased (and where consequently no excess value, as is the case in an advance, remains to the reversioner as an inducement to prove the death), it is the practice of Companies to consult the newspapers regularly and to institute inquiries from time to time.

The preceding considerations, modified to suit the different nature of the interests, apply to Life Estates and Contingent reversions. In the latter especially it is important that the protecting Policy should be rendered world-wide from the outset by the necessary deduction from the price. In all these interests, and prominently so in Life Estates, a supreme consideration is the probability of being able to obtain and furnish evidence, with ease, of the existence of the life-tenant as a condition precedent to periodical receipt of the income purchased. The possibility of trouble upon this point should be considered in the price, with its consequent depression.

It is sometimes deemed expedient in the purchase of Life Estates (in possession and reversion) to limit the investment to a portion only of the income in order that the vendor may retain an annual pecuniary inducement to aid the purchaser in the proof of continued existence in immediate Life Estates, and of the death of the prior beneficiary in the case of Reversionary Life Estates. It is probable, however, that this course is not generally very efficacious, as the vendor may dispose by sale in other quarters of the remaining annual balance. A practical mode would be to arrange, where necessary, with the vendor from the outset that he should present himself or furnish adequate proof of existence at each periodical payment of the income in consideration of his receiving a gift in money from the purchaser on each occasion. The buyer in settling the price to be paid for the Life Estate would deduct these successive payments from the income to be valued.

It is hardly needful to add as a general business axiom, and as probably indicative of the character and satisfactory issue of any loan, that that borrower, *cæteris paribus*, is the desirable one whose object in procuring an advance is not the discharge of existing indebtedness, or the transfer of a security to an easier lender at a reduced rate, but the acquisition of capital for the extension of business or for purposes of a permanent and productive nature.

A wise practice never requires the completion of a Life Policy as an accessory to a loan where an Assurance is not demanded by the necessities of the security, or desired by the borrower for prudential objects. It is always a mark of want of administrative sagacity to burden a borrower beyond the legitimate requirements of the case, and thus to hamper in any degree, by needless obligations, the ready discharge of his commitments.

III. An applicant desires an advance upon Personal Security. Judiciously selected and restricted within reasonable limits in relation both to the individual and aggregate amount, these transactions are desirable, and conjoin the advantage of augmenting the Life business of the Company with the additional prospect of a profit from the Assurance. In these loans the character and solvency of the borrower and sureties are of paramount consequence : and the object also (tactfully ascertained) proposed to be secured by the advance.

Two sureties at least join in the bond for guaranteeing principal, interest, and Life premiums : the suretyship is not proportional, but each is liable, as though he were a principal debtor, for the whole of the sums involved. Excellent references to each would of course be required ; men well known and stable in financial position ; and especially is it imperative, without any invidious implication, that the Company should institute exhaustive independent inquiries through its business connexions and friends respecting the character and financial standing of both borrower and sureties and the character of the several Referees.

The loan should be kept moderate in amount, adapted to the circumstances and prospects in each instance, and as suggested by the special nature of the transaction. An imperative condition prescribes the limitation of the advance to a comparatively brief period, say 5 or 7 years,—and the exaction of yearly or half-yearly equal instalments in reduction of the principal in every case. Even in the most superior and desirable advances of this character the changing possibilities of life are in full operation, and the opulent borrower or surety of to-day may be transformed into the struggling or impoverished trader or merchant, or suffer a substantial diminution of resources within a limited term of years. Hence the requirement of periodical repayments during the currency of the advance so that, if misfortune should affect

any or all of the persons involved in the transaction, the reduced loan may bear approximately the same relation to their diminished pecuniary position as the original advance sustained in comparison with their financial soundness at the outset.

Having regard to the fact that private lenders are not, from the nature of the case, largely interested in this mode of security, transactions of this character are more adapted to Corporations, and consequently the rate of interest obtained is enhanced by this restricted supply of facilities. Moreover, Corporations, on account of their wide ramifications, are better provided with the means of making those special and independent inquiries which these loans necessarily require.

It is well at reasonable periods, and in a judicious manner which avoids the danger of impairing credit, to renew these inquiries respecting the continued stability of the obligants.

These advances of a select nature are usually arranged with facility, since the guaranteeing friends, who can trust the borrower's integrity and his capacity to relieve them of the burdens of the contract so long as he continues alive, find the remaining difficulty removed by the accompanying Policy on the borrower's life (which is invariably required) which, if the borrower should die during the currency of the loan, discharges the balance of indebtedness and effects the final cancelment of their liability.

It is a primary element in the soundness of these loans that the borrower and guarantors should not be engaged in the same, or the same description of, business, or in allied businesses. If their businesses be thus quite distinct in nature, and financial trouble or depreciation should affect the class of business or commerce in which one guarantor is concerned, the forms of business pursued by the others may remain uninjured, and thus the aggregate integrity of the security may be maintained at an efficient level.

It is always important to ascertain, as a guide to decision, whether the applicant and his sureties have been bankrupt or insolvent, or have previously incurred, or are at present subject to, obligations as debtor or guarantor. A frequent borrower and sureties easy in lending their names (as former transactions may disclose) are generally undesirable clients.

The objection sometimes urged against this species of security,

and against other forms which yield a comparatively superior rate of interest, is not infrequently based upon the thoughtless acceptance of a famous (though rather stupid) dictum that high interest implies inferior security. Like most epigrams which have won popular favour, the maxim sacrifices a portion of the entire truth for the purpose of securing artistic symmetry and compact antithesis of verbal expression. The superiority of the rate may be due to the scanty supply of capital in any particular mode of investment: it may thus be produced by the skill and energy of the administrator in discovering fresh forms of security which others have not possessed the wit or enterprise to explore; and, in any event, if the course be deemed requisite, a portion of the interest may be utilised as a sinking fund in reduction of the principal invested. It will also be observed that, like all popular proverbs, the terms in which this dictum is expressed are vague and confused, since a proper appreciation of the statement would demand the definition of the term "high." For purposes of genuine guidance statements of this loose nature are idle, and generally form the equipment merely of minds to whom rigorous and accurate thought and expression are alien.

The mode of accounting in connexion with reversions and contingent interests purchased may be referred to. Three courses are possible: (1) Setting up in the books the requisite values of annuities as part of the Company's Annuity transactions for provision of the interest on absolute reversions and the interest and premiums appertaining to contingent reversionary interests. No investor naturally would adopt the course (though it is recorded that one Company at least in earlier years pursued the practice) of purchasing Annuities from another office for these purposes. This plan merely transfers the profit or loss to another department, and the total result of the transaction remains unaffected; (2) Debiting the purchase account annually with a rate of interest inferior to the return expected to be realized on the whole upon the calculated values. In absolute reversions the debit would amount to merely 2 or $2\frac{1}{2}$ per cent. per annum, being the average annual increment of value consequent on the advancing age of the life-tenant; while in contingent reversionary interests, where the premiums for the necessary protecting Life Assurances must be annually added to the amount invested, the debit of even a

nominal rate of interest would entail a reduction of the asset at the valuation; or (3) Debiting no interest during the current valuation period and then, on a revaluation of the reversions at the close of each quinquennial term, transferring the surplus-value to the interest account or distributing it equally over the ensuing 5 years. The last mode appears to be the most feasible and satisfactory; and the question then arises respecting the treatment of the surplus realized when reversions fall in during the early years of their currency and thus exhibit a considerable excess in the amount of the fund or corpus obtained beyond the original price on purchase (and the Life premiums, if any, discharged) accumulated on the basis of the anticipated return. In considering this question the conception of the reversionary investments as a mass must be steadily retained in mind. The whole of the reversions are purchased to secure a return upon the entire transactions of a given rate of interest, and consequently the surplus beyond that rate secured by the early realization of some purchases should (at all events, partially and adequately) be utilised to compensate those instances where the reversion, in consequence of the prolonged existence of the life-tenant, will become payable at a considerably deferred date and thus yield in the difference between the accumulated purchase-money and the amount of the fund then lapsing a reduced return compared with the amount which these investments in the mass were anticipated to produce. For like all aggregations of events, there will occur delayed realizations and realizations of briefer deferment. It is sometimes decided that the whole of the surplus secured on the premature termination of the lives of life-tenants (or, rather, on speedy terminations, since the notion of prematureness is excluded from the general assumption in these cases as a whole) should be regarded as divisible profit on the ground that, in the closing of reversionary transactions in any year or quinquennium, the protracted and the early incidence of realization may be treated as occurring together. But it may well happen that during a quinquennial period those reversions only fall in which have endured for a brief currency, or fall in for proportionately more substantial amounts, and if the entire surplus disclosed be distributed the future will be denuded of a portion of profit through the insufficient provision retained, or the absence

of that provision, from these more fortunate results for supplementing the deficient return which in many instances must inevitably occur. The calculated rate applies to the entire future history of the whole of these investments, and a reasonable regard for the future fortunes of the Company (which its later history is entitled to receive) should clearly be recognized in dealing with the results of these purchases, more especially when we contemplate the general decline in the rate of interest obtainable from other securities; and a husbanding of a part of the products of speedy realizations should be preserved for the undoubted diminution of return which will be experienced as the date of acquisition is postponed. In order thus to equalise the general return over the entire duration of the reversionary business, and to provide the future with its adequate share of the total yield, a proportion at least of comparatively early realizations (after providing for the market-price granted and its accumulation to date) should be retained undivided: and a suspense-account might be formed whence the diminished return on long-deferred reversions might be supplemented.

A reference may here be made to the mode of estimating the value of a Life Estate in possession, and an absolute reversion. The student is frequently puzzled by the immediate inclusion of d (the discount upon 1 due at the end of a year) in these formulas; and the deduction of the values in an elementary form with the adoption of i (or the interest due per unit at the termination of a year) may be serviceable.

Let 1 be the annual amount of the life estate, or annuity, proposed to be purchased: let p be the contract premium to ensure the restoration on the death of the life-tenant of the total amount invested by the buyer. He requires to receive interest from the income at the rate i upon the entire outlay, but assuming, for simplicity, that the income from the property or fund is only receivable at the end of each year, he will lose interest for the year in which death occurs unless a provision be arranged. (The student can readily adapt the resulting formula to instances where the life-income is payable half-yearly or quarterly and is apportionable to the date of death).

Let x represent the entire sum invested in the purchase. The annual return required is xi: the possible loss of xi or interest

for the year in which death occurs must be protected by a Policy for that amount; and the return of the aggregate investment of x must also be provided by means of a Policy of x.

Hence the entire annual payment to the investor which the income must yield consists of $xi + pxi + px$ or $x\ (i + pi + p)$: and this should equal the income or $x\ (i + p + pi) = 1$

$$\therefore\ x = \frac{1}{i + p\ (1 + i)}$$

To transform this expression into the customary formula, we note that $\frac{i}{1+i} = d$, that is $i \times \frac{1}{1+i} = d$ or $i \times v = d$, and since $v = 1 - d$, $i = \frac{d}{1-d}$: substituting this value we obtain as the total amount invested, or x,

$$\frac{1}{\frac{d}{1-d} + p\left(1 + \frac{d}{1-d}\right)} = \frac{1-d}{d+p}$$

Now since there is no payment of the Life income available at the outset, on the supposition, to provide the first year's premiums of pxi and px, the purchaser must himself disburse these amounts (and they are consequently included in the entire amount of the investment, or x,—this amount comprising also the cash value paid to the vendor): hence $\frac{1-d}{d+p} - px\ (1 + i)$ expresses the cash which the seller receives; that is to say,

$$\frac{1-d}{d+p} - p\left(\frac{1-d}{d+p}\right)\left(1 + \frac{d}{1-d}\right) = \frac{1-d}{d+p} - \frac{p}{d+p} = \frac{1-d-p}{d+p}$$

$$= \frac{1}{d+p} - 1, \text{ the usual formula.}$$

The history of this mode of valuation of a life interest in possession is deserving of chronicle. In primitive periods of Actuarial investigation, the value furnished was that of an ordinary annuity where no protection was involved against loss by death; and only at a subsequent date was the inclusion of a life policy at the current rate of premium adopted. The student will observe at the same time that the value of an annuity in the ordinary way is precisely identical with the formula just presented. For

$a_x = \frac{1}{d + p_x} - 1$, and the difference of result consists in the facts (1) that in the ordinary annuity the rate of interest involved is the same throughout, while the p_x is the pure premium at that rate and on the basis of the mortality prevailing among the annuitants, while (2) in the adaptation of the formula for market-values the interest employed, or d, expresses the return which the investor requires to realize, and the p_x is the loaded premium (constructed upon the mortality of assured lives) which a company would actually charge for the grant of the policy.

By a similar process the sum can be obtained as the purchase-price for an absolute reversion payable on the death of a life-tenant.

In this case provision must be introduced into the calculation for interest at the required rate of i during the duration of the life-estate, with one year's interest on the receipt of the reversion. For, on the usual assumption, the death of the life-tenant will occur in the middle of the year of death, and hence 6 months' interest must be allowed-for in respect of the succeeding interval; and again, as the annuity-value for provision of interest while the life-tenant exists (not to be applied in actual purchase, but simply to be included in the outlay) will make its last payment at the end of the year preceding the year of death, the intervening 6 months must be provided with interest, or (with the 6 months first mentioned) a final payment of one year's interest must be reserved. This concluding payment may be arranged by substituting for the reversion of 1 its value one year prior to its receipt, or v. Hence we need only deal with the annuity-value representing the interest during the currency of the life-estate. We thus obtain (if x be the total amount invested in the purchase) $x - axi : x ::$ (the price to be paid to the vendor) $: v$. Or the sum to be handed to the seller

$$= \frac{(x - axi)\, v}{x} = (1 - ai)\, v = \left(1 - \frac{ad}{1 - d}\right)(1 - d) = 1 - d - ad$$

$= 1 - d\,(1 + a)$, the customary formula, where a is the value of an annuity during the existence of the life-tenant.

Besides the amount granted to the seller, the purchaser, as has been stated, must reserve the present sum of avi (equivalent to the value of the interest during the continuance of the life-estate), and the two items constitute the entire amount invested. Hence

we obtain $1 - d\,(1+a)$, the price paid to the seller, $+ avi = 1 - d\,(1 + a) + a\,(1 - d)\,\frac{d}{1-d} = 1 - d\,(1 + a) + ad = 1 - d$, or the sum invested consists of the ultimate amount of the reversion diminished by one year's discount.

We might have left the reversion at 1 (without adopting its value one year prior), and provided for the loss of interest in the final year by effecting a policy for xi (at a single premium) and deducting that premium from the price handed to the vendor, but since the formula for a single premium is necessarily identical in structure with that for an absolute reversion, we should, by this arrangement, have been committing the logical material fallacy of " begging the question." (For practical use, the formula must necessarily be adapted to the conditions of each problem.)

As in dealing with complex problems in Life Survivorships, the most intricate reversion may be reduced to a simple and manageable form on careful and thoughtful analysis of its conditions. The student is advised to pursue the principle of the course, usually described as Moral Algebra when applied to personal and social decisions of difficulty, of inserting in one column the conditions favourable to the receipt of the reversionary interest, and in another those which defeat its acquisition. He will thus perceive at a glance the various contingencies, and be able to form the several functions (for incorporation into the appropriate formula) which are necessary to secure all interests involved. Regard, as an example, a comparatively elementary case. A is entitled to a reversionary life estate in freehold property producing £x per annum, expectant on the death of B, the present life-tenant, and in the event of C dying at any time (before or after B's death) during A's lifetime, A then becomes entitled to the fee-simple of the property on his own death. The value of the entire expectation of A is the present value of an absolute reversion at B's death to the fee-simple, less the single premium for an assurance to be payable on the death of the survivor of A and B provided A die before C,—the difference being multiplied by the present value as a perpetuity of the income derived from the property. The appended Table exhibits an analysis of the several events affecting the succession, as a basis for the formula just furnished :—

The possible events.	The effect of each possible event upon	
	the succession to the property.	the Policy.
i. *A* may survive *B*, and		
(1) *C* may have died before *B*.	The fee is gained at *B's* death, for *A* then acquires the life estate and the reversion at his own death, which together constitute the fee.	Cancelled since *C* died before *A*.
(2) *C* may also survive *B*, and		
(3) *C*, surviving *B*, may then die before *A*.	The fee is acquired at *B's* death under the circumstances described in i (1).	Cancelled as above.
(4) *C*, surviving *B*, may also survive *A*.	*A* secures the life estate, and the Policy money at his own death, the two together constituting the fee as at *B's* death.	Payable.
ii. *A* may die before *B*, and		
(1) *C* may have died before *A*.	*A* obtains the fee at *B's* death.	Cancelled.
(2) *C* may die after *A* and in *B's* lifetime.	The life interest and reversion are lost, but by means of the Policy the fee is really acquired as at *B's* death.	Payable.
(3) *C* may survive *B*.	As in ii. (2).	Payable.

Hence, by the instrumentality of the Policy in the suggested form, the formula for valuation is ascertained to be complete.

If again, referring (for simplicity) to an absolute reversion, the corpus is burdened in any case submitted for a professional opinion with a contingent legacy of $£x$ payable to a person only if he survive the present life-tenant, the value of the reversion will be diminished by the pure single premium (based of course upon the lower rate of interest at which a contingent Policy of this nature would be calculated by a Life office) for provision of the amount on the life of the life-tenant against that of the legatee. If the fund, too, is subject to a reversionary annuity to be payable during the subsequent lifetime of a person provided he survive the life-tenant, the market-value of the reversion will be reduced by the cost of such an annuity $(a_x - a_{xy})$ at the lower rate of interest which an Annuity Company would charge for the grant of such a bond.

A practical caution may be added. The author remembers an instance of a reversionary life estate, involving a large income, where the reversionary life-tenant was very young, and in the army. The case was submitted to several Actuaries, and one result was considerably divergent from the rest. This arose from the fact that, to the ordinary premium, that Actuary had added the assumed annual cost of a whole-world and professional licence, and since the ordinary Assurance premium itself was small at that age, the addition in question constituted a very considerable proportion of that rate. Hence it is advisable to express the value in the customary form, with the addition of the qualification that the value must be diminished by the sum required for the necessary licences in complete protection of the Policy.

It may be repeated, as a general conclusion to this chapter, that the administrators of Life Assurance finance are not restricted in their range of investments and advances within the narrow confines usually prescribed to Trustees: their aim should be, by constant watchfulness of financial affairs, to discover sound and suitable openings for loans and purchases which are not so adapted to private investors or Trustees as to Corporations: they should remember that surplus interest (the principal being protected) furnishes the basis not merely of increasing profit, but also of more stringent reserves and they are supported by the fact

that since the Policyholders expect them to provide bonuses augmenting if possible with the extended duration of their Assurances, those Policyholders have implicitly entrusted them with the duty of vigilantly and widely exploring and utilising all promising sources of investment which will contribute safely and enhancingly to this contemplated end. Earnest, enterprising, and sagacious search with remunerative results in general is more to be commended, even though some unhappy experience should occasionally intervene, than a supine and easy adherence to routine courses which demand no special ability or knowledge to pursue.

CHAPTER VIII

The Interpretation of the Board of Trade Returns

Prior to the enactment of the Life Assurance Companies' Act in 1870, Actuaries and the public possessed no effective means for definite comprehension and appreciation of a Company's financial position and prospects (so far as its technical estimates were concerned) beyond the fuller or more meagre indications afforded by the particular office's reports and accounts, which might be brief or commendably complete, but even in the latter event might furnish imperfect actuarial data for adequacy of judgment. This deficiency and obscurity of assessment were the more serious when we consider the vital manner in which, from the special character of Assurance operations, its precautionary provisions are related to a distant future. The existing position might be fairly gauged from inspection of the Revenue Account and balance sheet, but the essential element of forecast, by precise disclosure of its effective previsionary methods, was largely incompetent of survey and appraisement. The Act of 1870 enlightened this darkness, and through the instrumentality of the data compulsorily required, particularly in relation to the mode and extent in which future requirements have been anticipated and secured, supplied the machinery for the first time of reasonably adequate estimation, by an external observer, of the measure of solidity and prosperity, or the reverse, which a Company might be expected to maintain and improve.

The provisions of the Act, though capable of useful amendment in some respects, were founded wisely on the preservation of corporate initiation and scope of administration, adapted to the circumstances of each institution, while demanding the published presentation in a systematic form of the methods and their results thus voluntarily adopted. Freedom of enterprise and publicity of record are thus happily combined. The Act, unlike many enactments affecting industrial undertakings, has proved eminently serviceable in the exaction of substantial information only and the

avoidance of insignificant and vexatious details, in consequence of the sagacious manner in which it was originated and framed. Not devised exclusively by men practically unacquainted with the nature and complications of the business to which the law was to be applied, the form of the Act of 1870 was from the outset determined and controlled by the advice of competent Actuaries; the result thus stands out in signal and conspicuous contrast to the legislative failures which, through the absence of skilled constructive counsel, too frequently impede, instead of purifying and strengthening, the development of commercial affairs. Our legislators do not yet possess the practical wisdom of the "spacious days" of Queen Elizabeth, when (as Thos. Fuller informs us in his *History of the Worthies of England*), the Lord Treasurer, Burleigh, "always consulted artificers in their own art."

It has sometimes been asserted that the prescription by Statute of some selected and uniform basis of valuation would entail upon the Government the assumption of financial responsibility for any failure of solvency which might occur. This conclusion as a direct deduction is clearly a *non sequitur*: the basis might be sound, and might, though this supposition is open to grave controversy, apply universally, and yet the decadence of a Company, in consequence of excessive expenditure, for example, would lie beyond the Government's control: still, even admitting for a moment that a uniform scheme of valuation might be appropriate, it is true, as a practical consideration, that, whatever might constitute the cause of failure, the public would undoubtedly judge that interference by Government in one department of execution involved supervision, and consequent responsibility, in the general administration of the Company. It would naturally be contended that any course of action (enlarged expenditure, let us assume) which tended to defeat the integrity of the established method of valuation entailed implicitly an obligation upon the State, by reason of its prescription of the one prominent and integral element on which essentially the entire framework depended. And it is evident that a serious attack would be made upon the spontaneous and enterprising development of Life Assurance if any interference were attempted with the valuation methods and provisions which the judgment and foresight of responsible administrators might conceive to be expedient in the interests of their Policy-holders. Improper or

injudicious actions are revealed by the published Returns, and thus enable the public to withdraw their confidence, and refrain from membership. The authors of that Act accordingly displayed a sound and sagacious judgment and appreciation of the true basis of British commercial enterprise, and the avoidance of a possible (or probable) grave and perilous position, when they scrupulously adhered to the maintenance of corporate liberty of organization and method with the public declaration of such prominent details as would enable an opinion to be formed upon the practical wisdom and judiciousness of the administration.

It is proposed to explain the manner in which the data contained in the Returns can be readily utilized in deciphering the indications of prosperous issues.

No need exists of reference to the specimens of Bonus in subsection (3) of the 9th section in the 5th Schedule, as these proclaim their own story, and clearly exhibit the actual benefits derived by the Policy-holders from the operations of the valuation period. Our object is specifically to determine the probable expectations of the future as founded upon adequacy of provision. The Returns (sections 3 and 4 of the 5th Schedule) present the table of mortality and the rate of interest employed in the valuation ; but upon these points no special comment is requisite, inasmuch as the general practice of Companies in these respects may be accepted as sound. The only observation which appears to offer an aspect of interest to the student—though its practical consequence is insignificant—is the course which has very occasionally been adopted of valuing non-participating Policies at a different rate of interest from that assumed for Assurances on the profit scale, for which no adequate justification seems to exist. Where contracts as a whole possess a briefer period of duration by reason of the advanced ages than those appertaining to younger lives, a somewhat higher rate of interest (that is, a rate more closely approximating to the realized return) might theoretically (though not expediently) be adopted in the former case, since prevision of the probable future value of money is obviously more competent of assessment than is feasible where the future range of survey is considerably prolonged.

With regard to any discrimination in the rates of mortality employed, we would assert that, looking to the constitution of Assurance business generally, and to the fact that the whole of the

lives, whether their Policies have been effected on the participating or non-participating scale of premiums, are selected with equal care and discernment, no reasonable ground exists, as a question of sound judgment and expediency, for basing the reserves to be retained for these respective classes of whole-life contracts upon different data of mortality. It may be admitted, however, that on the whole Assurances completed on the profit scale exhibit the more extended duration, since no doubt a self-selection is involved, to some degree, in favour of a higher longevity in the choice of a participating Policy. A partial indication in this direction is afforded by the published experience of a particular Company where, massing together the whole of its participating Assurances, the actual deaths amounted to 2,618, compared with the number of 2,958 expected by the H^m Table, or a diminution of about $11\frac{1}{2}$ per cent.

In the experience of mortality recently prepared by the Institute of Actuaries and the Faculty of Actuaries, it appears that, taking quinquennial ages from 20 to 90, the average annual rate of mortality among males assured under whole-life policies (q_x), was, in respect of Assurances participating in profits, about $6\frac{1}{5}$ per cent. inferior to that exhibited by the non-participating policies.

The two fundamental elements to be scrutinized in the examination of a Company's Returns as an index to future prosperity consist of (1) the ratio of loading (or margin of premiums reserved unanticipated in the valuation for expenses, fluctuations and bonus) to the actual rate of expenditure (commission and charges for management) deduced from the Revenue Account,—the balance affording a measure of the provision preserved for future profits; and (2) the rate of interest assumed in the computation of the reserves in comparison with the net return actually realized. It is evident that the greater be the excess of these two reserved funds beyond the experienced requirements, the more propitious is the prospect of stability and success.

For the purpose of illustration an actual instance is here selected from the Board of Trade Returns, where the figures have all been proportionately modified in order to avert identification, since this book is intended to embody and indicate general principles and processes only, without the expression of predilection for any particular Company or class of Companies, or attaching a preferential value, except so far as the tendency or results involved may suggest,

to any special method of organization, administration, or valuation.

We deal first with the provision appropriated to future expenses and other recurrent and permanent demands.

In the form appended to the 5th Schedule (section 7) the value of the gross or contract premiums (there erroneously termed the "office" premiums)—excluding, of course, all extra charges for the risks of foreign residence and occupation, and deducting the portion of the liabilities which had been reassured—was stated to amount to £7,295,734, while the present value of the corresponding pure premiums (described improperly as net premiums in the Act) was expressed at £5,907,273: the difference of £1,388,461 represented the present value of the loading for the entire future duration of the aggregate Assurance obligations of the Company, including the provision for profits. The proportion of this reserved loading to the value of the contract premiums is 19·03 per cent., that is, an annual fund is retained for expenses, fluctuations and profits of 19·03 per cent. upon the yearly premiums receivable. Turning now to the Revenue Account for the year of valuation, we find that the net premium income amounted to £622,069, while the actual expenditure in commission and costs of management absorbed £83,994; the percentage of the latter amount to the former was consequently 13·5. Hence, after providing for commission and expenses of administration, the surplus loading for accumulation into profits amounted to 19·03 − 13·5, or 5·53 per cent. upon the premium revenue. Thus the Company had maintained unvalued for future requirements an annual fund of this percentage in excess of the experienced demands, and the expectation of profits by the assured was consequently measured, as regards this particular source, by the sum of £34,400 per annum, being the ratio in question assessed upon the revenue from premiums.

The more appropriate mode of comparison (considering that fluctuations may naturally and sporadically occur from year to year) would be to adopt as the basis of actual experience in this respect the Company's Consolidated Revenue Account for the preceding five years as required by section 6 of the 5th Schedule. The net premium income for the quinquennial period was there shown to be £2,993,407, and the total commission and expenses for that term £400,999, or a percentage of 13·39. Comparing this result with the 19·03 per cent. previously obtained, we perceive an annual surplus

of loading of 5·64 per cent. upon the premium revenue. Assessing this excess loading of 5·64 per cent. upon the yearly premium income of £622,069, we ascertain that the annual margin (after discharging expenses) for accumulation at compound interest into profits was £35,085.

The course just pursued is undoubtedly the suitable one to adopt in discovering the extent of the Company's reserved provision, since a period of five years necessarily includes account of any accidental variations (comprising, as is requisite, the special charges incurred in the valuation and distribution), and presents accordingly a more stable and adequate basis of comparison.

Examining now the extent of the source of profit derived from the surplus-interest which exists for augmentation into bonus, we find (from section 4 of the 5th Schedule) that the valuation was based upon the rate of 3 per cent., and for the purposes of comparison the net realized return upon the investments, after deduction of income tax, may be thus obtained from the Revenue Account for the valuation year.

In the analysis of a rate of interest, a portion consists of a provision, or sinking fund, for possible losses and depreciations of value on the investments and securities, so that any deficiencies of this nature which have occurred should be deducted from the interest revenue in order to ascertain the clear return. This is not requisite in the case of Life Assurance Companies as a matter of practice, since they generally create (and should in all instances erect) an independent Investment Reserve Fund, from which these adverse events are satisfied. It will be ascertained also that the interest-income of a Company as exhibited in its Revenue Account has been diminished by income tax, as the tax in the vast proportion of investments has been deducted at its source.

In dealing with the accounts of a Company for the purpose under consideration, we naturally regard the " effective " rate of interest in place of the " nominal " rate. Interest is usually payable half-yearly, and if consequently a mortgage or investment bears 4 per cent. (the nominal rate) the entire or effective return in a year is 4·04, or about £4 0*s.* 10*d.* per cent. The first half-year's interest, it will be observed, itself secures interest at 4 per cent. for the succeeding half year, so that during the first half of the year the gain on each £100 invested or advanced amounts to £2 ; at the end

of the second half of that year the Company also obtains, in addition to the further £2 upon the principal for that period, a return of 4 per cent. for half a year upon the prior interest of £2, or 10*d.*, making together £4 0*s.* 10*d.* If n be the number of intervals in a year at which interest is converted into principal (for the purpose of earning interest at the nominal rate of i per unit per annum) the effective rate is $\left(1 + \frac{i}{n}\right)^n - 1$, where, if 2 be substituted for n, and ·04 for i, the return above mentioned is obtained.

Let I be the interest revenue for the year, free of tax and of any losses or depreciations of value in investments; let A represent the amount of the Life Assurance Fund at the beginning of that year, and B its amount at the close; B − A then furnishes the excess (including interest) of income over expenditure of every description. The net effective rate of return is deduced by the formula $\frac{2I}{A + B - I}$.

The *rationale* of this expression may be thus demonstrated.

Adopting the preceding symbols: A obviously bears interest (assumed to be i per unit per annum) for one year; B − A, on the supposition of a practically uniform distribution of receipts and payments during the year, yields interest at i for half the year, or produces $(B-A)\frac{i}{2}$ by the close of the year. But $(B - A)\frac{i}{2}$ also comprises the interest for half the year upon the included I (or total interest revenue): and since our object is to compare the rate on the fund with the aggregate interest received (that is, the interest itself which is paid plus the interest which it has also earned by investment) we must either exclude $I\frac{i}{2}$ from the expression $(B - A)\frac{i}{2}$ on the one side of the equation, or add its amount to the I on the other side. We thus obtain two identical equations for deducing i or the realized effective rate required, namely

$$Ai + (B - A)\frac{i}{2} - I\frac{i}{2} = I, \text{ or } Ai + (B - A)\frac{i}{2} = I + I\frac{i}{2}.$$

Either equation of course furnishes

$$Ai + B\frac{i}{2} - A\frac{i}{2} = I + I\frac{i}{2}$$

$$A\frac{i}{2} + B\frac{i}{2} - I\frac{i}{2} = I.$$

Multiply each side by 2, then

$$Ai + Bi - Ii = 2I$$

$$i\,(A + B - I) = 2I$$

and i (the rate required)

$$= \frac{2I}{A + B - I}.$$

The realized rate is sometimes derived by dividing the interest revenue by the average of the two amounts of the fund, that is, by $\frac{A + B}{2}$, so that the formula becomes $\frac{I}{\frac{A + B}{2}} = \frac{2I}{A + B}$.

This result is obviously inferior to that furnished by the formula we have adopted, and does not so adequately express the actual experience of a Company whose mortgages and investments bear interest at intervals never less than twice a year.

The Life Assurance Fund of the Company in question amounted at the beginning of the valuation year to £6,519,586, and at its termination to £6,691,570; the interest included in the account was £262,517: hence the application of the formula shows

$$\frac{2\,(262{,}517)}{6{,}519{,}586 + 6{,}691{,}570 - 262{,}517},$$ or ·04055 per £1, and, therefore, 4·055 per cent. The valuation rate being 3 per cent., there remained an annual surplus of interest of 4·055 − 3, or 1·055 per cent., which, assessed upon the fund at the close of the year (£6,691,570), would yield £70,596 per annum for accumulation into profits.

We thus perceive that the aggregate yearly surplus for the future from both sources amounted to £35,085 plus £70,596, or £105,681, as an excellent measure of the favourable prospects which might be reasonably anticipated from the company's operations in succeeding years.

Here again it would be more congruent to ascertain the actually realised rate from the Consolidated Revenue Account for the preceding valuation term, since a comparison would be more justly based upon a series of years where any accidental fluctuations in individual years may be assumed to balance. The application of the formula, however, in this mode to the Consolidated Account would merely furnish the rate for the entire period, while we require

the annual return. It would be inappropriate to divide the aggregate rate by 5 (the duration in years of the term) and accept the quotient as the yearly result, for the progression of the ratio year by year will probably fail in uniformity. Assuming that the series of terms approximately assumes a geometrical nature, we only possess for the determination of the "run" of the rates, so far as the Consolidated Account is concerned, the elements of s, l and n, and hence a solution cannot with exactness be obtained. It will be sufficient for all practical purposes to adopt the course of computation which has already been presented in connexion with the account for the final year of the valuation period, notwithstanding the possibility that this ultimate term of the series may exhibit exceptional features.

It should be added that in Section 9 of the 6th Schedule the average rates of interest realized are furnished for each of the preceding 5 years, and the agreement with the results which the formula presents will depend upon the method of assessment adopted by the Company. A difference will naturally be produced if the returns be deduced from the ratio of the interest revenue to the average of the amounts of the Assurance Fund at the commencement and termination of each year.

NOTE :—Referring to the demonstration on page 274, the following investigation appears to be more complete :—

A = the fund at the beginning of the year ;
B = the fund at the expiry of the year ;
R = the income of every description (*i.e.*, the Revenue from premiums and miscellaneous receipts) with the exception of the interest since this is the element on which we desire to operate ;
E = the expenditure of every kind—claims, surrenders, expenses, commission, and miscellaneous payments ;
I = the total Interest Revenue, including the interest which the re-investment of receipts of interest produces.

Then, obviously,

$$A + R - E + I = B \ldots \text{(i)}$$

I (on the assumption of a fairly uniform distribution of receipts and payments)

$$= Ai + R\tfrac{i}{2} - E\tfrac{i}{2} \ldots \text{(ii)}$$

From (i), $R - E = B - A - I$:
Multiplying by $\frac{i}{2}$,

$$R\tfrac{i}{2} - E\tfrac{i}{2} = B\tfrac{i}{2} - A\tfrac{i}{2} - I\tfrac{i}{2} \ldots \text{(iii)}$$

Substituting (iii) in equation ii., we find—

$$I = Ai + B\tfrac{i}{2} - A\tfrac{i}{2} - I\tfrac{i}{2}$$

$$= A\tfrac{i}{2} + B\tfrac{i}{2} - I\tfrac{i}{2}$$

$$\therefore 2I = i(A + B - I)$$

$$\therefore i = \frac{2I}{A + B - I}$$

CHAPTER IX

The Winding-up, Transfer, and Amalgamation of Assurance Companies

I.—Winding-up

The necessity of this course may theoretically be entailed by (1) the insufficiency of the premiums charged for the risks.

This origin may, however, be omitted from consideration, since inadequate premiums have never practically constituted an efficient cause of insolvency.

(2) A reckless course of business resulting in the admission of a preponderant mass of inferior lives. This cause again may be dismissed from view: financial unsoundness terminating in insolvency does not appear to have arisen from excessive mortality.

(3) Losses upon investments and severe depreciation of values. This may occasionally have proved in the past a contributory cause, but not the primary one. One Company indeed seriously jeopardised its possibility of independent existence in consequence of the investment of a considerable proportion of its funds in a single leasehold security of largely unproductive character. But on a general review of the history of institutions, whose separate career has terminated, this factor in the production of a grave impoverishment of resources may also be neglected.

(4) The efficient and actual cause of weakened reserves in the past which has rendered a winding-up imperative has invariably consisted of excessive expenditure in the attempted acquisition of an extensive and needless amount of new business.

By the provisions of the Life Assurance Companies' Act of 1870, the status of the Policy-holder in the preservation of his jeopardised interests, was for the first time recognized. By section 21, the Court may order the winding-up of a Company on the application of one or more of the Assured, provided it be shown to the Court's satisfaction that the Company is insolvent. In its determination upon this point the Court considers (not the Company's ability to discharge current claims and demands out of accruing income or funds, but) the value of its prospective liabilities under the contracts in relation to the accumulated assets. The Court,

however, will not assent to a public hearing of the petition until, as evidence of good faith, the applicant has deposited costs for such an amount as the judge to whom the petition is first submitted may deem to be adequate, nor unless the judge is satisfied that a primâ facie case has been established. In the event of the petition proceeding into Court, the Court will suspend further action for a reasonable time,—if the Company possess an uncalled Proprietary Capital sufficient in amount to supply, with the future premiums and the invested assets, the ascertained deficiency—in order that the capital or an adequate proportion may be realized. If this effort should fail, the Court will make an order upon the petition as though the Company had been proved insolvent.

The preliminary proceedings before the judge (presumably in private) form a just provision, since if there exist a possibility of reformation and a restoration of solvency, it is most expedient that the attempt should not be frustrated by the damage to credit which a petition heard first of all in public would produce.

By section 22, the Court may on demonstration of insolvency reduce the amount of the contracts of the Company in place of issuing an order for winding-up: that is to say, the several sums assured (the premiums continuing at their existing amounts) would be diminished until the present value of the reduced policies, less the present value of the future premiums (after allowing for the necessary costs of administration), were brought into equality with the assets in possession. In these circumstances the policies of longest duration suffer the heaviest diminution, and the necessary cause of this apparently anomalous result often fails to be understood. It appears at first sight to be manifestly unjust that those who have contributed most largely in premiums, and could only secure protection elsewhere at the highest sacrifice, should sustain a severer loss than members who were more recently admitted. The unavoidable character of this relative position will be clear from the following analysis :—

A policy was effected at age x, n years ago, for £S : on the expiration of that period the reserve should amount to $[A_{x+n} - \pi_x (1 + a_{x+n})] S$, and the Company is also entitled to the future premiums possessing a value of $S. \pi_x (1 + a_{x+n})$: its assets consequently, actual and contingent, amount to $S. A_{x+n}$ (or the sum of the two quantities), that is to say, the single premium for the Assurance at the advanced

age ; in other words, at age $x + n$, the Assurance of £S is provided by (1) a single premium of the reserve, and (2) the continued annual payment of π_x. Expressing this result in a concrete form, let x be 30 : n = 10 years ; then the reserve per unit assured will be ·46425 — ·0179 (18·394) or ·135 (O^m 3 per cent., and assuming the premium to be immediately due) ; and the future premium is ·0179. Now the pure single premium for 1 at age 40 is ·46425. The reserve of ·135 treated as a single premium will therefore maintain an Assurance of ·46425 : 1 : : ·135 : x, or ·2908 ; and, the annual pure premium at 40 being ·02524, the actual premium of ·0179 will provide an Assurance of ·02524 : 1 : : ·0179 : x or ·709 : the two amounts of ·2908 + ·709 forming together the sum of ·9998, or the sum assured of 1 furnished by the single premium at the age attained. (This result will be evident, on reflection, by inspection of the formula.)

Assume then that at the date of winding-up, a policy had been effected for £500, 30 years prior to that date, at the age of 35 ; and that another for the same amount had been completed at the same age 5 years ago. The pure premium on the O^m Table at 3 per cent. at age 35 is ·02116 per unit assured, or 10·58 for the Assurance of £500,—each Policy-holder paying the same annual sum at the present ages of 65 and 40 : the Company in relation to the first Policy-holder should thus possess the reserve-value of 500 $[A_{65} - \pi_{35} \ (1 + a_{65})]$ or £500 [·72181 — ·02116 (9·551)] or £259·856, with the retention also of the future annual premium of 10·58 ; while, as regards the second Policy-holder, the reserve should be retained of £500 [·46425 — ·02116 (18·394)] or £37·52, together with the future premiums of 10·58.

Now, following the preceding analysis, the reserve of 259·856 will, regarded as a single premium at age 65, provide £360 of the total assured sum of £500, while the premium of 10·58 (the annual premium per unit of Assurance at age 65 being ·07557) will, by proportion, sustain the balance of £140 : the corresponding amounts in the case of the recent policyholder (where the annual premium at age 40 is ·02524) will be £81 and £419.

Assume that the condition of insolvency involves the loss of one half of the assets (that is, the reserves) which in a sound company would remain in possession ; then in respect of the first policy there exists an actual reserve of £129·928, and, in connexion with

the second, one of 18·76,—each policyholder continuing to discharge the original premium of 10·58. In the first case, the total sum assured now provided by the diminished reserve and full premium is 180 + 140 or £320 : in the case of the second policyholder, the total amount assured still possible will be 41 + 419 or £460, or the later entrant (although he has only contributed for 5 years) obtains in the reconstruction a policy of about $43\frac{3}{4}$ per cent. in excess of that allotted to the policyholder of 6 times the duration.

The reason and justification are evident : the uniform premium provides a reduced Assurance at the older age of 65 than at the younger age of 40 : the balance in each instance should be constituted by the larger proportionate reserve attached to the former policy, but it is precisely in consequence of the wasted reserves which express the insolvency that this compensation has been rendered impossible.

In the preceding exposition we have assumed, with a view to simplicity, that the valuation at a reconstruction implying a reduction of contracts would be based upon 3 per cent. interest, and that pure premiums would be employed. In practice, however, this course would be obviously inappropriate. Since no new business is thenceforth to be acquired nor any profit specifically anticipated (and therefore provided for), the premium to be valued will be in excess of the usual pure premium, and the rate of interest (with a margin for fluctuations, possible losses and depreciations of securities) will approximate more closely to the actual return expected to be realized upon the assets. If that rate be $3\frac{3}{4}$ per cent., the rate employed in the valuation of the liabilities may probably be adopted at $3\frac{1}{2}$ or $3\frac{3}{8}$ per cent., and the premium valued as a prospective asset may perhaps be obtained by deducting 10 per cent. only from the contract premium. In respect of the latter course, however, care will require to be exercised that negative values do not appear, and thus produce a fictitious amount of assets, since as regards the more recent Assurances and for many years of duration a percentage of 10 will produce this fallacious effect. In an instance which came under the cognisance of the author, several years ago, the element of negative values was overlooked, and the computation consequently of the reduced contracts, as a basis of proposed reconstruction on insolvency, was largely vitiated.

The table of mortality to be selected will demand special consideration. The distrust occasioned by the failure of the company will necessarily entail the effect of depriving the office of its young and vigorous lives who will seek Assurance protection from other institutions of unimpaired and prosperous condition : a considerable proportion accordingly of the company's members after reconstruction will consist of lives of advanced ages and of those who deem it impracticable, on account of deteriorated constitution and prospects of longevity, to obtain substituted policies elsewhere, and hence are compelled to incur the chances that exist. Moreover, as no new business will be sought, the enhancing rate of mortality will not be influenced by the diminished death-rate of superior lives incorporated from time to time. The mortality consequently to be anticipated in the future, due to this abstraction of the sounder lives, will undoubtedly be augmented, and the table of mortality adopted as the basis of valuation in a winding-up or reconstruction must embody this adverse expectation and provide for the increased mortality experience which will prevail.

II.—Transfers

Among the reasons which render a transfer of some company to a more vigorous competitor expedient or imperative, the cause will frequently prove to be the enfeebled (though solvent) condition of the office, produced by inefficient or inert administration, which suggests the hopelessness (without extravagant expenditure) of maintaining an independent existence among enterprising and alert rivals.

It is desirable that precise definitions should be attached, and consistently employed, to the terms, Transfer and Amalgamation.

A transfer occurs where the assets and liabilities of one company are accepted by another and retained as a separate series distinct from the second company's own business ; so that the series is worked out to extinction as a closed account, possessing, in the event of its independent assets ultimately proving insufficient for its independent obligations, the additional protection of the resources of the accepting office. In the course of time the number of these contracts unrecruited (as will be the case) by new entrants will become inadequate as a basis for separate existence, and provision will then be required to be made for merging them on equitable

terms with the general contracts of the transferee company itself, which possesses the necessary foundation of a multiplicity of risks.

In negociating a transfer, the sum to be handed over as the consideration-money to the accepting office will necessarily be less than that produced by a pure premium valuation, since the heavy cost of acquiring new business will now be extinguished. And in all probability a higher rate than 3 per cent. will be employed, though the rate will be maintained sufficiently low to provide a reasonable bonus and consequently promote a feeling of content in the transferred policyholders.

Besides the consideration-money (or the aggregate reserve for the existing obligations) and premiums, the accepting company receives the good-will possessed by the transferred office in the form of its business connexions and agents, whose services will be utilized in extending the purchasing company's own operations.

By the Act of 1870 (section 14) every policyholder in an office proposed to be transferred must be furnished with a statement of the nature of the transaction, an abstract of the material facts embodied in the deed by which the change is effected, and a copy of the actuarial or other reports on which the arrangement is founded. The Court, again, will not approve the bargain if policyholders representing one-tenth or more of the total amount assured in the company intended to be transferred dissent from the proposal.

It appears to be an unfortunate omission from the Act that similar information is not required to be supplied to the policyholders of the accepting office, since obviously the transfer in question may affect their position, either in consequence of increased expenditure being charged upon their income (a result which may sometimes occur where the accepting company agrees to conduct the business of the separate series on a diminished scale of cost) or of a possible ultimate additional strain upon their resources by the protection furnished by their own funds. Moreover, the specific security provided by their own capital is to an extent reduced by the importation of these additional obligations.

The limits within which the valuation reserve theoretically may range are expressed by a pure premium valuation and one based upon gross or contract premiums. Let S be the total sum assured, P the aggregate premium revenue actually payable, π the total pure premium income, then $P - \pi$ or $\dot{\phi}$ expresses the

aggregate annual source for payment of expenses and creation of profits. Let these symbols accented represent the present values of the several items. The reserve then on a pure premium valuation will be $S' - \pi'$, and on a gross premium basis, S'—P'. The latter reserve is of course much inferior to the former by the anticipation as a realized asset of the whole of the loading—the equivalent $S' - (\pi' + \phi')$ being lower than $S' - \pi'$. The transferred company cannot probably afford to offer $S' - \pi'$, since if it possessed this adequate reserve it would hardly require as a rule to abandon independent existence. And moreover, there is no necessity or justification for the purchasing company to demand so substantial a sum, since the expenditure on the transfer of the purchased office will materially be reduced in consequence of the relinquishment of its separate branches, inspectors, advertising, and the customary organization of a continuing concern. On the other hand, the accepting company cannot in justice to its own policyholders (even though it acquire a serviceable goodwill in the form of agents and commercial connexions) consent to receive a reserve of $S' - P'$, since by the absorption of the loading it would be undertaking the conduct of the transferred business at the exclusive cost of its own members. Some intermediate basis of valuation between these extremes would accordingly, as we have just remarked, be arranged in adaptation to the actual conditions and prospects presented in each instance.

III.—Amalgamation

This term is properly applied where the assets and liabilities of two or more companies are merged together for the purpose of constituting a combined and homogeneous institution, and where the separate titles of the offices may be associated, or fused into some substituted title expressive of the unified and organic body. It would extend beyond the limits and purpose of this book to enter into the details of arrangement with respect to the adjustment of the capitals of the amalgamating companies or of their resources generally. The object to be secured in the transaction is an equalization of the position, risks, and benefits of each company involved, which is frequently a process of a delicate and difficult nature. Assuming that two companies amalgamate, the combined capitals are liable thenceforth for the united liabilities, and their

respective paid-up and uncalled amounts will demand proportionate adjustment. If, for example, Company A possess a business and a paid-up and uncalled capital considerably smaller than the obligations and capital of Company B with which an amalgamation is arranged, it would be unjust that the lesser capital should be jointly responsible with that of B for the more expanded liabilities of the latter, and hence the need of some prior reassessment upon that point. Such an instance, however, is not very likely to occur.

It has sometimes happened that in an amalgamation the modes of distribution of profits differ in the two companies, and an adaptation must be effected. The author recalls an instance, though it is probably of rare occurrence, where the practical results of the method of division of profits adopted by the one office in the form of an annual cash percentage abatement of the premiums proved equivalent with those produced by the compound reversionary bonus system which the other institution employed, so that the amalgamation demanded no necessity of modification of process in this aspect.

The liabilities for Assurance and Annuity contracts in each company will be valued, as the primary proceeding, upon the same principles and an identical table of mortality and rate of interest, so that the financial position of each office at the epoch of amalgamation will be represented by the reserves thus obtained, increased by the paid-up capitals and outstanding obligations respectively. Deducting each total from the realized funds, a surplus is disclosed in each instance. In one method of investigation, it was found convenient to value the actual contract premiums as a prospective asset, so that the resulting surplus (S in one company, and S' in the other) expressed the funds from which respectively the expenses of future administration and bonus additions would be provided. This particular item (S and S') consequently affords, with others, an index of the comparative condition of the two offices. A pure premium valuation of course would produce identical results, for the two sets of assets would then comprise separately the capitalized value of the future loading.

Now the necessities of a company with regard to future expenditure may be measured by the amount and probable duration of its annual premiums generally (which are to be administered), while the needs which will occur in the distribution of profits will

similarly be determined by the amount and probable continuance of the premiums upon the participating policies alone; in other words, taking the question of future expenses as an example, the yearly requirements of the future and the duration of those requirements will obviously be measured by the amount and duration of the policies from whose premiums the provision for expenses is to be obtained and during whose continuance those expenses will require to be discharged. In forming this estimate the pure premium values were employed; and if P and P' be the present values of the participating pure premiums respectively in the two companies, and NP and $N'P'$ those of the non-profit premiums, the relations necessary to constitute an equality of condition between the offices consist of S (the surplus) $= \frac{P}{m} + \frac{NP}{n}$ and $S' = \frac{P'}{m} + \frac{N'P'}{n}$: n representing the denominator of the fraction which expresses the proportion of the surplus (S and S') requisite for expenses and profits to shareholders, while m, besides including these provisions, comprises the demands for periodical returns of bonus to the policyholders.

It will be observed that these expressions equate the total surplus ascertained to the present value (measured in the mode above described) of the future annual charges which that surplus is required to satisfy. If we regard the annual surplus as consisting of the loading contained in the premiums (neglecting the source of surplus which is formed by the excess interest realized upon the premiums beyond the rate upon which they are computed), we can determine by an inspection of the company's accounts and an acquaintance with its bonus regulations the denominators in question. Thus: if 10 per cent. be necessary for expenses and shareholders' profits on the non-participating policies, and 20 per cent. for these requirements (including also the future bonuses to the assured) in respect of the participating Assurances, the m will be 5, and the n, 10.

Assuming that the surpluses of S and S' are equally proportional to the respective requirements of the companies for future expenditure and profits, the offices are thus shown to be competent to combine upon equivalence of terms; and the bonuses in the united office will accordingly be declared upon a uniform basis. This

equality naturally will rarely exist, and one equation will exhibit an excess, which will require to be arranged prior to the amalgamation by appropriation of its amount among the members of the company in which it is disclosed. An equivalence of condition being thus established by this process, the two companies can be equitably merged into one organic institution, administering the combined business upon an equality of method in every respect.

Probably a disproportion subsists between the amounts of the paid-up capitals, so that the larger amount will require an additional annual provision for dividends. This inequality will first need to be redressed, and a mode has been proposed that the standard to be applied should be the value of the surplus retained for all purposes beyond the allowance for bonus to the policyholders; that is to say, the relation should exist (distinguishing the capitals as A and A') of $A : A' :: \frac{P + NP}{n} : \frac{P' + N'P'}{n}$. Presumably a reduction of total expenditure will result from the union; different branch offices, for example, will not be required, and economy in other directions can be effected, so that the profits of the combined companies will increase to the advantage of the entire body of policyholders.

In this method of procedure, S and S' are adopted as proportional to the values of the pure premiums, with the result that the profits in future will be distributed in the unified company upon a uniform principle and independent of any difference in the rate of loading contained in the respective premiums charged by each office. Such a principle of allotment, it has been pointed out, would be the grant of additions to policies at a uniform rate per cent. per annum upon the sum assured. If the participating pure premiums in one of the two companies are not equally loaded with those in the other, the premiums in the one involving a proportionately larger addition, the policyholders of that office are entitled to increased bonuses, and the process already mentioned of an immediate adjustment and distribution for the rectification of this divergence affords the Assured in the company in question a present advantage corresponding to their enhanced rates, so that during the sequent duration of the combined offices they will receive the same proportionate return as that which is assigned to the members of the second company, and thus complete equality

in this respect will, as a consequence of the preliminary arrangement, prevail.

Assume however, as another authority has observed, that the method of distribution which the company adopts after the amalgamation be one which includes consideration of any inequality in the loadings—suppose, for example, a cash distribution proportioned to the premiums paid or to the loadings—it is clear in that event that the plan previously adopted of reserving a surplus proportional to the pure premiums would not be equitable. If the values of the pure premiums in the two offices be equal ($P = P'$)—P being the premium in the one office and P' that in the other—with however a loading in the second company which is double that adopted in the first, then by the plan above described the same reserve would be created in each case ; but if the profits are distributed among the Assured in proportion to the loading, the Assured in the second company would receive twice as large a bonus as that allotted in the first, although the reserves retained for the two classes of bonus were equal. Consequently if the bonuses are to be proportional to loadings, the reserves for such bonuses must be proportional to the value of the loading ; if proportioned to the contract premiums, they should be proportioned to the value of those premiums. If L and L' be the values of the loadings of the policies on the participating scale in the two companies, the union may be effected on equivalence of terms, if the bonus is proportional to the loading, provided $S = \frac{P + NP}{n} + \frac{L}{m}$, and $S' = \frac{P' + N'P'}{n} + \frac{L'}{m}$. If, on the other hand, the bonuses are to be proportionate to the contract premiums, $P + L$ and $P' + L'$ should be substituted in these equations for L and L'.

The consideration of these questions is highly technical and involved, but the student at all events, whatever be his difficulty in grasping the details without the aid of actual experience, will perceive the principles of assessment with a view to producing that equality of condition and benefit which should govern these transactions.

By section 14 of the Act of 1870 the same process is prescribed in connexion with amalgamations as that provided for transfers, but the copies of the statements on which the arrangement is

founded must be forwarded to the policyholders of both companies concerned, divergent accordingly, without apparent reason, from the practice enforced in the case of transfers.

Some companies by the provisions of their Deeds of Constitution or otherwise possess express power to transfer their obligations to another office, or to effect amalgamations. Several cases were formerly submitted to the Courts where such transfers had occurred, when it was decided that the policyholder(however he might dissent) possessed no remedy against the company in which he originally assured and was only in a position to claim upon the transferee office to which had been handed the funds out of which all the claims should be provided. It has also been held on many occasions that, in fact, the policyholder had accepted the responsibility of the new company in substitution for that of the original office. Where the question is one of fact it is difficult to deduce any definite principles relating to Novation, as it is technically termed—that is, the acceptance by the policyholder of the new or substituted fount of liability for his claim, or, in more general terms, the assent of a creditor to substitute one person for another as his debtor. Such an acknowledgment of altered liability as a fact, consisted, it has been judicially affirmed, in the acceptance of a bonus from the fresh company, or the circumstance of bringing an action for the settlement of a claim against the latter. The adequate mode involves an endorsement upon the policy, followed by the payment of premiums to the substituted office. For the mere payment of premiums to the new company, unless accompanied by other facts which prove that the Assured was cognisant of the transfer of his policy and assented to the change, did not necessarily demonstrate that a fresh contract of Assurance had been substituted for the original policy.

Amalgamations and transfers conducted in accordance with the provisions of the Act of 1870 obviate all these possible difficulties and sources of contention.

Amalgamations are often of permanent value : but their number and extent may prove in time to be inharmonious with the legitimate expansion of life assurance and the facilities of selection which should be afforded to the public. If the number of independent offices be greatly diminished in the future, the choice of the public may become unduly narrowed ; the capacity of existence of admirable offices of smaller magnitude and fulfilling useful

functions may be jeopardized; the healthy spirit of a vigorous and reasonable competition may be limited in its serviceable scope; and combined companies may ultimately become so unwieldy in dimension that they may fail to be as effectively and profitably administered by a single mind as the several companies, while continuing independent, were competent of being managed by separate minds, rendered alert and responsive by a generous rivalry.

Though the student will accept the following maxim as generally applicable in business and not specially confined to Assurance institutions, he will remember that extent of size is not necessarily coincident with range of profit.

CHAPTER X

The Uses of Life Assurance to the Business Man

Besides the provision of a Family Fund, created in the usual form of a policy continuing for the whole period of life, the following adaptations of Assurance to the varying needs of the Assured himself, his family, and his commercial necessities, exist. For convenience of reference, they are arranged under distinctive headings :—

For the Assured Himself.

A policy, termed a Pure Endowment, may be effected which is exclusively payable if the applicant attain a specified deferred age. In consequence of this limitation of risk, the annual premium is very moderate.

For the Assured and His Family.

An Endowment Assurance constituted for a definite term comprises a temporary Assurance of the full amount for the period extending to the deferred age on which the Assured may determine, and a pure endowment of the same sum to be claimable on the attainment of that age. By this combination the sum assured is received by the representatives if death occur during the period in question, while if the Assured survive to the age upon which he originally fixed the amount is payable to himself.

These Assurances can be so arranged that if, after the lapse of a year or two, the policyholder should prefer to discontinue the contract (without accepting a cash surrender-value) a proportionate part of the sum assured, based upon the number of annual premiums already discharged, can still remain in force on the conditions of the original Assurance ; if, for example, the interval between the age on admission and the deferred age be 30 years, and 3 years' premiums have been paid, the free policy granted in substitution would be $\frac{3}{30}$ths or $\frac{1}{10}$th of the original amount. The form of the Assurance can also be settled in such a mode that, if death occur prior to the deferred age, a certain sum will become payable to the family, while if that age be attained double that amount will be receivable by the policyholder himself.

Or, on reaching the specified age, the Assured can, at his option, either obtain payment of the amount assured or an annuity for

the remainder of his life of a prescribed amount, with the receipt by his family of the original sum expressed in the policy on the policyholder's subsequent death; or a substantially increased annuity can be promised to the Assured during the sequent continuance of his life without any final payment to the family.

For the Family Generally

On the death of the Assured, the policy-monies may be permitted to remain in the company's guardianship for distribution to the family in the form of specified instalments over a fixed term of years. The advantage of this arrangement lies in the fact that the fund is meantime managed by a solvent trustee, and saves the widow the trouble and possible losses incidental to investment.

Or, the proceeds of the Assurance may continue after death in the possession of the company, and, until it is discharged, the company will allow a settled rate of interest to a nominee (for life or for a period), to be named on the original completion of the contract.

An Assurance may also be effected by a husband and wife provided the former possess an insurable interest in his wife's life, to become payable at the first death which occurs.

If the wife or husband possess a life-estate in property or funds which would be diverted at death to another branch of the family or elsewhere, a portion at least of the capital-value may be preserved in the family by means of a policy.

For the Children

For the purpose of providing the requisite funds for an expensive professional education to a son, and creating a marriage portion for a daughter, or the starting of a boy in life, a policy, without medical examination, may be effected at birth or at any other infantile age which shall be payable on the attainment of age 21, or any other fixed age. In such instances the Assurance may be completed either on the condition that the premiums paid shall be forfeited, or shall be refunded, (without interest), in the event of death occurring prior to the specified age. It may also be provided that, if the father be ascertained to possess average health, the payment of the premiums, moderately increased to compensate the additional risk, shall cease in the contingency of the parent

dying before the child has reached the age determined on, so that no further burden will then be entailed, and the policy will still remain payable in full on survivance to that age.

Assurances may also be completed without medical examination either in the form of a deferred Life Policy or an Endowment Assurance upon the life of a child at any infantile age, with the provision that the liability of the company shall not commence until the child has reached the age of 21 or 25, or some other specified age. The Assurance is then payable if death occur after the age in question (or, in the case of an Endowment Assurance, if the deferred age be attained), and the advantage is thus secured of presenting the child, when he reaches the age of 21 or 25, with a complete Assurance subject to a considerably reduced premium.

These policies are naturally effected in the name of the parent or guardian, and it is customary to state in the proposal form that no insurable interest exists and that the Assurance is simply completed for the ultimate benefit of the child. We refer to the legal aspect of these contracts in Chapter XI.

The premiums in the event of death happening prior to the adult age agreed upon may be either non-returnable (and consequently fixed at a still lower rate) or returnable in full (without interest) to the parent or guardian.

For Commercial Uses and Purposes of Security

The Assured may require a policy upon his life for a term of years only in order to protect some specific interest or undertaking connected with his business or expectations which it is important to cover for a definite period; and should he desire at any time prior (to the extent of at least one year) to the expiry of the term for which the Assurance was effected to convert the contract into one for the whole duration of life, he can usually conclude this arrangement in respect of the original sum assured and without medical examination at the ordinary rate of premium for his age next birthday at the date of conversion. The Assurance would be thereafter regarded for all purposes as a life policy completed at the time of change.

A very important and serviceable mode of Assurance for partners in a business firm consists of a joint-life policy upon their lives, which in this aspect may be described as a Partnership Policy.

In newly established firms particularly the early death of a monied partner and the consequent abstraction of his share of the capital may prove a serious strain upon the business resources and enterprise, and this possible difficulty is effectively prevented by the completion of a policy payable upon the first death that may occur among the members of the firm.

The policy would be treated as a partnership asset, and the premium discharged out of the business income. On the death of a partner, consequently, his share of the capital is restored by the receipt of the sum assured. And when a partnership is dissolved, and the policy should no longer be required, its surrender-value can be obtained and distributed.

A person, *A*, is entitled to a sum of money contingent upon his surviving another person, *B*, probably the life-tenant of an estate. Except as a speculation, largely dependent upon the condition of the health of *B* or of the present beneficiary of the fund, the contingent reversion possesses no value. It can be converted, however, into a marketable investment by the completion of a policy payable only in the event of *A* dying during the lifetime of *B*, and subject to annual premiums during the joint duration of the two lives, or rendered unburdened (and thereby made absolute) by payment of a single premium. In this protected form the security becomes available also for purposes of a loan.

Similar remarks apply if *A* be entitled to the income for life of a certain fund provided he survive the present recipient of the income, *B*. In this case the protecting and completing Assurance must be effected for the entire continuance of *A's* life.

A person is entitled to a certain fund or estate provided the present beneficiary fails to leave children by his existing or any future wife. This contingent reversionary interest can also be rendered complete by effecting a policy payable in the event of children being left to defeat the succession.

Similarly the possession of an estate may be contingent on the present owner retaining the Name and Arms, as his name and arms, of the person from whom the estate has been derived. The ownership can be made secure to a purchaser or mortgagee (in the event of this condition becoming violated) by a policy to be payable if the name and arms be abandoned, so that whether this result occur or not the value of the estate is preserved.

An estate may be receivable in expectancy if the present holder, a lunatic, fail to recover sanity of mind sufficient to execute a valid will or devise which would defeat the expectation of succession of the reversioner. Companies will issue an Assurance to guarantee this possible forfeiture of future ownership.

An investor has purchased a leasehold property for an unexpired period of *n* years ; or he has bought a bond in some Government stock or commercial undertaking at a premium which is redeemable after a fixed term at par. He can fully provide the restoration of his capital in the first instance, or the premium in the second, by effecting what is termed a Leasehold Redemption or Sinking Fund Policy (without the contingency of life being involved) for the period in question.

A debtor may owe an important sum, and in all probability the occurrence of his death would prevent repayment or entail a serious burden upon his family. The interest of his creditor and the relief of the family can be secured by an Assurance upon the debtor's life.

Various Purposes and Modes

By the Married Women's Property Acts of 1870 and 1882, a married man may effect a policy for the benefit (expressed in the contract) of his wife, or of his wife and children, or of his children only, and provided it be completed when his financial position was solvent, the Assurance will be protected by law from the claims of any subsequent creditors.

The Trustee or Trustees who are to receive the amount of the policy on the death of the Assured can be appointed either in the policy itself or by a memorandum signed by the Assured ; and if no Trustees have thus been constituted, the proceeds of the Assurance are payable (under the Trust) to the legal personal representatives. But in order to avert the expense to the widow of taking out Administration to her husband in the absence of Trustees, it can be arranged, by a special form of policy, that the wife herself shall be appointed the Trustee, and thus receive and retain the money for her own benefit where the Trust has been created in her favour. This cumbrous machinery, however, may be avoided by the wife (under section 11 of the Act of 1882) effecting the Assurance in her own name upon her husband's life.

A married woman, too, may contract for an Assurance either upon her own life or upon that of her husband for her separate use as though she were a single woman (*feme sole*).

The policy on her own life can also be expressed to be for the benefit of her husband or of her children or both.

Policies are also issued which are exempt from the payment of premiums during temporary or permanent incapacity produced by accident or bodily and mental disorder. To the professional man, whose income is dependent upon his personal attention to his professional pursuits, an Assurance of this description is peculiarly valuable. The ordinary rate of premium is moderately increased: the life must be eligible upon the usual terms, and the profession or occupation must not subject the applicant to exceptional exposure to risk or danger.

The provision of a fund for the payment of estate duty under the Finance Act of 1894 can be arranged by a policy which, effected for this specific purpose, will be prepared in such a form that the company will undertake to pay to the Commissioners of the Inland Revenue Office either the whole of the sum assured or the necessary proportion required for the discharge of the duty, and this course will be adopted before probate or letters of administration have been obtained. There frequently occurs at death, especially where the estate is extensive or complicated, a temporary difficulty in providing the amount required for probate, or if this charge be met there may in consequence result an insufficient fund for current purposes before the estate is realized, and thus a valuable aid is furnished to the executors, administrators or trustees by this form of policy. The ordinary rate of premium is charged, and the power of the Assured to assign or mortgage the policy continues unaffected. The author, with another actuary, unsuccessfully, at a private interview with the Chancellor of the Exchequer, endeavoured, as appeared to them to be just, to obtain (on certain conditions) an exemption from estate duty of these specific Assurances themselves.

Policies effected at what are termed minimum premiums are also issued. This method assumes two forms: (1) probably $\frac{3}{4}$ths of the ordinary premium alone will be exacted annually, while the remaining $\frac{1}{4}$th (with compound interest at a specified rate) will continue outstanding as a first charge upon

the policy in favour of the issuing company, and the amount will be liquidated by the bonuses which are successively allotted. No personal liability is entailed upon the policyholder for the unpaid quarters (and interest), as the sole security to the company for such advances (since this is the true form of the transaction) consists of the policy itself; (2) another mode is adopted by companies which distribute their profits in the shape of a uniform reversionary addition to the sum assured. A certain rate of bonus is assumed (somewhat less than that actually realized in the past); this is discounted and converted into an equivalent annual amount for the entire period of life; assume that this equalized sum is 20 per cent. of the full premium, the policy holder then pays $\frac{4}{5}$ths of his contract premium only, and should the future profits be sufficient to assign a share in excess of the amount employed in the discounted process just mentioned, the surplus will be applied to a further reduction of the premium or to the increase of the sum assured. Should, however, the realized bonus in future prove inferior to the rate assumed in obtaining the discounted value, the Assured would be subject thenceforth (until the divergence had been rectified by augmented bonuses) to an enhancement of the diminished premium adequate to compensate the difference between the abatement previously allowed and that which the fresh bonus will alone permit.

Assurances are granted without medical examination at entry, but evidently on condition that the applicant presents the external appearance and symptoms of sound health. If death occur during the first year, $\frac{1}{3}$rd only of the sum assured is paid: if during the second year $\frac{2}{3}$rds; while after that period the policy is constituted at its full amount. This preliminary reduction is necessary in compensation for the possibility of the Assured possessing internal and unapparent defects or latent sources of disease.

Annuities

These may be either payable immediately for a term of years or for life, or continuing during the joint lives of different persons; or during the joint lives and the life of the survivor; or during the remainder of the lifetime of a person if he (or she) survive another person; or during the continued lifetime of a person after the lapse of a definite number of years or after the applicant has attained a specified age.

CHAPTER XI

A Brief Summary of some chief legal principles affecting Life Assurance.

It will be found useful to specify some of the more important legal principles relating to the contract of Assurance and to various connected circumstances.

The Contract

The contract of Life Assurance in its usual form may be defined as one in which the company agrees to pay a certain sum upon the occurrence of a particular event contingent upon the duration of human life in consideration of the immediate payment either of a smaller sum (by way of a single premium) or of certain equivalent periodical payments (or premiums) by the Assured either during his entire lifetime or for a specified portion of his life.

The Proposal

The principle on which the maxim of *caveat emptor* (the buyer, or, in this case, the grantor of the policy, must look after himself) is based does not apply; the proposer is required to exhibit in his statements the completest good faith, so that the contract may be perfectly fair and open on both sides. The materiality of his answers to the questions submitted does not depend upon his own personal conception of the importance or insignificance of the facts to be disclosed; the statements he presents must be material in themselves, and should any dispute arise, the decision upon the fulness, relevancy, or accuracy of the answers furnished will, as matters of fact, rest with the judgment of a jury. (Though, on a verdict contrary to the evidence, application may be made to the Court to set it aside or order a new trial, still the courts, as a rule, are unwilling to interfere, in these matters, with a jury's decision.) The applicant, consequently, must not merely return true answers to the questions proposed by the company, but must voluntarily disclose any information exclusively within his own personal knowledge of which it is material that the company should be cognisant in order to form an accurate estimate of the risk submitted.

Specific questions obviously may fail to cover the whole of the necessary facts, and hence in the proposal form a general inquiry is always included, the intention of which is to comprise every important circumstance which a series of limited and particular questions may not succeed in compassing. If on any point the proposer does not possess positive evidence, he should qualify his statements by the addition of the words " to the best of my knowledge and belief," and in this case the warranty relates to the belief and not to the fact, and it is sufficient that the applicant honestly believes the assertion to be true. If a proposal has been submitted to another company, the date of that application, the name of the office, and the result should be scrupulously furnished. If the proposer employ another person as his agent for effecting the Assurance, the same obligation of conveying all information of a material character is obviously incumbent. The knowledge of the agent will be also the knowledge of the principal, and the concealment of an important circumstance known to the agent, although entirely unknown to the principal, will avoid the contract. So will the non-disclosure of facts in the possession of the principal, although the agent be unacquainted with them.

The statement of facts relating to the proposer's health, habits, and other circumstances, is termed the Declaration, which, either expressly or by reference, is embodied in the policy, and its terms, where unconditional and stated as facts, then constitute legal warranties and must be strictly and literally true. The whole of the requisite information precedent to an Assurance is not, however, contained in the declaration; other statements are made by the proposer (for example, in the record prefixed to the medical report by the company's own adviser) which form Representations, and constitute a portion of the evidence on which the company judges the eligibility of the risk and the amount of premium to be charged.

The distinction between a warranty and a representation—both of which are involved in the Assurance contract—is based upon the condition that the former must be literally true, while in the latter it is sufficient that its accuracy is substantial. Where any statements are to be rendered binding as warranties, they must appear upon the face of the policy by which the Assurance contract is effected; either expressly mentioned or incorporated by reference. If this explicit or implicit inclusion be not adopted, the affirmations

are simply representations; and it is not universally true that every statement in the proposal, although the latter may be referred to in the contract, can be construed as a warranty. In order that it may be so interpreted the assertion must amount to an affirmative statement of certain facts essential as a basis of the contract. As in all contracts it will be observed that certain general expressions must be employed whose precise and literal import and connotation cannot be minutely defined; like the wide phrase of "reasonable" wear and tear in ordinary agreements, they must in the last resort, if this should unhappily become necessary, be construed by the common-sense judgment, based upon the customary usages of language and social transactions, of a jury.

The Referees

The referees named by the proposer (to whom questions are submitted by the company relative to his health and habits) must reply to the full extent of their knowledge without the slightest evasion; and should they furnish a wilfully untrue statement, or act in collusion with the proposer, they may, although personally in no way interested in the contract, render themselves liable to an action if loss should occur.

The replies of the referees are not privileged communications, and can therefore be produced by the company if legal proceedings should become necessary. A curious incident happened in the professional experience of the author, illustrating the divided state of mind of a referee who wished to represent the truth and yet in so illusive a form as probably to protect his friend, the proposer, and also enforcing upon the actuary a careful scrutiny of the whole of the papers connected with a proposal. It was the author's habit not merely to examine the documents with vigilance before they were submitted to the doctor and directors, but to subject them again to a revised review prior to the issue of a decision. The particulars relating to the applicant appeared in every respect to be favourable, but in his final reading the author was struck by the fact that in two places, widely separated from each other, the referee had, as though it were incidentally, referred to the proposer's bodily health as thoroughly sound. This qualification then appeared strange; independent inquiry was therefore instituted, and it was ascertained that for a certain period of time the proposer

had been confined in a private lunatic asylum : hence the employment by the referee of "bodily," as distinguished from mental, health. And yet the proposer himself was entirely innocent of deception, for the malady from which he had suffered was of so peculiar a nature that, in the doctor's judgment, the patient on removal would absolutely retain no remembrance of the event so far as his own consciousness was concerned.

The proposer, in furnishing the names of referees most competent to supply information (which the company should test if necessary), is not responsible for the office's neglect on this point or (in the absence of a condition that any untrue statement shall avoid the contract) for frauds on the part of the referees of which he is innocent.

Insurable Interest

It is obvious that the system of Life Assurance is liable to abuse, and may be employed as an instrument of gambling transactions. Hence public policy demands that a legitimate ground should exist for competence to complete a contract ; and on account of the serious extent to which mere speculative Assurances were effected, an Act, usually termed the Gambling Act, was passed in 1774 (14 Geo. III. c. 48) prohibiting contracts by way of gaming or wagering, and requiring an interest by the proposer in the life offered for Assurance as a condition precedent to a valid bargain.

The insurable interest prescribed by the Act (which must be existent at the date of completing the policy) must possess a pecuniary nature ; ties of blood and affection are insufficient to constitute a legal interest. The interest must depend upon some existing right of property which may be adversely affected by the occurrence of the death of the person proposed to be assured, and which, whether it be in possession or in reversion, or be of a contingent character, would entitle the Assured to a standing in a Court of Law if the title to the property should be in question. Thus a parent is not competent to effect a valid Assurance upon the life of his child unless a pecuniary interest exist, that is, unless on the death of the child, property, which attaches to himself during the child's lifetime, would be diverted. On the other hand, a wife, apart from the Statute presently to be mentioned, is presumed to possess an insurable interest in the life of her husband.

The costs of education incurred by a parent do not of themselves constitute an interest. But a promise to maintain and educate a child, given by a person upon whom no legal responsibility existed in this respect, would confer upon that person a right to assure the child's life.

In Chapter X. we have mentioned Assurances upon the lives of children effected in the names of the parent or guardian. These contracts are really in their nature extra-legal though justifiable in fact, that is, they could not be enforced in a Court of Law in consequence of the absence of an insurable interest, but the honour of Assurance companies is so absolute and invariable that no apprehension can possibly exist respecting the complete and final settlement of a claim. Public policy is not contravened by these contracts, for if the child were killed by the parent or guardian while under the age of 21 no benefit would accrue, and it is not within the bounds of the most exiguous reason to anticipate that, at the age of 1 or 2 years, when the policy may be effected, the father is contemplating the destruction of the child on his attainment of manhood. The object of these salutary and praiseworthy Assurances is to secure a definite benefit for the child itself in the ultimate possession of a provision subject to a materially diminished premium.

The heir or next-of-kin again, who possesses an expectation of property on the death of a certain person intestate, does not possess in virtue of that expectancy an insurable interest in such person's life, even although the premature death of the latter might deprive him of property which might otherwise devolve upon him. The reason is evident.

A person expectant of a devise upon the death of a testator is not competent to assure the latter's life in order to protect the value of a promised devise : the promise may be revoked.

A creditor possesses an insurable interest in the life of his debtor, since the chance of obtaining payment is considered to be reduced by the debtor's death, and this capacity to assure is not affected by the fact that the creditor may hold some real security.

A creditor has promised his debtor that he will not enforce the debt during his (the creditor's) lifetime, yet the debtor does not possess in law an insurable interest in the creditor's life.

A clerk may assure the life of his employer (as some protection

against the deprivation of salary by the latter's death), but the term for which such a policy can be effected cannot be extended beyond the period of service contracted with his principal.

A surety for a principal (in connexion, for example, with an advance upon personal security) is entitled to assure the principal's life for the purpose of protecting the liability which he (the surety) has incurred.

An insurable interest pertains to a trustee in respect of the legal right vested in him in connexion with the trust which may require protection, but the proceeds of the policy will assume the nature of a trust on the principle that a trustee is not entitled to derive any personal benefit from the administration of the trust estate.

A sister, it has been held, may possess an insurable interest in the life of a brother who supported her ; and a father in the life of a son under age to whose earnings he would have a legal claim, though he would not possess such an interest if the son were of age, and, consequently, legally entitled to his own earnings.

Assuming that a legitimate insurable interest existed when the policy was completed, the holder of the Assurance may yet legally maintain it in force and secure its ultimate benefit, notwithstanding the circumstance that the debt or consideration which constituted the original insurable interest may have been discharged or extinguished. Hence the insurable interest and the beneficial ownership of a policy need not remain in the same person, but an assignee possessing no such interest in the life will be entitled to the proceeds of the Assurance, and will be protected by the insurable interest of the vendor—the original Assured.

By the Married Women's Property Acts of 1870 and 1882, a married woman may effect a policy upon her own life or upon that of her husband for her separate use, and the benefit, if the policy be expressed upon its face as so effected, will exclusively belong to her and will be as valid as though completed by an unmarried woman.

The nature of the insurable interest should be accurately described in the declaration. It is not the practice of companies, when they are fully satisfied with the bona fides of a proposed contract, to examine into details upon this point unless that course appears to be necessary for a complete apprehension of the nature and purport of the risk. A general and explicit disclosure of the interest is sufficient in the majority of instances.

The Form of the Policy

It is sufficient here to observe that, in accordance with the general rules of construction, any ambiguity of form or expression, seeing that the policy is prepared by the company itself, will be interpreted, where a double construction is possible, in the sense which is the more favourable to the Assured.

The forfeiture of a contract where death is occasioned by the hands of justice or by suicide results from considerations of public polity, and not from the arbitrary decision of the issuing company. Where, however, the policy has been assigned or mortgaged for valuable consideration, the interests of the assignee or mortgagee are fully preserved, notwithstanding the termination of the Assurance by any of these modes of death. Where a mortgage or conveyance has been executed, it will be noted that the ground for the retention of this condition is removed by the fact of the Assured now possessing no beneficial interest in the policy. The condition, however, avoiding the Assurance on the occurrence of the preceding modes of death obviously fails to operate where the policy is effected on the life of a nominee ; where, for example, A assures the life of B, since in that case the insurance can constitute no inducement to commission of the criminal act, and the tables of mortality employed in the calculation of the premium include this possible cause of death, which accordingly is implicitly assured against in the rate which is charged.

And seeing that duelling has vanished from fashion ; that death by the hands of justice rarely occurs in Assurance experience ; and that suicide largely indicates the existence of a disordered mind, all companies discharge these claims when these events occur, and even where the policies have not been legally dealt with, provided death happens after the lapse of one or, in some instances, two years from the completion of the contract.

When, too, an error has been committed in the statement of the age and documentary proof should hereafter prove that the age originally furnished was less than the actual age, so that an insufficient premium has been paid, the company will either deduct from the sum assured at death the difference of premium with accumulation at interest (allowing also, where necessary, for an adjustment of the bonuses allotted), or will modify the sum assured

by ascertaining the amount which the premium actually received would for the true age have originally provided.

The Assured should have his age admitted when the contract is completed, since this course avoids possible delay and trouble to the representatives who naturally may find a difficulty in procuring the necessary evidence. The exact accuracy of the age forms the essential basis of premiums, valuation reserves, and distribution of profits. It is worth while parenthetically mentioning that a certain shrewd actuary always required the age to be proved, for obvious reasons, when a surrender-value was applied for.

It may be added that all contracts—whether a clause to this effect be inserted or omitted in the policies—are indisputable in law except in case of fraud. With a view, however, to constituting complete explicitness in the contract (seeing that the observance of the conditions expressed in the policy would still remain) it is usually provided that if the age has been proved; if the assured is not engaged in the naval, military or seafaring professions; and if, during the lapse of a specified brief term of years, he has not incurred additional risk by proceeding to countries where an extra premium would be exigible, the Assurance shall thenceforth be disburdened of conditions beyond the payment of the stipulated premium and the production of adequate evidence of death. The fact of the Assured having failed to change the profession or occupation in which he was engaged at entry for a more hazardous one, and the circumstance that he has not meantime incurred additional risk, furnish a sufficient ground for supposing (regarding also the ages at which Assurances are usually effected) that, after the expiry of the prescribed period of probation, the Assured's habits and purposes of life in these respects may be reasonably assumed to be definitely formed, so that the contingency of extra risk may be treated as practically cancelled. It may be added that the sum assured by a policy consists of a "chose in action," that is to say, in legal language, a thing of which the owner has not the actual possession, but simply a right to recover it by a suit at law.

The Risk

The risk commences from the time when a complete contract has been executed by payment of the requisite premium, and the policy furnishes the written evidence of that contract.

With respect to the epoch of the termination of a policy effected for a specific period—an Assurance, for example, which is to endure for twelve calendar months from a stated day—the term excludes that day, but includes the corresponding day of the ensuing year.

The Premium

The fact that an agent of a company has given credit to the Assured for overdue premiums or has even actually received them subsequent to the date (including the days of grace) stipulated in the contract, does not necessarily bind the company to regard the policy as continuing; and to avoid the possibility of forgetfulness in the payment of the premium during the days of grace, where, for example, the serious illness of the Assured might readily entail this result, it is the practice of offices to provide that if death occur during the currency of this term the premium, if unpaid, shall be set off against the amount assured and the difference discharged.

Renewal

It should be carefully remembered by the policyholder that no excuse for non-payment of a premium within the prescribed time can be urged in the fact that a renewal notice reminding him of the amount due was not issued or received. For the convenience of the Assured these notices are forwarded (termed renewal notices), but no legal obligation is incumbent upon the company to adopt or continue this course.

Assignments

A consideration for the assignment must be expressed in the deed of conveyance, and considerations comprise valuable considerations, good, moral or meritorious considerations, voluntary considerations, and illegal considerations, the last of which will render a deed void although otherwise possessed of validity. Money or money's worth, future marriage or the agreement to perform or assent to any act which is beneficial to the assignor or detrimental to the assignee, will constitute a valuable consideration in favour of the latter. A good or meritorious consideration consists of the natural love and affection which a man is assumed to feel for those who are closely related to him by blood or marriage.

A voluntary consideration originates from affection or friendship only, where the act is one of simple bounty and is not performed in consequence of any legal obligation entailed upon the donor.

Illegal considerations exist where their execution is opposed to public policy or the statute law. A security granted for the satisfaction of a gambling debt, for example, is included within this category.

Notice of Assignment to the Company

In order that a contract of assignment may be fully protected definite notice of its execution must be served upon the company for registration in its books, and the policy should be delivered to the purchaser. It is evident that although an assignee may have his ownership of the policy constituted by an adequate deed, yet if he fail to obtain the custody of the policy and omit also to notify the company, the vendor might afterwards fraudulently dispose of the policy to the office by way of surrender, and thus defeat the conveyance, and no remedy obviously could exist against the company, whose integrity of action would be completely established by the clearness of the title and the exchange of the policy for the surrender-value. And in accordance with the Policies of Assurance Act of 1867 (section 6), a company, on the request in writing of any person by whom a notice has been served or signed, or of his executors or administrators, and on receipt of a fee not exceeding five shillings (which companies generally do not exact, though fully justified in this requirement if only on the ground of trouble), must deliver an acknowledgement in writing of the receipt of the notice signed by a principal officer.

Hence, in addition to immediate notice to the company (having first ascertained from the office that no record of any former charge exists), the purchaser or assignee, for the purpose of completing his security, should obtain possession of the policy.

Where a prior assignee (if more than one conveyance has been executed) notifies the company but fails to secure custody of the policy, he possesses precedence over a subsequent purchaser who gained delivery of the policy, but without notice of the previous charge.

Where the first assignee obtains possession of the policy, but omits to serve notice, and a subsequent incumbrancer does furnish

intimation of his lien, the latter will (except under special circumstances) be entitled to the benefit of the Assurance : preference is accorded to the person whose notice stands first even though his interest in the policy was created by a subsequent deed. The Statute was, however, passed for the protection of Assurance Companies and not for the purpose of regulating the priorities of assignees of policies *inter se*, and the assignee, consequently, of a policy who had given notice to the company but who had notice himself of a prior encumbrance would not obtain priority over the first encumbrancer notwithstanding the failure by the latter to furnish due notice to the Office. Each incumbrancer may, and should, inquire for the delivery of the policy, but should he fail in this course, and yet act in a bonâ fide manner, or if his application is evaded by the mortgagor, it has been submitted (by an able legal writer) that the fact of the policy being withheld does not constitute notice to him of a prior deposit.

The Act of 1867 only applies to assignments executed subsequent to its date. With reference to the titles to policies which were affected prior to the Act the student should consult the accredited text-books. Section 3 of that Act expressly provides that the date on which the notice is received by the company shall regulate the priority of all claims. It has been justly contended that the section on this point is very obscure. Where notice, it may be added, is served of a deed creating a general charge upon a policy, the company should demand its production when the claim occurs, since a notice of this description implies notice of its contents ; where, however, the notice is expressed in a limited form, such as the assignment of a policy to secure a specific sum, the company would appear not to be affected with additional notice nor would greater priority be given against a subsequent mortgagee or assignee. The company, as a matter of caution, should, however, require the inspection of every deed, and this course is rendered imperative on the grounds now to be described. Here the student is reminded of more recent legislation—the Customs and Inland Revenue Act of 1888—which casts the onus upon the company of observing the adequate stamping of all deeds of which notice has been received and which constitute the chain of title. Assurance offices are thus improperly made the servants of the Revenue authorities for the collection of taxes. By section 19 of this Act a payment made by

a company upon an assignment unstamped or insufficiently stamped renders the office liable for the stamp duty and a penalty, while in the opinion of counsel a contract to indemnify the company against this liability would be void. It will be remembered that a deed is valid, even if unstamped or inadequately stamped, so long as the contracting persons abide by its terms, but if its interpretation or enforcement requires to be submitted to a Court of Law, the deed can only be produced provided it be first impressed with the proper stamp, which alone brings it within the purview of the tribunal. But this fresh Act places the position in a much more serious and onerous form. Suppose the following two cases, as examples of the stringency and importance of this enactment: (1) *A* has assigned a policy to *B* by an unstamped document; it is considered by counsel that, in pursuance of this Act, the difficulty is incapable of removal by the execution of a fresh deed in *B*'s favour with the full stamp impressed, for it is held that under section 19, *B* would still claim under the first instrument as well as the second, thus necessitating the stamping of the former document; (2) it is also considered that a remedy is not supplied by a reassignment to *A* from *B*, and the execution of a new and stamped transfer from *A* to *B*. Every deed consequently in the course of title, although its efficiency may have disappeared by subsequent dealings, is required by this Act to bear its relevant stamp. It should be added that the Board of Inland Revenue stated, in November, 1891, that, in respect of the scope of Section 19, they had not been advised to regard that section as extending to the payment of stamp-duty upon instruments which, although required by Assurance Companies to be produced for their satisfaction and security, were instruments which in fact record a mortgage transaction closed prior to the transaction in which the claiming assignee was concerned, and as such did not appear to be included within the terms of the section. According to present practice, therefore, when a policy has merely been mortgaged by the Assured to a third party and subsequently, on the discharge of the debt, has been reconveyed to the assured, the Company is not bound to see that those deeds are properly stamped; they simply drop out of the title and do not constitute links in the chain. Attention might be called to a loose current usage of language where the word "assignment" is constantly employed to describe an absolute assignment (which finally

conveys the entire interest in the property) and also an assignment merely by way of mortgage. The student should carefully study the provisions of the Act itself.

A reference may be made to the position of an assignment which is followed by the insolvency or bankruptcy of the assignor. It was formerly necessary, with a view to preventing the policy in this event being liable to the claims of the assignor's creditors, as remaining, according to the legal expression, in " the possession order or disposition " of the bankrupt, and therefore vesting in his assignees in bankruptcy, that notice should be served upon the company and the policy delivered to the purchaser. But since the Bankruptcy Act of 1883, policies as choses in action are not deemed to be in his order and disposition as was formerly the case. Notice consequently is not requisite as against the creditors, but the necessity of notice for perfection of the transfer and the regulation of priorities between different incumbrancers still remains.

Where the Assurance is unencumbered and bankruptcy occurs, the policy becomes vested in the trustee, and it is incumbent upon the trustee to serve notice upon the office. If he fail to do so, and the bankrupt dispose of the policy to a third party, it has been decided that the purchaser, ignorant of the insolvency and notifying the company of his lien, should be preferred to the trustee. Justice would evidently be defeated if the position of an assignee who had ascertained upon application at the office that the title was clear, had received no intimation of the insolvency, and had registered his own notice, should be affected by a bankruptcy of which he was completely incognisant. Presumably, however, he should be especially careful to obtain the custody of the policy since its absence, without reasonably rebutting circumstances, might imply a prior charge.

The assignment of a policy will involve the assignment also of the bonuses whether already allotted or in future accruing to the sum assured, unless an exceptive intention be expressed in the deed of conveyance.

All assignments should at once be stamped.

A brief form of assignment (implying the sale of the policy) is appended to the Policies of Assurance Act (1867) as follows :— " I (*A. B.*, of, etc.), in consideration of, etc., do hereby assign unto *C. D.*, of, etc., his executors, administrators and assigns the (within)

policy of Assurance, granted, etc. (here the policy should be described). In witness whereof, etc."

(If the assignment be endorsed upon the policy, the term " within " [enclosed in brackets in the form] should be expressed ; if a separate deed be executed, the word should be omitted). For the avoidance of possible difficulties, an assignee or mortgagee is counselled to consult a solicitor in any dealings with policies.

The Death

The onus of proof of death rests upon those beneficially entitled to the policy, and adequate evidence must be tendered. In some instances difficulty occurs in consequence of the uncertainty of the fact or time of death, where the original Assured has assigned his beneficial interest to purchasers or mortgagees. An Assured, for example, may have proceeded abroad, and no intelligence may subsequently have been obtained of his existence or death ; in these instances, the rule of law prescribes that a presumption of death arises at the expiration of seven years from the date of final disappearance : not that the death occurred at the beginning or end of any determinate or particular period during that term, but that after the term has elapsed the person is not alive. But in all such cases sufficient inquiry to satisfy a jury must have been instituted. In one instance, in which the arrival of the Assured at a certain foreign city had been ascertained, the mere facts that he had not been heard of since that date for a period of seven years, and that a letter addressed to him under an assumed name had failed to reach him, were held to constitute an insufficient ground for a presumption of death within the seven years, and much less for such a presumption at any definite period. On the other hand, presumptive evidence may be adequate to resolve the perplexity. In one instance, the Assured sailed for the Cape in a small vessel of war, which must have encountered a known storm of so violent a character that it was with extreme difficulty that much larger ships escaped from destruction. During two years from the date of the tempest, no intelligence of the vessel was received, and a jury, at the end of that period, considered the evidence to be sufficient to warrant them in deciding that this vessel had foundered during the storm in question and that the Assured, who was on board, had also perished.

It has proved difficult from the cases decided in the Courts to deduce any definite rules respecting the adequacy of evidence as the basis of the presumption of death. At all events, for the purpose of creating such a probability, exhaustive search and inquiry must be instituted among those persons who, if the Assured were still alive, would be the most likely to receive news of him. A striking instance in his experience recurs to the recollection of the author. A debtor absconded from England ; and the assignee of the policy, after paying the premiums for twelve years subsequent to that event, applied for payment of the sum assured on the presumption of death, since enquiry had been made in every available quarter and no one who had known the Assured had ever received any tidings of his existence or death. The author suggested that an attractive advertisement should be inserted in the newspapers of a certain city where debtors not infrequently make their final and obscure home ; the advertisement, prepared by the assignee (without the author's knowledge of its form) was furnished to the effect that the Assured on application to C. D. (the name of the assignee of course not being divulged) would "hear of something to his advantage." The curious result was that the veritable Assured at once responded; and perhaps it is worth mentioning that the advantage he received in return was the statement conveyed to him by C. D. that "honesty was the best policy." Here then, after twelve years' disappearance and of absolute and continuous ignorance respecting the Assured during the whole of that time, and the fruitless result of minute and prolonged inquiries, discreetly and judiciously made among persons who could by any possibility possess any fragment of knowledge upon the subject, the presumptive evidence signally failed.

In discharging the claim the company is entitled to delivery of the policy; it does not at all follow, however, that inability to produce the document (which is merely the written evidence of the contract), interposes any serious difficulty in a settlement. Satisfactory enquiry having been instituted, a claim may be paid upon an adequate indemnity, or it may be shown, with reasonable conclusiveness (where bona fides is established), that the policy was unwittingly burnt or destroyed, or became missing consequent upon removal from one residence to another.

Difficulties occasionally occur in the determination of the proper persons to whom the proceeds of a life policy (admitted by the

Company to be due) should be paid. In these circumstances, the Company, by the Life Assurance Companies' (Payment into Court) Act, 1896, can effectively dispose of the complexities of title which, in the judgment of the Board of Directors, prevent an adequate discharge being otherwise obtained, by paying the sum assured into Court. It need only be added that Life Assurance premiums are exempt from income-tax to the extent of one-sixth of the taxable income of the Assured.

CHAPTER XII

FIRE INSURANCE

The practicability of reducing (as a basis of expected claims and the consequent construction of rates of premium) the vast accumulated statistics possessed by Fire Insurance Institutions to a scientific form, corresponding with that so admirably developed in Life Assurance estimates, affords a very interesting and difficult enquiry.

We are confronted at once by the radical distinction, forming a definite demarcation between the one department of experience and the other, of a partial loss, while in the business of Life Assurance a total loss must ensue in every instance if the contract be continued. This difficulty, however, might possibly be surmounted by the construction of two tables of Fire experience in place of the single table of mortality employed in Life calculations—tables involving claims which consist of total and partial losses respectively.

The element usually termed the Moral hazard is prominent in Fire Insurance considerations : the probability, that is to say, of intentional or incendiary fires being produced by the insured and the consequent chance of fraudulent claims. This is of course a question of moral character which admits of infinite degrees of gradation, dependent upon the original and acquired vigour of moral habit and the resulting urgency or impotence of motive prompted by the changes and pressure of self-development and experience. An Assured may possess a sufficient amount of moral strength to persevere in a course of honour during propitious circumstances, but the imminent occurrence of financial trouble or disaster, in its suggestion of retrieval of material fortunes at the cost of a wealthy corporation, may prove too powerful upon a struggling and inferior mind.

This feature is obviously paralleled in Life Assurance : it may be noted in the suppression of material facts from the proposal preliminary to the contract of Assurance ; in the formation of modes of life likely to entail a loss upon the company (though a distinction here exists between an unintentional and, in Fire Insurance, a voluntary infliction of damage)—passing in the extreme form into suicide, although here again the discrimination just mentioned remains. Still on the whole this phenomenon in Fire experience does not appear to present an insuperable barrier to the scientific method of construction.

A third apparent dissimilarity may be suggested : the neighbourhood, namely, of a risk, eligible in itself at a minimum rate of premium, to a building or manufactory where dangerous processes are conducted. Here again, however, an analogy is discoverable with Life Assurance experience in the possibility of a proposer becoming engaged in some hazardous occupation. The additional risk is compensated in Life Assurance by an increased rate, and a corresponding course would be pursued in Fire assessments. This difficulty consequently does not seem to afford a final bar; the comparison between the total number of ordinary risks of a specific class and the aggregate number of total losses (and in a separate table, the partial losses would be tabulated) would exhibit the basis of the minimum rates of premium, to which would be added the loading for expenses, profits, and general fluctuations (or special fluctuations incident to any particular description of risks in the schedule of classification) ; and the vicinity of any dangerous building or manufactory would be counterbalanced by the increase of the normal rate determined in the preceding mode.

To what extent companies or any combination of companies have actually proceeded in this precise direction is unknown to the external observer ; but obviously they possess a most valuable and instructive accumulation of materials available, under suitable classification, to this scientific end.

Whether however any system of tabular construction corresponding to the table of mortality adopted by life offices exist or not, the fire manager must obviously be guided in his settlement of rates by an examination of the particular class of risks in relation to the losses sustained, or in another form by a comparison between the claims experienced and the premiums received upon the included insurances. In all businesses a process of computation of the value of contingencies must exist, either based upon a formal inspection of actual facts, or guided, without any scientific tabulation of those facts, by the application of practical insight, acquired and cultivated by the vivid and accurate recollection of events in any section of experience, to similar facts in insurance presented from time to time for judgment. In every scheme of commercial enterprise it is competent to a sagacious and observant mind, after prolonged and extensive experience, to pronounce an adequate opinion, with the force and vividness of an intuition, upon the value of the risk

involved in any contingency which may arise in his specialized department of business. In many instances the possessor of this capacity is entirely unable to present any reasons for this determination of the form of judgment, and, carefully analysed, it will be found that the apparent intuition, or instantaneous soundness of conjecture, is simply the consolidated result of numberless actual experiences vigilantly noted and firmly deposited in memory of congruent kinds of problems. And the rapidity of judgment upon any question which may be submitted in the course of business is due to the clearness and precision with which parallel events (and their consequences) in the different classes of experience are retained in mind, and the mental swiftness of comparison (and thus of judgment) of the novel proposition with the section of remembered cases to which it most closely corresponds.

Although it is maintained that an approximately scientific form is capable of being attached to Fire Insurance statistics, it is only possible here, in the absence of definite information of the course actually pursued, to describe the nature of two attempts which have been made in this direction. The statistics utilised are now antiquated, but the feature of interest lies in the processes employed.

Over a recorded general experience during two periods of 10 and 7 years respectively, it was ascertained that, in the baking trade in London and its environs—and assuming that each property was insured for its full value, and that the average value was £100 per house—the statistics collected for these years showed, for every 100 fires which had occurred, the several losses of $3\frac{1}{3} \times$ £100, $25 \times$ £25, and $71\frac{2}{3} \times$ £10, in respect of buildings "totally destroyed," "considerably damaged," and "slightly injured," or an aggregate loss of £1,674·7: the proportion of fires to the extent of the trade was officially stated to be 75 in 10,000, so that every 10,000 houses would be liable annually to loss by fire of $\left(\frac{1{,}674{\cdot}7}{100} \times 75\right)$ or £1,256: the premium, consequently, apart from any addition for expenses, fluctuations of experience, and profits, would amount to $\frac{£\,1{,}256}{10{,}000}$ or 2*s*. 6*d*. per cent. If on the average one-half only of the property were insured, the premium thus obtained would merely be half sufficient to provide the claims and would require to be doubled.

The process here indicated is no doubt in principle correct, but the results are purely theoretical, as the data do not appear to be adequately extensive as a foundation, or duly classified.

A premium deduced on such a basis—an appropriate mode with sufficient materials for computation—would of course be affected by the locality of the property, the construction of the premises, the close vicinity of fire engines, the adequacy of the water supply, the situation of hazardous neighbouring risks, and the chance of loss from a fire occurring in the adjacent buildings ; but if, as should be the case, a general rate could be thus calculated as applicable under ordinary conditions and circumstances, the additional hazards just mentioned could be compensated, as in Life Assurance, by an appropriate addition to the normal terms.

The method above adopted, though accurate in principle, was an absolute one, and failed to recognize (1) the difficulty of ascertaining the number of persons engaged in the trade during the years of observation selected ; (2) the necessarily arbitrary assumption of the proportional value of a " considerable " and a " slight " damage in comparison with a total loss, and (3) the intricacy of classifying the vast varieties of fire risks as a basis of computation.

With the object of avoiding in a partial degree the difficulties enumerated, a further investigation was attempted. If the official returns furnish for each trade the total number of fires, with their causes, occurring respectively in the business portion and the private part of a tradesman's premises, a scheme could be prepared for deduction of the premium. Selecting the trade of bakers, it was ascertained from the returns for 24 years that 10 per cent. of the fires were due to overheated ovens, etc. ; 50 per cent. to various other causes, both occurring in the business ; while 40 per cent. were produced by causes connected with the building itself as a private dwelling ; that is to say, 60 per cent. were consequent on causes related to the baking trade, and 40 per cent. on those associated with the dwelling. If it be assumed that the adequate rate for a private house be 1*s.* 6*d.* per cent.—consisting of 9*d.* for the risk, and 9*d.* for expenses and profits—the baking risk (presuming the quantitative value of each cause to be identical—a wide assumption) could be then compared with the risk attached to a private house. The risk of the baker comprising the whole of the hazard of a private dwelling plus the specific danger of the trade,

the proportion of 40 : 100 : : 9*d.* : *x* furnishes the risk premium, for the total baking hazard, of 1*s.* 11*d.*; so that the entire premium would consist of—

	s.	*d.*
The value of the real risk	1	11
Expenses and profits	0	9
Total premium	2	8 per cent.

It may be noted that the margin here added is identical with that required for a private house, but this mode of loading obviously fails to take into account the commercial principle that as the nature of the risk increases the margin for fluctuations and profits should proportionately be augmented.

This form of procedure would merely furnish the rate for the average baker's risk, and would require to be modified in accordance with the special considerations and circumstances which have already been enumerated.

Although the attempt assumes the character of a speculative or theoretical process, it may possibly be ultimately ascertained to be feasible, with suitable modifications, if the accumulated data can be classified in such a form as to provide its appropriate basis. It is, however, pointed out, in this proposition, as fundamentally affecting the practicability of adopting the systematic and scientific deduction of rates that, in the classification of the varieties of fire risks, there necessarily exist as many different risks as the sum of all the combinations of all the several possibilities of construction, situation, occupation, and a number of other elements. Even on this point, however, the suggestion may again be ventured that if a process of classification could be devised, appropriate approximately to the computation of the premium for what might be loosely termed the normally constituted risks, any deviations and attendant additional hazards might be met, with probably reasonable accuracy of measurement, by the imposition of corresponding extra premiums upon the general rate.

It does not appear to be inconceivable that some similar method to that pursued with practical certainty on the whole in Life Assurance might be adopted in fire classification: the number of risks of each special class insured; the total sum insured; and the aggregate premiums received; and on the other side of the account the

number of risks on which a loss has occurred during the year over a sufficiently extensive term; and the amount of losses discharged,—introducing an adjustment of the general premium derived for any exceptional circumstances of hazard; and dividing these compared statistics into those involving total losses and those where the losses have proved to be of a partial nature. A combined and uniform classification of risks would evidently facilitate the analysis and comparison of the entire experience in each section at stated periods, and thus constitute the preliminary basis at all events of some practical scheme approximating to a scientific method of construction of rates.

We proceed now to the consideration of the

Nature of the Contract,

which expresses that the company undertakes to pay or make good to the insured any loss or damage by fire, which may occur during a particular period to specified property, which shall not exceed the amount named as the limit of the insurance or of each item comprised, in consideration of a fixed and immediate payment or premium. A fire policy may then be generally described as a contract of indemnity, or the restoration to the insured of the value of the loss he may sustain within the limit expressed in the policy, so that he may not realize any profit by the occurrence of the fire. The sum specified in the contract does not furnish the measure but simply the limit of the amount which the insured is entitled to recover.

The Proposal

In submitting an application to a company the completest good faith and the voluntary disclosure of all material facts must be observed, together with exact replies to the several enquiries in the proposal form. The property must be accurately described in order to place the company in a position to enter into the contract with a full knowledge of the risk involved. As in the case of Life Assurance, the maxim of *caveat emptor* is excluded from the bargain; and the facts disclosed must not simply be those which the applicant conceives to be material, but those which are in reality of that character; those which a jury would pronounce to be essential to a correct estimate of the risk and the foundation therefore of a fair and open contract.

The proposer, where the risk is not particularly hazardous, may obtain provisional protection from the Company, and pay a deposit for which he will receive an acknowledgment covering the property for a definite period until the proposal has been formally and finally accepted or declined. Within the specified period the company would institute its inquiries, and, if deemed necessary, inspect the premises; and should a fire occur prior to the expiry of that term the company, unless it had previously rejected the risk, would be responsible, assuming, of course, that no fraud had been committed or material circumstance concealed.

The deposit receipt is not in itself a policy, but simply a contract to issue one upon the usual conditions if the proposal be entertained. The policy is prepared and delivered when the company has been satisfied upon the eligibility of the applicant and the nature of the risk.

It may occasionally occur that when a proposal is submitted to insure a property which is situated at a distance, a fire may have actually destroyed the premises at the time, although both the company and the applicant were ignorant of the fact. If the proposer were cognizant of the destruction, a fraud would obviously exist and vitiate the intended contract; but in itself there is no reason against each side agreeing to contract on equal terms for an insurance thus antedated. Assuming perfect good faith on the part of the applicant, the question on such an occurrence would involve the nature of the proposed contract, and the date when it was intended that the protection should commence. In the event of the property being destroyed before the date at which the policy began, and no provision had been introduced for an antedated insurance, the bargain would be void.

So far as the proposal and its statements are mentioned and expressly or by implication embodied in the policy, they assume the form of warranties, and thus must be rigorously true; if, on the other hand, these statements are simply representations, their truth must be substantial; where there is attached to a representation in the proposal the qualifying words that the fact is merely to the best of the knowledge and belief of the applicant, the subsequent discovery of error will not vitiate the contract unless the assertion has been fraudulently made.

When the survey of a risk is deemed necessary by the company

prior to acceptance or rejection of the proposal, an official, the surveyor, makes an examination of the property, and it can well be imagined, especially in manufacturing risks, with their manifold sources of danger and the multiplicity of processes, each with its attendant hazard, that this post involves an exceptional scale of skilled and technical knowledge and capacity of judgment. It has already been incidentally mentioned that the assessment of the danger of a risk, and the amount of the deduced premium, depend upon many elements, such as the height and construction of the building, the vicinity of any hazardous risks, the efficiency of the water-supply, the nature of the processes carried on in the premises, the fire brigade appliances, and the apparent vigilance which is exercised to prevent or reduce a fire.

The Insurable Interest

The statute of George III. for the prevention of wagering contracts, already mentioned, and generally known as the Gambling Act, applies, since it has never been doubted that fire insurance is comprised within its scope. Hence the provisions of that Act control the nature of the insurance contract, and a proper insurable interest must exist as the foundation of the agreement. The right of insurance must be legal or equitable in nature : a legal right existing where the insured is the absolute owner of the property ; an equitable right accruing where he has advanced money upon its security, or has contracted to purchase, or has acquired some limited interest as contrasted with complete ownership. It may be incidentally remarked that an insurable interest does not attach to a simple expectancy, even though it be that of an heir apparent whose succession amounts to a moral certainty, where, for example, the ancestor in title is aged ninety-five, is lunatic, and intestate, and death is momently imminent.

A trustee may effect an insurance in connexion with the legal estate or right of possession of a property with which he is vested, although the name of the beneficiary or person for whom the property is held in trust be not inserted in the policy. But in such an instance where the insurance is effected by a trustee who is not beneficially entitled, notice of the circumstance should be expressed upon the policy, but only if so stipulated by the policy.

A mortgagee of premises possesses an insurable interest, and

an executor or administrator may also insure any property included in the testator's estate, but though entitled to insure (and as an act of expediency and full preservation of the property under his fiduciary administration he ought to insure), it would appear that no obligation is entailed upon him in this respect, or that any financial responsibility would be involved if he neglected to provide this protection.

The insurance, it has been judicially held, is to the person insured, so that he must possess the insurable right both at the date of the policy and the date of occurrence of fire. If a house be sold, the vendor ceases after the sale to preserve an insurable interest, and is not entitled to recover. And if the purchaser has no contract with the company, he is unable to claim under the seller's policy, unless, prior to the fire, the insurance was transferred to him, and the transfer admitted by the company. It would appear also to be the result of legal decisions that the same principle applies as between a mortgagor and mortgagee. The ground of this condition is reasonable. The moral hazard or the character of the insured is equally an element in the value and assessment of the risk as the construction of the buildings, and if assignments were permitted without the assent of the company, this important, and indeed essential, feature of protection would be abolished.

The Policy.

A specific insurance designates one where a specific sum is insured upon a specified property, and where the expressed amount is payable if the damage reach that limit irrespective of the value of the property. If, for example, a building be worth £10,000, and be insured for £1,000 only, and a loss occur to the extent of £1,000, that sum must be paid notwithstanding the excessive proportion of the uninsured value; in other words, a specific policy is one which does not contain any conditions of average. Hence an apportionment of damage does not arise, and the process applicable is one merely of the adjustment of the damage sustained within the contracted amount. The objection to this form of contract lies in its encouragement to effect insurances to an extent considerably inferior to the value of the premises without contributing any corresponding advantage to the

company. This fact may accordingly influence the rate of premium. The insured also incurs the possible contingency of a serious and unprotected loss. If more than one specific policy exist, the loss is distributed *pro ratâ* to the aggregate amount insured.

The complicated cases where specific policies coexist with insurances containing the average clause, and with floating policies, need not be discussed, as presenting too technical a character, even if the author possessed the requisite knowledge and experience.

The average clause embodied in a policy implies that the company is only liable to pay in the event of damage by fire such a fraction of the loss (not exceeding the value of the property destroyed) as represents the ratio borne by the amount of the insurance to the value of the property insured. Thus, assuming the value of a certain stock of merchandise to be £60,000, which is insured for £30,000, or half its value, and damage by fire is occasioned to the extent of £30,000, the company is not liable for £30,000, but only for £15,000, or one-half of the loss, seeing that the insurance was effected for that proportion of the value of the goods; the value of £60,000 is to the amount of the policy of £30,000 as the loss of £30,000 is to x (the company's liability), whence $x =$ £15,000.

It is evident that the omission of the average clause from a contract is equivalent to a substantial, though undefined, reduction in the rate of premium. Hence insurances containing this clause can be issued at lower (apparent) rates than those from which it is excluded. The practical effect of the condition of average is the reasonable one that the insured is placed in the position of an insurer himself for the excess value of the premises unprotected by the policy, that is, the difference between the sum insured and the total value of the property, so that in the event of loss he himself contributes towards its amount by a proportionate abatement of his claim upon the company.

A floating policy, or a "floater," covers one or several kinds of goods deposited or stored in different localities under one sum for one premium and in relation to the same owner. A merchant, for example, may insure £200,000 on merchandise in a number of different docks and warehouses, where the quantities of the goods deposited in each will fluctuate from day to day, some being

increased, and others diminished, dependent upon sales or consumption and consequent removal, and replacement or augmentation by additional imports—the insurance "floating" with these changing circumstances of the risk. Policies of this description accordingly differ from specific insurances, and all floating policies by their nature constitute average insurances.

The term Valued Policies sometimes occurs in the literature of the subject, and may be explained. Though common in America, valued policies are of extremely rare occurrence in English practice. The general rule limiting the claim to the amount of the loss may be qualified by the two parties to the contract agreeing upon a merely arbitrary estimate of the value of the property insured, and in the absence of fraud this estimate will constitute the measure of the liability of the company. In the valued policy an attempt is made to cast the onus of proof of the amount of loss upon the company. The advocates of this system argue that offices possess no right to receive the premium, for example, for an insurance of £1,000 and afterwards to contend that the property is only worth £500, although that sum may express its market value at the occurrence of the fire. It is obvious that if this plan were adopted the company, in consequence of the augmentation of its risk, would require to increase the rates, and the insured would probably and justly suffer severely compared with the position which he possesses under the existing equitable mode of business. Where pictures and other valuable articles are insured according to a valued schedule, it might be contended, in the absence of any qualification in the policy, that the contract assumed the nature of a valued insurance, and that no proof of value could be demanded in the event of loss; but it will almost invariably be found that the conditions of the policy cast the burden of producing such evidence upon the insured. In the case of a valued policy it is evident that the contract ceases to present the character of an indemnity, and hence is detrimental to the interests both of the company, the insured, and of public policy. An American writer has, in scathing terms, exposed the tendency to widespread fraud which the adoption of the valued policy system necessarily promotes. The abrogation which this class of policy effects of the rational principle of the actual loss sustained, and the substitution of the doctrine that the existing

amount of insurance should constitute the measure of the value of the premises and the liability of the company, regardless of their actual value, has been found to conduce distinctly to constant fraud. It has been ascertained from experience in America that this scheme of insurance, wherever adopted, has increased the cost of fire protection to the public, so that the innocent insured are heavily burdened in the compensatory rates required for the intentional destruction of property which the plan of valued policies encourages. The objections to the system may be summarized: the invidious position in which the company is placed; the motive to perpetration of illegal and immoral acts; the consequent additional tax imposed upon the right-minded policyholders; and the resulting tendency to litigation.

The Period of Protection by the Policy

The protection afforded by a fire insurance endures during the term which the contract specifies, and neither the office nor the insured without an express condition to that effect can, assuming that the provisions of the policy have been faithfully observed, and that no fraud is involved, terminate the agreement by offering, or requiring, a return of the premium. Where the time is not specified at which the protection of the policy ceases, it will be fixed at the midnight of the day of the expiration of the period of insurance. If a fire occur within the usual days of grace allowed for payment, the insured should at once tender the amount, since the company is not bound to set-off the premium (as in Life Assurance practice) against any claim. And if the premium should fail to be paid within the days of grace and a fire occur, the subsequent acceptance of the amount by the office in ignorance of the destruction will not revive the insurance.

The Term of Insurance

In the United Kingdom fire insurances are usually effected by yearly contracts—contracts entered into for a year with the right of renewal or discontinuance by either side at the termination of each period. But contracts can be arranged for a briefer term than twelve months' duration, and are then designated "short period" policies; or they may be effected (and are then named "long period" insurances) for any number of years exceeding one year without continuing in perpetuity. In some cases a 7 years' policy

is granted in consideration of the annual premiums for 6 years being paid down, and usually a deduction of 5 per cent. is allowed from all the premiums beyond that received in the first year in respect of the number of years for which they are discharged in advance.

Renewal of Insurance

When fire contracts are not constituted for terms of years (and the remark applies equally upon the expiration of any periods arranged), the company is not compelled to renew nor is the insured. But the intention of the Company not to renew must in general be expressed prior to the expiration of the agreed period of protection.

The Alteration of Premises

A difficult element arises in connexion with alterations effected in the structure, or use of the buildings, subsequent to the date of the policy, either with respect to the trades carried on or the nature of the goods deposited. The decisions in such cases have usually depended upon the construction of the policy conditions. Notice should be furnished to the company of any proposed alteration or the introduction of any stove or furnace or other appliances, however necessary to the convenience of the building, and indeed conducive to its improvement in character, so that a fresh survey, if deemed desirable, may be made. Until this reasonable course has been pursued the protection by the insurance is properly cancelled. The insured again may erect a building upon ground stated to be vacant when the policy was completed, which may prove dangerous in respect of the possibility of fire to the premises insured, and repairs also may be undertaken which, although substantially improving the risk, involve, during the operations, additional hazard from the introduction, for example, of a carpenter's bench and the necessary heating apparatus for the execution of the work. In all such incidents, the conditions of the policy should be studied and prompt and adequate notice afforded to the company of any contemplated changes, since equity requires that both contracting parties should be placed on an equality of knowledge and risk. The author recalls an instance where a church organ, fully insured, was transferred for repairs to a manufactory without intimation, through innocent omission, of the fact to the company. A fire occurred in the manufacturing premises, which entirely destroyed the organ,

and the company's liability was thereby cancelled. And justly so, since the office had failed to be accorded an equality of risk as determined and defined by the altered conditions. Had they been informed, they might possibly have declined to continue the insurance altogether on account of the vastly augmented hazard, or would certainly have required an additional premium which the insured might have refused to pay.

The Forfeiture of a Policy

Forfeiture of the contract is incident to the non-payment of the premium; a misdescription of the risk; the alteration of premises without prior notice and assent; the introduction of additional or prohibited hazards, such as pipe-stoves and explosive or inflammable substances; the removal of insured property without consent; alienation or change of ownership without the company's approval of the substituted possessor; intentional burning; the submission of claims which in fact amount to fraud; and the failure to furnish reasonable and required evidence of loss sustained. These causes of forfeiture are equitably based upon the fundamental principle that the grantor of the insurance should be kept in possession of complete continuous intelligence of any alteration in the nature and extent of the risk which was originally accepted.

Assignments

The contract of fire insurance purports to make good to the insured, his executors or administrators, the loss or damage he may sustain in connexion with the specified property, usually without the addition of the term "assigns," and always with a condition inserted providing for the transfer of the policy to a third party in one particular mode only, namely, by endorsement with the consent of the company issuing the policy. The reputation of the insured, as we have observed, is as integral an element in the assessment of the risk accepted as the construction of the building, and indeed, without suggesting an intention to commit a fraud, a company may justly refuse an insurance, however admirable may be the intrinsic merits of the risk itself, on the mere ground, for example, of the insured having suffered many previous losses by fire, indicative of negligence, or of having submitted excessive claims. The contract is a purely personal one, of necessity, and not transferable without the assent of the grantor; and the company should

be provided in fairness with the means of ascertaining that any conveyance of interest passes to equally responsible persons, where the moral hazard is not augmented, or this essential condition of the contract may be defeated. Hence even where an insured property has been sold in a bonâ fide manner, the interest in the policy effected by the vendor does not necessarily accompany the conveyance unless the consent of the company to its transfer has been previously obtained.

The Loss

It is to be observed that the loss entitling to payment must be occasioned by fire as the destroying agency : either by the actual ignition of the property itself, or some substance situated near to it and not intended to create heat. And, generally speaking, there must be an accident by fire, and this must be taken in the ordinary acceptation of the term, to form the foundation of a claim : something must have caught fire and damage have been thereby occasioned to the property insured.

Any losses consequent upon a fire beyond the amount of the damage actually produced upon the property, a loss of profits for example, or the injury occasioned by the derangement of business administration, would not be comprised within the definition of loss or damage by fire.

If an explosion occur, where a fire has happened upon the insured premises, and gunpowder or other explosive substance has ignited and augmented the damage, it has been considered that in the absence of any special condition in the policy affecting such a contingency, a claim would be valid.

The employment of gas for lighting and warming is permitted where the gas is not manufactured within the building mentioned in the policy or in any building communicating with it. And electric lighting is allowed as an ordinary risk where the mode and structure of the installation have been surveyed and approved.

It need scarcely be added that injury by fire resulting from lightning is included within the liability of the company ; and it may be further added that companies (for obvious reasons—the absence of adequate evidence of the value of loss) will not afford protection against the destruction by fire of books of accounts (regarded in

this aspect of value as documents), banknotes, bills of exchange, bonds or written securities.

With respect to fires produced by foreign invasion, riot or civil commotion, no claim exists.

With the object of impeding the progress of a fire, neighbouring buildings may require to be demolished, and this is generally effected by the employment of gunpowder. It has been legally held that such a proceeding is not inexcusable in cases of necessity, and appears to be within the range of the policy, though an eminent writer has properly suggested that equity would be more precisely established by the apportionment of the resulting loss among all the persons whose properties had been preserved or benefited by the act. The expenses of extinguishing fires are usually very onerous, and the customary practice appears to be that companies will discharge a rateable proportion of the cost in relation to the amount of their interest as insurers in the premises or goods endangered unless the fire occur within the rating area controlled by a local authority ; within such an area, only the expenses " out of pocket " of a brigade are sometimes paid : the ratepayers, obviously, being entitled to protection by the Authority in respect of fire as well as protection to life.

Immediately a fire happens, notice should be served upon the company, when the extent of damage will be inspected officially ; and on the occurrence of a fire the duty is obviously incumbent upon the insured to adopt every available and effective means for its extinction or the reduction of its area of destruction. All claims for loss require, on demand, to be substantiated by such vouchers, proofs and explanations as may be reasonably requested, and may even need to be accompanied by statutory declarations of the truthfulness of the statements submitted.

A total loss in fire insurance practice does not signify the entire destruction of the property, but its destruction or range of injury to so great an extent as to render the company responsible for payment of the full sum insured ; a partial loss occurs when, as usually happens, the insured property is not entirely destroyed.

Where goods are only partially damaged or are saved, the insured receives the salvage and allows for its assessed value in the settlement of the loss, or otherwise the Company receives it and realises its value by sale.

The claim under a fire insurance, as has been stated, is of the nature of an indemnity, and consequently the amount to be recovered must be estimated upon the basis of the condition or state in which the property existed at the time of the fire or immediately preceding its occurrence, and not the expense of erecting, in the case of a building, a fresh structure. Where, for example, old machinery or goods deteriorated by wear and tear are consumed, the company is simply liable for such an amount as shall be found equivalent, on the day of the calamity and by the market test, to their actual value, without any regard to the sum which the properties cost when they were purchased as new. Thus, a picture which originally cost £1,000 when the painter's productions were in demand, might, when destroyed by fire, possess a market value of £200 only, in consequence of a decline in the artist's reputation and the diminished inquiry for his works; and the sum of £200 would alone be claimable. This condition obviously is a necessary rule, since the payment of the original price would operate as a premium upon fires and tend consequently to public detriment. If the insured merchandise (which is destroyed) should be stored in a dock or wharf, its market price (within the limit of the sum insured) would furnish the test of value, since it would be necessary to effect restoration by purchase in the market; but where the insured articles (for example, soap) are manufactured upon the premises, the cost of producing them affords the standard, since the contract of insurance is one of indemnity and the payment of the market-value to the manufacturer would involve the superaddition of an anticipated profit.

On the destruction again of an important building by fire, the whole of the original amount insured, or even an increased sum, might be required to re-erect premises of equal dimensions and description; if, however, the building had become old and dilapidated, the measure of damage, for the reasons already adduced, would be furnished by the actual value at the time of loss.

Reverting to the illustration of pictures just mentioned, it actually occurred, in one instance, that a wealthy collector (capacity of artistic insight and the accumulative mania not infrequently existing in inverse ratio) had paid several thousand pounds for an ancient picture, although warned by an expert that the painting was merely a counterfeit. The collector adhered to his personal view (stubbornness of conviction and accumulation being often in

direct ratio) and completed a policy corresponding to the price; it was afterwards discovered beyond doubt that the expert's judgment was accurate, and the picture a mere copy, worth only a nominal sum; an arrangement was therefore effected with the dealer from whom the production had been bought, which resulted in the refund of the purchase money, and the lapse of the insurance. Had the picture been destroyed by fire prior to this discovery, the company would have been entitled to inquire fully into its reputed value, and hence, although a premium had been paid for many years upon the several thousands of pounds represented by the policy, the claim would only have been discharged to the extent of a few pounds.

New furniture is purchased for £500: the premiums are paid on that amount for 20 years: the articles then are consumed by fire, and since, in consequence of deterioration by use they would now command a value of £100, the insured would receive merely the latter sum. This result is equitable, as we have observed, on the grounds of public polity already specified; and since an insurance is simply a contract of indemnity, the amount discharged is necessarily the value at the date of payment of the real loss without any accretion of profit to the insured.

An important consideration here occurs in connexion with goods. If the measure of the indemnity by the company were not precisely expressed as the value of the property at the date of the fire or immediately preceding its occurrence, it might well happen that in an extensive conflagration the market value of the merchandise consumed would be considerably enhanced by reason of the abstraction of so appreciable a proportion of the supply of such goods with the consequently inflated augmentation of the company's liability: the stock being thus largely diminished by the fire, the market price would at once advance; hence the necessity of the qualifying phrase, which prescribes the date at which the market value of the consumed property is taken.

The Company always possesses the right of reinstatement or repairment of the premises in place of discharging the loss in cash. This course frequently avoids many perplexing questions, such as those where the measure of damages cannot be agreed upon, or where doubt exists as to the person entitled to receive the amount of claim. The option of requiring reinstatement does not reside

with the insured, and justly so, for the right is necessary to be reserved to the company, where difficulties occur in the completion of its contract as above described.

Where the company and the insured disagree upon the extent of the loss, and no fraud is imputed, the question is usually referred to arbitration, for the purpose of determining the measure of damage or the sum at which the loss should be adjusted anterior to any action. A single arbitrator would be agreed upon by both sides, or if difficulty of selection exists, an arbitrator would be appointed by each, and in the event of difference of judgment between the arbitrators an umpire (appointed by the latter prior to entry upon the reference, who would sit during the proceedings with the arbitrators) would decide.

The adjustment of losses means the determination of the amount payable by the company as the resulting liability by fire in connexion with the conditions of the contract and the interest of the holder of the policy. The adjustment of losses other than the most insignificant damage is effected by a trained and skilled assessor of claims—an office which demands no less knowledge, broadmindedness and a concern for the interests of the company which he represents than tact, courtesy, and judgment, and a due regard also to the real interests of the insured.

The apportionment of losses (where different concurrent policies on the same property have been effected with various offices, or where the property belongs to more than one owner insured in the same or in different companies) consists of the process of determining the rateable proportion in which each policy must, under its individual terms and conditions, contribute towards the discharge of the ascertained general loss—the loss being divided in the ratio of the several sums insured.

The insured can only recover under the whole of the policies the amount of the actual loss, and he can claim from no single company (under one or several policies) a sum in excess of its rateable proportion of the entire loss.

If more than one fire should occur in the same property during the course of any single year, the office is not liable to pay in the aggregate a sum in excess of the original amount insured. A policy, for example, is issued for £1,000, and early in its currency injury is produced by fire to the extent of £500; the sum is

discharged. If a second fire arise during the remainder of the year's currency, and damage be inflicted to the amount of £1,000, the company would only be responsible for £500. Thus the total damage in the year is £1,500 in connexion with an actual insurance of £1,000 only; and the amount contributed at each successive fire is the balance of the original sum insured which remains unpaid upon the policy.

In double insurance, each grantor of a policy sustains a *pro ratâ* amount of the admitted loss in relation to the sum covered by its policy. If one company has insured £1,000, and another £2,000 on the same property, and the damage extends to £1,500, the first will contribute £500 (that is 1,000 : 3,000 : : x [the share of loss] : 1,500), and the second £1,000.

Rent

The rule that the claim must be restricted to the value of the destroyed property at the time of loss, and excludes consequential damages, involves the condition that the insurance cannot be extended to cover rent or interest on the value until reinstatement has been effected. Rent accordingly must be specifically insured. The rate charged is the same as that required for the insurance of the building.

The Tenant

A caution may be given to the tenant of premises that a fire does not affect the position of the tenancy, and consequently, until its legal determination, he is liable to defray the rent, notwithstanding the fact that the premises may thus be lying waste, even although the landlord may have insured and received the insurance money, when the lease contains an express covenant for the discharge of rent during the currency of the tenancy. This liability may of course be negatived by a specific contract, where, for example, a provision is inserted in the lease that the rent shall be payable, "damage by fire excepted." In such an event the lessee has been held entitled to a proportional abatement of rent when a fire has destroyed a part of the buildings.

The Tariff

For the protection of common interests and uniformity of practice, so far as this course is consistent with individual enterprise and administration, the principal Fire Companies have united

into a body termed the Fire Offices' Committee, which fixes minimum rates for certain descriptions of risks of the more dangerous kind, with a concurrent agreement that no company shall entertain these risks upon reduced terms.

This Association has rendered great service (involving distinct benefit to the community of the insured) by an improved classification of risks, and a consequent discrimination in the appropriate rates of premium.

The regulations affecting the conduct of fire business generally are also naturally under its purview and settlement. We have just referred to the resulting advantage to the insuring public from the institution of such a body. It may, however, be urged that its primary object is the preservation of adequate rates; but this course, proper in itself, involves a simultaneous benefit to the public by assessing for each description of risk its proportionate premium, so that those who submit for insurance the less hazardous properties should not be burdened with an average premium (deduced from the experience of all kinds of risks) in order to compensate the insufficiency of those premiums which fail to represent the measure of an increased risk. Additional advantages conferred upon the public consist of the evident benefit of a completer uniformity of practice among the companies in their relations with the insured, and the distinctly appreciable service of the assessment of rates upon the basis of the united experience of the whole of the principal Fire Offices. These Offices tabulate, in respect of each description of risk (for example, the premises of a jeweller, and a warehouse of a particular kind), the amount of premiums received over an adequate series of years (in order to exclude accidental annual fluctuations) and the losses sustained: the data thus furnished by each company are massed together under their several categories, and, on the foundation of such extensive numbers and prolonged experience, the appropriate premiums can be deduced. And no rate is increased for any particular species of risk except on the indications of these average results. Hence by this scientific and practical method, the several rates of premium charged are in no degree fanciful or arbitrary, but strictly provide the actual measure of value of the different risks. Nor, in the interests of public convenience and welfare, should a reference be omitted to the service competent of being rendered, in however

informal a manner, by the existence of such a body as a kind of supreme tribunal of supervision and implicit power of moral control.

The London Salvage Corps

is an institution founded by the Associated Fire Companies for the purpose of protecting goods, merchandise, and effects from destruction by fire either by removing them, or by accepting their charge after removal. This body is distinct from the Fire Brigade, though as far as practicable unity of action is adopted.

The Non-Tariff Companies, disregarding classification of risks to a large extent, and without the advantage of combined experience, agree to accept all risks assessed upon their several merits. As the name implies, they do not belong to the Association, and prefer to exercise individual judgment in place of common action, based upon associated experience.

Preservation of Property from Fire

The conservation of property from fire obviously forms a national concern, especially when fires by their extent may amount to conflagrations. This view of the subject is emphasized on national financial grounds for the maintenance of prominent sources from which taxation may be secured towards the general administrative purposes of the country and municipalities, and with the object also of the arrest of fraud. The attention of the legislature has accordingly been frequently directed to this protective end, and many statutes have been passed.

On the first establishment of Fire Companies, and for a lengthened subsequent period, the offices undertook the duty of the extinction of fires ; and in the earlier history of the business the various companies maintained their own men and engines ; these servants, divided into firemen with instruments to extinguish fires, and porters to remove goods, were severally distinguished by a distinctive badge upon their coats, emblematic of the offices they represented, while corresponding metal plates were affixed upon the outside of the buildings insured by the individual offices (some of which are still observable) in order that the servants of each particular company might be able, by this indication, to direct their efforts specially to the benefit of the buildings protected by their respective offices.

This individual and collective function has now been abandoned, and the just conception has been adopted that the extinction of fires with the attendant costs should constitute a national responsibility upon the public themselves acting through their municipal authorities.

Fire Inquests

A reference may be added to the grave necessity of instituting a general systematic official enquiry into the origin of fires in special cases,—an enquiry usually designated as a fire inquest. The number of fires which annually occur, for which no discoverable cause can be assigned by ordinary methods of investigation, demand this legislative action both in the interests of the companies themselves and no less in those of the public, who would appreciably, in all probability, derive advantage from an accordant revision of rates, consequent upon the comparative absence of wilful fires, which on the adoption of this scheme would undoubtedly ensue. This suggestion, however, is simply and exclusively of value provided an exhaustive investigation into the whole of the facts can be made. Were this feasible, the plan would possess the benefits we have named; but as a practical question and under existing conditions such inquests, we fear, will generally fail to effect the purposes which they ought to subserve. Without in any way impeaching the impartiality of juries, the verdict will usually be "accidental," through the absence of a complete presentation of facts and a consequently exact enquiry, so that a really criminal insured may be acquitted and the company be accordingly compelled to pay. It should be remembered also that the crime of arson frequently destroys not merely the evidence which would aid in its establishment but the circumstances themselves also which would indicate the commission of a crime. Hence until a thoroughly efficient and organised system of investigation can be devised (and no other will suffice in the public interests) the position of the companies and that of the public are better secured by leaving the Offices to deal with such instances directly.

Municipal Insurance

The subject of municipal insurance is frequently discussed, especially by those who regard the question with superficiality of knowledge and judgment. This project would assume the form

of general insurance by the ratepayers themselves by means of taxation. In a recent instance it is pointed out as an argument in favour of this mode of municipal enterprise that during a particular year the property of 377 municipal authorities was insured for a total sum exceeding £23,000,000, subject to annual premiums of upwards of £27,000. The amount of compensation received from the companies for damages from fires during twenty years was slightly over £100,000, while the sum expended in premiums during that period was upwards of £445,000, showing an excess of premiums discharged beyond the amount obtained for losses by fire of upwards of £344,000. The grave error involved in these contentions lies in the failure to recognise the import of the wide extent of operations by which Fire Companies are able to secure average results in consequence of the distribution of their business over a vast area—the whole of the United Kingdom, and, indeed, the world—so that excesses of claims occurring in one portion of their field of work are balanced by reduced losses in another; and the more extended the area of operations the greater the probability of equilibrium being produced. But a municipality must necessarily confine its operations to the limited area of certain towns or cities without the counterpoising benefit of business in remoter and more extensive spheres. Hence if a conflagration occurred the loss would prove enormous, and would impose a severe and uncompensated strain upon the resources of the municipal bodies; while a company could afford to sustain the risk of serious fires or a series of fires in the same locality by reason of the counterbalancing influence upon local results of a widespread distribution of risks elsewhere. Fluctuations in the narrower case would be unrelieved by width of risk, which disappear in the latter instance in the combined results over a spacious range. A very vital consideration also resides in the fact that, in consequence of the method of reinsurance practised between companies, the aggregate risk in municipalities is divided among a large number of offices so that each individual company's loss is comparatively small; while, on the plan of municipal insurance, the entire burden of destruction would, without the compensation of reinsurance, devolve exclusively upon the resources of one corporate body.

It is of interest to mention that in 1681 the authorities of the City of London published a notice that a committee had been

appointed to perfect a scheme " for the better securing and insuring the inhabitants of this city and liberties from dangerous accidents happening from fire." Notwithstanding the fact that a sum of £100,000 in lands and ground-rents was arranged to be settled as a protecting fund, the project would not " take perhaps because the credit of the city at this time was but low."

A curious relic of history has been preserved in one of the offices. It will be remembered that when the change of style for the rectification of the calendar was effected a popular clamour arose—" Give us back our eleven days." In 1752 it was decided that the renewal receipts should be made eleven days " forwarder " in the succeeding year in place of the eleven days omitted or " annihilated " in 1752, so that the insured might possess a complete year.

In the practical conduct of fire insurance no information requires to be furnished beyond an explanation of the mode of forming the reserve which appertains to the unexpired risk at the close of each year. We will assume for convenience that the revenue consists of annual insurances, and that the premiums are distributed over the year in approximately uniform proportions. On this assumption, the total income may be supposed to become due on the 24th of June, and since the premiums then paid cover one year's insurance, there remains on the 31st of December half a year's risk unexpired, for which a proportionate part of the premiums received, after deduction of expenses, should be retained in hand. Assuming further that the commission and expenses amount to $33\frac{1}{3}$ per cent. of the premium revenue, the balance of premiums remaining is $66\frac{2}{3}$ per cent., so that one half of this balance or $33\frac{1}{3}$ per cent. of the entire premium income would constitute the required reserve, which would accurately express the continued risk (if the assumed ratio of expenses be correct) on condition that the business comprised only annual policies renewable in equal proportions at the term dates, and if these terms were divided evenly. Presumably these considerations formed the basis upon which the provision for the unexpired risk was usually erected. But examine the figures on the basis of the actual term-days at which premiums are payable in England, namely, the 25th of March, the 24th of June, the 29th of September, and the 24th of December :—

On Policies renewable at	The number of days unexpired at the 31st of December are	Being fractions of a year of
Lady Day	83	·2274
Midsummer	174	·4767
Michaelmas	271	·7425
Christmas	358	·9808
	886	2·4274

The average unexpired risk accordingly embraces 221½ days, or ·6068 of a year.

On the assumption then of an even distribution of the business over the several actual terms, the rate of reserve on annual policies would be 60·68 per cent. if calculated on the premiums less the assumed 33⅓ per cent. for expenses, or 40·45 per cent. (that is, ⅔rds of 60·68) if calculated upon the full annual premium revenue.

But in practice the business is not equally apportioned between the different terms, and the Christmas term is considerably heavier than any of the other three. In one company the proportion of business, calculated upon a period of years, amounted to 23 per cent. at Lady Day, 22 per cent. at Midsummer; 23 per cent. at Michaelmas, and 32 per cent. at Christmas; and adopting these proportions, the reserve for unexpired risks on the 31st of December would be 42·79 per cent. if assessed upon the full actual premiums.

A different proportionate reserve would be produced if long term and short term policies were included.

The preceding statement is restricted to risks upon property in the United Kingdom.

The following interesting statement was recently made in public by the Chairman of one of our most stable and prosperous Fire Companies. During the period from 1893 to 1903, the total premiums received by the Fire Insurance Companies of the United Kingdom amounted to £197,350,000. In 1893, the Companies, in the aggregate, sustained a trading loss of £1 17s. 2d. per cent. upon

the premiums as the result of the year's working; and in the succeeding years their percentage of profit was £8 18s. 4d. in 1894; £9 16s. 6d. in 1895; £9 7s. 5d. in 1896; £8 14s. 7d. in 1897; £4 18s. 1d. in 1898; £3 0s. 5d. in 1899; £4 7s. 5d. in 1900; £1 13s. 5d. in 1901; and £13 7s. 8d. in 1902. The average annual percentage of profits during the 10 years was consequently £6 5s. 0d.—a consideration for the enormous risks incurred and the commercial protection afforded which cannot certainly be deemed excessive.

A significant comment upon the preceding statement has been afforded by the recent conflagration in San Francisco, where the aggregate amount at risk, under the policies of British Companies, exceeded £14,000,000. This disaster has emphatically enforced two practical criteria of sagacious administration: (i.) the necessity of constructing reserves, during periods of prosperity, competent to sustain, without disturbance of the Company's financial history, exceptional stresses of an inevitably periodic character, and (ii.) the vital importance of a discriminative and constantly-revised distribution of the amounts at risk in the several cities and localities comprised within a Company's area of work. Soundness of perception, and rigour of retention of appropriate averages apply to all businesses, which competition, unhappily, not infrequently obscures. Many years ago one of our principal Insurance Institutions adopted the wise and provident plan of forming a special Conflagration Fund, in addition to the ordinary Reserve, in anticipation of catastrophes similar to the San Francisco fire. Simply by the observance of these conditions, (cæteris paribus), can that stability be secured in Fire Insurance administration which ensures a steady uniformity of value for the shares and averts the spasmodic panic which an exceptional experience usually entails.

CHAPTER XIII

MARINE INSURANCE

THE difficulties which we have described in Chapter XII apply also to the preparation of statistics—showing the number of risks incurred and the proportions which result in claims—in a formal manner correspondent with that exhibited in tables of mortality. A valuable mass of experience—no doubt analysed—is possessed at Lloyd's and in the records of Joint Stock Marine Companies, and constitute the necessary basis of the actual computation of rates ; but apparently in no department of Insurance business do successful results depend so largely as in marine transactions upon a specialized native sagacity, cultivated by vigilant observation and practice, and adapted naturally to the system of estimate which this description of Insurance involves.

The history of Marine Insurance is rendered famous not simply from the wide extent to which honourable mutual trust is displayed in all transactions, but also from the interesting and unique story of the gradual concentration of the business in the renowned centre known as Lloyd's.

We learn that a seaman's coffee-house owned by Edward Lloyd was already in existence in 1668. It was situated in Tower Street, the resort of seafaring men, as this street constituted the principal thoroughfare between Wapping and the centre of the shipping activity on Thames side and the City of London ; hence the coffee-house became identified with shipping affairs. In 1692 Lloyd migrated to the corner of Lombard Street and Abchurch Lane ; and thus by closer situation to the centre of commerce in the City, he lost the patronage of seafaring persons, but gained the substantial advantage of securing the custom of merchants. Although Lloyd's was only one of numerous coffee-houses—a rival was Garraway's—he, by his energy, enterprise, and intelligence, speedily obtained an extensive connexion, and sales of vessels constantly took place at his tavern. He had established a wide system of home and foreign correspondents in the principal ports who supplied him with news, which he posted in the tavern for the information of his customers, of the movements of vessels and of maritime affairs generally.

This display of news formed the origin of the present *Lloyd's List*; and the tavern with its special devotion to shipping concerns proved to be the foundation of the subsequent greatness of the Corporation of Lloyd's.

In 1696, Lloyd started a newspaper under the title of *Lloyd's News*, which was published three times a week, and furnished shipping and commercial intelligence. In the seventy-sixth number of the paper some information was inserted relating to certain proceedings in the House of Lords with regard to silk goods, and he was summoned to appear before the Bar of the House for the offence. Instead of rectifying his statement, as the House had required him to do, he suppressed the future issue of the paper, and substituted handwriting for printing in the dissemination of maritime information, so that a written news-sheet continued to be exhibited in the tavern. In 1726, however, the old form of journal was revived under the designation of *Lloyd's List*, which (though now published, since 1884, under a different title) is the oldest newspaper now extant with the sole exception of the official *London Gazette*.

The suppression of the newspaper just mentioned did not interfere with the success of the coffee-house. Lloyd's gradually became recognized as the centre of Marine Insurance on account of the prominence devoted to shipping affairs; this branch of business was subordinate however to the collection and dissemination of maritime news.

The importance to merchants of a sound and systematic scheme of insurance against the losses incident to their properties at sea increased, and the transaction of marine insurance was at first adopted by individuals who on the strength of their personal reputation for good faith and solvency, undertook the risk, while companies were gradually established for the same purpose. Hence the origin of the designation Underwriter lies in the fact that the persons who accept marine risks subscribe or underwrite their names at the foot of the document which furnishes evidence of the contract.

In connexion with the issue of Marine Policies by Corporations, it may be added that in 1720 the exclusive privilege (when the fortunes realized in marine insurance operations stimulated a general desire to extend the lucrative business beyond the confines of Lloyd's) was conceded to two of our oldest offices for the

transaction of this class of risk (in addition to Lloyd's), and as the consideration for this monopoly these companies contributed a considerable sum to the exchequer towards relief of the debt upon the Civil List. One-half of this amount, at least in the case of one of these companies, was remitted in 1771—the original payment, stipulated to be made by instalments, being £300,000 by each Corporation. By an Act passed in the reign of George IV., the monopoly of marine insurance between Lloyd's and the Corporations mentioned was abolished, so that any combination of individuals formed into a joint stock company could engage in the business.

The original document constituting the evidence of the contract was in writing; but ambiguities and difficulties of interpretation gradually became so apparent, and the perils attendant on sea voyages so increased with the complex ramifications of commerce, that clauses were subsequently introduced for the purpose of more exactly defining the underwriters' liability. And since with the enlarging intricacy and extent of commercial transactions the difficulties of conducting the business direct between the merchant and underwriter similarly augmented, there arose in consequence the need of a skilled intermediary named the Marine Insurance Broker.

Lloyd's coffee-house, we have seen, was found to afford a convenient place of exchange between merchants, underwriters and brokers, and thus formed a centre for the conduct of marine insurance, though the frequenters assembled at their own pleasure without subjection to any rules or regulations. The coffee-house, in consequence of the increase of complexity in marine operations, proved in time to be incommodious; and about 1770 the brokers and underwriters (after the death of Lloyd, and apparently dissatisfied with the speculative bargains that were frequently occurring in the coffee-house) removed their place of meeting to temporary premises in Pope's Head Alley. An enlarged abode again became necessary, and the acquisition of suitable premises was considered at a conference of fellow members of the coffee-house towards the end of 1771; and finally, in 1774, it was decided to lease rooms over the north-west side of the Royal Exchange from the Mercers' Company.

It is exceedingly interesting thus to note, as an illustration of the mode in which extensive and systematized schemes of business evolve, that the patrons of a humble coffee-house established themselves ultimately in the centre of the most powerful City as a unique

and vast organization which it may be affirmed with precision of truth possesses no parallel in the records of the commerce of the world.

In the days of the coffee-house the merchants and underwriters, as we have seen, assembled at will and without the control of any regulations relating to the conduct of their transactions. This freedom of meeting and intercourse naturally attracted adventurous and speculative minds to Lloyd's, and their ancient spirit of speculation in a modified and business form has survived to some degree. In early times, for example, the chance of the notorious John Wilkes being elected Member of Parliament for London was valued at five guineas to fifty guineas per cent., and transactions were effected. In 1813 an insurance was completed by which certain unde writers agreed to pay £300 if Napoleon Buonaparte should cease to live or should be taken prisoner before or on the 21st of June, 1813—the policy was to endure for one month and the premium was fixed at three guineas per cent.

The protection of bank deposits is frequently insured at Lloyd's ; the Baring Guarantee at the financial crisis in 1891 was extensively covered there at a premium of ten guineas per cent. ; the chance was insured against the celebrated racehorse, Orme, being scratched out of the Derby runners of 1892 ; insurances against the birth of twins have been effected ; and quite recently a small amount of insurance was completed at Lloyd's on the chances of an alteration of the Income tax in the ensuing Budget—the premium for an insurance against no reduction is stated to have been fixed at five guineas per cent., while for an insurance against a reduction the odds were given as ninety guineas per cent.

The historic and national service rendered by Lloyd's in the establishment of the lifeboat organization should not be omitted from their chronicle. This valuable system would probably have failed in accomplishment—at all events in the days of the originator, Henry Greathead, a boat builder—had not the public spirit and pecuniary aid of members of Lloyd's intervened. Besides assisting Greathead to complete his invention and providing the funds for the construction of the first lifeboat, the Corporation during the year 1802 and for upwards of twenty succeeding years maintained the entire lifeboat service of the country in existence until its transfer to the National Lifeboat Institution.

Lloyd's also rendered signal service to the nation during the years which terminated the eighteenth and commenced the nineteenth century. In 1803, after war had been declared against France, a patriotic fund was originated by Lloyd's for the relief and comfort of the soldiers engaged, the loss of health and limbs they sustained, and the sufferings inflicted upon their widows and orphans. The Corporation itself subscribed £20,000, and having invited public contributions, they collected by the year 1826 the sum of nearly £630,000.

It has also been asserted on adequate grounds that Lloyd's is entitled to the eminent honour of having been the first institution in Great Britain which originated the popular destruction of the slave trade. Cargoes of slaves were constantly conveyed from West Africa to our Colonies of North America (as they then were) and to our West Indian possessions. Insurances were effected upon the slaves at Lloyd's in the same manner as a cargo of cattle. One such cargo was insured with the underwriters subject to the condition that if the slaves perished by the perils of the sea (if, for example, the ship foundered), the underwriters should be liable, while if the deaths occurred from disease the owners of the cargo should suffer the loss. In a certain vessel conveying slaves from West Africa, disease became rife among them, and for the purpose of avoiding a loss to the shippers and transferring it to the underwriters the captain cast the whole cargo of slaves overboard, and represented that drowning, and not disease, had proved the cause of death. The underwriters at Lloyd's discovered the fraud, and resisted payment ; and finally the case was submitted to the Courts of Law. Public attention was aroused to the entire subject, and the widespread indignation thus excited became the sustained origin of a popular agitation of so profound a character that the slave trade was ultimately annihilated.

In 1779 the printed form of policy still in general use was prepared, and the only subsequent change in its wording was introduced in 1850, when the previous preamble " In the name of God, Amen," was cancelled (under a sense of reverence) and the phrase, " Be it known that," was substituted.

The power conferred upon Joint Stock Companies to undertake the business of marine insurance does not appear to have affected the transactions conducted at Lloyd's. The value of commercial

ventures where the perils of the sea are involved, has increased so enormously with the development and complexity of modern commercial interchange that the larger consequent business distributed over a wider area leaves still sufficient for the production of adequate and prosperous results to all.

As a specimen of rates which were current during the seven years' war between 1757 and 1763, we are informed that for the voyage between North America and Jamaica the premium was twelve guineas per cent., while the brief voyage between Liverpool and Gibraltar—a route especially exposed to attack from the French and Spanish fleets—was charged a rate of twenty guineas per cent. In 1782, when practically the whole of the naval powers of Europe were in conflict with Great Britain, ten per cent. was required for the voyage from London to the West Indies under the protection of a convoy ; London to Cork or Dublin, six guineas per cent. ; Liverpool, Bristol and Glasgow to New York, £25 to £30 per cent., with a refund of £16 per cent. if the ships were guarded by a vessel of war.

In 1871, Lloyd's obtained a Charter of Incorporation, and one of the objects defined was the collection and publication of information relating to shipping. This service, it has been noticed, had always formed a prominent feature of the old coffee-house.

The intelligence department of Lloyd's possesses a considerable number of agents in every part of the world to which vessels proceed, and speedy information is despatched by these representatives to Lloyd's, respecting all arrivals, departures, wrecks, and casualties occurring to shipping. The position of a Lloyd's agent is one of high importance and prestige, and is eagerly sought. The agent offers assistance when required to the masters of vessels ashore or in distress ; he takes charge, in the event of a wreck, of the ship's materials and stores ; prevents pillage and waste, and adopts precaution, when repairs to a vessel are necessary, that such repairs, where effected at the cost of the underwriters, are confined to damages received during the voyage insured, without the inclusion of those entailed by a former voyage or defects produced by age and ordinary wear and tear. He guards the interests of the underwriters.

The information thus transmitted from every portion of the globe is analysed and distributed for the benefit of the underwriters and of the public generally.

The captains of vessels are also desired by Lloyd's to supply

particulars relating to ships which may be spoken with on their voyages, and when they become cognizant of any wreck or vessel in distress or overdue to forward intelligence to Lloyd's agent at the first port of call.

A Register of Captains is also maintained, containing a record of the entire body of certified commanders in the mercantile marine service of Great Britain, and including a statement of the vessels which have sustained damage or wreck, or proved exempt from casualty, during the several commands.

An annual publication is also issued by *Lloyd's Register of British and Foreign Shipping* (founded in 1834), furnishing particulars of every British vessel of 100 tons and upwards.

A reference to this register reveals at once a vessel's condition and its fitness for carrying any particular cargo or successfully making any specified voyage. The term "A1" attached to a vessel's name signifies the highest class of wooden ships, while the best type of iron vessels is indicated by "100 A1."

The details thus furnished in the different registers—disclosing the nature of the construction, the ownership, the history of the vessel, and the allotment of symbols—enable the underwriter to assess the appropriate rate of premium.

A Record of Losses—named customarily the Black Book—naturally exists; and it is recorded that, during the extensive and disastrous gale of 1881, 108 casualties to shipping were entered within the course of a single day.

When a vessel is so considerably overdue as to be deemed hopeless by the owner, an application is submitted to the Committee of Lloyd's to "post" it as "missing."

Among the ingenuities of fraud practised upon underwriters may be mentioned the instance of a vessel which was stated to be conveying specie of a considerable amount to the West Indies. She ran upon a reef close to the place of destination, and the captain (who was saved with the crew) reported that she had slid off the rock and had foundered in deep water. The underwriters decided to attempt salvage: an expedition was accordingly despatched, and the cargo of the ship was discovered to consist of stones.

Until the first year of Queen Anne's reign, the law did not comprise insurance frauds within the schedule of punishable offences, and a grievous burden was thus imposed upon underwriting interests.

The oldest policy preserved at Lloyd's is one dated the 20th of November, 1680, and insures for £1,200 (£200 on the vessel and £1,000 upon goods) the *Golden Fleece* for a voyage from Lisbon to Venice at a premium of £4 per cent.

Lloyd's as a Corporation sustains no financial liability for the solvency of its members ; its corporate duty is limited to the admission so far as is practicable of men of repute and stability ; to the provision of facilities to underwriters and brokers in the transaction of business, but the nature and volume of that business do not enter into the official cognizance of the Corporation; and the requirement by the Corporation of a deposit from members of at least £5,000 as a guarantee for the performance of their contracts. It is stated by a recent writer of authority that the total deposits amount to £3,500,000.

The history of Lloyd's for prompt and honourable dealing ranks it among the finest institutions which the commerce of any country has ever created.

CHAPTER XIV

Marine Insurance (*continued*)

The Contract

The contract implied in a marine insurance policy is one of indemnity, where the claimant is recouped for loss sustained (from specified dangers or perils), without the exaction of a profit, to the extent of the sum assured. But the construction of the term indemnity is ambiguous. A merchant employs his sagacity and skilled experience in trade which involves not simply the price of the goods he insures, but also the anticipated profit upon the enterprise. If he recover from the policy simply the amount he invested in the merchandise and the cost of shipping, he forfeits the benefit which the venture was undertaken to procure, had no perils of the sea existed, consisting of the augmented value or price which the project would have afforded had the articles reached their destination and been sold. An indemnity consequently which omitted the element of this calculated profit would evidently be an imperfect one, and would depress to an appreciable extent the incitement to a vigorous and enterprising trade with foreign lands. A course accordingly is frequently adopted which consists of the valuation of the goods at the invoice cost and a percentage representative of the expenditure actually incurred in the shipping and insurance, with the expected profit on realization. If in the event the policyholder protects the entire amount of his valuation, the result is either excessive or deficient insurance according to the prices ruling in the foreign market meantime falling or increasing. It is therefore the general practice to employ different descriptions of policies, which we will describe at a later stage.

The Conduct of the Business.

In the application for insurance, the condition of the fullest truthfulness of representation must be observed in order that both persons to the contract may be placed in a clear and complete position as regards their respective risks and benefits. And here another ambiguity of terms occurs, for the word

"risk" is employed both for the liability of the underwriter under the agreement, and in a more restricted sense for the perils which the contract covers, such as the risk of fire and of jettison. This criticism is not limited to marine insurance, but is of universal application in commerce; and it is an interesting observation to notice how terms of apparent looseness and vagueness of meaning can yet perform the function of definite and exact service in the commercial interchanges of life. The result is attained by the operation of two causes: the establishment of a fixed current conventional significance by the custom of the particular trade or trades, however confused may be the meaning to minds outside the business sphere; and the happy prevalence of mutual trust and personal honour which, by a liberal and just interpretation, avoid the perplexities and contentions which the existence of verbal ambiguity might produce.

The process of insurance consists in the broker presenting to the underwriter a piece of paper, technically termed the "slip," on which appears the broker's name in print, and usually a few printed clauses of general application. The broker inserts under his name the nature of the risk proposed to be covered, the name of the vessel, the intended voyage, and other essential particulars. Each underwriter then adds underneath the statement the amount he is prepared to accept, and appends his initials only. The slip does not in itself constitute the contract: it is but the preliminary form and serves as the basis of the final contract; in brief, the slip is simply a provisional agreement obliging the issue, in honour only, of a stamped policy on certain conditions. But the slip cannot be stamped and sued upon as a policy: at least, this appears to be the sounder opinion although other authorities consider that the question is not free from doubt. Although by Act of Parliament it is provided that no agreement for a marine insurance is valid until a policy expressing the contract has been issued, the practice in underwriting is honourably beyond this narrow legal boundary. The obligation in good faith and honour to effect to the utmost the agreement preliminarily embodied in the slip and initialled is treated in so large-minded a manner that even if any particulars connected with the risk may arrive subsequent to acceptance, which may really affect the nature of the hazard entertained, such information need not be communicated; as Lord

Blackburn expressed it, the insured is not bound to lead his underwriter into "temptation" by supplying information which ought to exercise no influence upon his decision, but which might expose him to a temptation to break a contract already concluded: though, with the probity and confidence which mediate between the contracting persons, any relevant facts would no doubt be at once intimated; and it is presumably this invariable experience which constitutes the ground of the practice we have described. At the same time it is to be observed that until the rate of premium or "quotation" named by the underwriter has been accepted by the applicant or by persons authorized to represent him, no legal obligation is entailed upon the underwriter; until this acquiescence in the terms has occurred no agreement obviously exists and the underwriter accordingly is entitled at any time, prior to acceptance, to withdraw his offer. In law, even where the offer assumes the form of what is termed in commercial transactions a "firm" offer, no such final responsibility rests on the underwriter to maintain it open until acceptance or rejection of the quotation has been expressed. Still it does not ensue that a quotation or firm offer fails to create an honourable obligation upon the underwriter. The character of any proposed contract for insurance may obviously be materially altered by the receipt of news subsequent to the completion of the slip; and the underwriter again, on renewed consideration may be disposed to revise his quoted terms. This is simply a fair position, for each side should be placed on an equality of risk and advantage, and since the applicant is free to consider the advisability or expediency of acceptance or refusal, so should the underwriter be accorded a corresponding right. Any difficulty of this character is always justly and fairly solved by the traditional and continued system of good faith which prevails as the basis of marine insurance commitments. When the terms of the proposed insurance have been definitely agreed, two methods of business ensue upon the initialling of the slip. At Lloyd's, the broker himself prepares the policy, and procures the signature of the underwriter. Insurance Companies, however, prepare their own policies, from a copy of the original slip made out by the broker for this purpose and containing full details in place of the previous abbreviations. This second slip has been termed "a request unto or mandate to the company to make out and

execute a policy" ; but, like the primary slip, with which it constitutes one entire and indivisible contract, it is unenforceable.

Among the facilities for the transaction of Marine Insurance business should be mentioned those which are afforded by Shipowners' Mutual Clubs and Protecting Associations. These bodies originated through the inadequacy of the means of effecting insurances during the period while the monopoly of the two chartered corporations existed ; since its repeal in 1824 they have maintained their position partly owing to the economy in the cost of insurance secured by the mutual principle and partly in consequence of the fact that they insure risks of a peculiar character, the amount of premium for which could not adequately be estimated in advance but is made dependent upon the occurrence of the event. Medical and funeral expenses in respect of the crew, the costs of litigation, raising of wrecks, damage caused to harbours, etc., small damages and items not allowed on the ordinary sea policy form a portion of the risks which such Institutions undertake. These Associations are now required to be registered under the Companies' Acts. Their method of business varies somewhat in details, but an owner desiring to effect insurance applies in writing for admission to the membership of the Club. If the insurance relates to a sea risk, as defined by the Stamp Act, a policy is issued and an initial premium paid. For the discharge of claims and expenses, calls are made, as required, upon the members in proportion to the values of their several interests insured.

The Insurable Interest.

Without the possibility of damage occurring to any venture, no person obviously would contract by insurance for an indemnity against loss ; and it is this element which distinguishes a policy of marine insurance from a mere wagering agreement, against the contraction of which the Act of 18 Geo. II., cap. 37 was passed. This Act, however, did not prohibit wagering agreements in respect of foreign ships or of goods laden thereon, but the better opinion is that these insurances would be rendered invalid by the Act of 8 and 9 Vict., cap. 109, Sect. 18. If this be so, it is not necessary to consider the extent to which such insurances would also be forbidden by 14 Geo. III. cap. 48 (1774),—a statute upon which no writer upon Marine Insurance would appear to rely.

From the completest form of insurable interest—that of exclusive

ownership—many grades of interest exist which justly constitute the foundation of a policy.

A lighterman charged with the custody of goods for conveyance from the land to a vessel, or from a vessel to the shore, is liable, apart from any express contract, for any loss which the goods may sustain in transit, unless the damage be occasioned by what is termed the Act of God (*force majeure*), or by the Sovereign's enemies. He accordingly possesses an insurable interest; the merchandise, it is true, may be protected by a policy effected by the proprietor, but even if such a policy exist, the lighterman is debarred from claiming its benefit,—the principle of Subrogation (the assignment to the person who has discharged an obligation of all rights and interest in the subject concerned) only applying to contracts of indemnity, while the liability of the lighterman is imposed by the common law relating to common carriers. The fundamental obligation of shipowners to shippers is identical with that of a lighterman, that is to say, to carry and deliver safely subject to the same general exceptions or any special stipulations; but the question whether the owner of a general ship is bound by the other obligations of a common carrier, namely, to carry goods at a reasonable rate for any person who may offer, has never been raised.

It is obvious that the owners of goods exposed to the perils of the sea possess a conclusive insurable interest. When, however, the goods have been purchased from the owner, the benefit of the policy which the owner may have effected is not transferred to the buyer unless the transfer is contained in the contract for sale, since once the property in the goods has passed on sale, the original insured ceases to retain any insurable interest, and hence possesses nothing to transfer. If a merchant to whom merchandise is consigned has advanced money upon its security, he is vested with a legal interest in protecting the forfeiture of his loan which the loss of the goods might produce.

With reference to the vessel itself the owner of the entirety of it can clearly insure; and, by a judicial decision, the proprietor of a share or shares in a ship is empowered legally to cover the value of his holding by insurance.

If a shipmaster remove his vessel for repairs to a port which is not the nearest convenient port, he is responsible for the vessel's safety and can insure its value.

A mortgagee of a vessel is not entitled to insure in excess of the amount he has advanced, unless the excess of this sum be insured on account of the mortgagor.

A vessel encounters an accident on the voyage which necessitates a return to the port whence she sailed, or to some intermediate port of refuge; while stationed there expenses are incurred and are discharged by the ship's agent in the interests of all who possess a holding in the vessel. The disburser of such charges is vested with an insurable interest to their extent from the port of refuge to the destination, where the costs will be defrayed by the ship, the cargo and the freight in certain proportions to be determined at the close of the voyage.

The profits anticipated from the venture of goods exposed to the dangers of the sea confer an insurable interest on those to whom they will accrue, and the amount of this interest, instead of being expressed separately in the policy, is usually added to the valuation of the merchandise, from the sale of which the profit will be derived.

Connected with this subject of insurable interest occurs the case where the same interest has been insured several times with different underwriters, or a multiple insurance exists. Where no fraudulent intention is involved, and the same amount of insurance has been effected with two underwriters, for example, the insured is entitled to claim the loss in full from the one he may select, and that underwriter in his turn is entitled to receive one-half of the sum from the other. In practice, however, the insured himself, in the case just cited, would invariably restrict his recovery to one-half of his claim from each underwriter.

The Policy

We proceed now to the nature and issue of the policy as the legal evidence of the preliminary contract.

Policies may be divided into Voyage Policies and Time Policies, where goods are respectively insured during their transmission from one place to another, or for a specific period of time.

In a Valued Policy, the value of the insured articles is expressly mentioned, and the burden of proof in any allegation of fraudulent excess valuation is imposed upon the underwriter; while open policies do not contain that statement, and in presenting a claim

under a policy of this character the insured must prove the value of the subject insured. The valuation of a vessel for insurance under an open policy is her actual worth at the beginning of the voyage, including the stores, provisions, and outfit, the money advanced for seamen's wages, and the cost of insurance. The evident difficulty of proving an exact value suggests therefore the expediency of valued policies.

In Floating Policies there is usually furnished no name of the vessel on which the risk is insured, and the wording is sufficiently b oad to comprise the interest insured whatever may be the steamer, ship or ships by which the goods are conveyed. The shipment in a floating policy is thus covered in respect of unknown vessels and affords the merchant protection in the event of a loss happening before he is able to effect a specific insurance. When he is placed in a position to adopt this course of business, he "declares" the particulars to the underwriter, when the amount and the name of the vessel are endorsed on the policy.

A few leading explanations may be furnished in connexion with the form of the policy. The common form of a Lloyd's Policy constitutes the basis of all British policies, and in an early part of it occurs the expression that the proposer makes assurance and causes himself to be insured "lost or not lost, at and from . . . upon any kind of goods and merchandise and also upon the body, tackle, boat, &c., of and in the good ship or vessel called the . . . whereof is master (under God) for this present voyage (name of master) . . ." The curious expression, "lost or not lost," is derived from ancient times, and its introduction is supposed to have been intended to provide for the case of missing ships, that is, of vessels acknowledged at the time they are insured to be so long at sea, unheard of, that doubt exists respecting their safety. In the earlier conditions of foreign trade and the deficient organization, and, indeed, the impossibility, of constant and prompt intelligence, it must frequently have occurred that when the policy was effected no knowledge could be possessed of the safety or loss of the vessel. The expression is peculiar to English and American policies. At the present time, of course, the words are of general significance, and if, perhaps, now superfluous, since the condition may be inferred from the spirit of the contract, they yet place it beyond dispute that if the subject insured be lost or have arrived in safety when the

contract is made, it is still valid or mutually enforceable if completed in ignorance of the event.

The description of the voyage is then inserted after the words " at and from " : the risk on the ship commencing when the ship is " at " the place in reasonable safety ; and upon the goods from the time of their being exposed to the perils of the sea, within the conditions contained in the policy.

The course of a vessel will naturally be affected by the weather, the season, political conditions abroad, and many other circumstances, so that variations in the customary route may be imperative and not voluntary. In law, the voyage stated to be covered by the insurance is the course at sea from the starting point to the terminating point as prescribed by the customary track of navigation, and the transit of the vessel insured must virtually correspond with that traditional or usual passage. Between any two ports, the course at sea is generally the path along which the one can be gained from the othe · in the briefest period of time compatible with safety.

Originally, goods alone appear to have been insured under a marine policy, that is, the merchants only were protected, but at an early stage the ship itself was also covered in the protection of the shipowner.

It is of interest to notice that if, for example, a vessel carries as a portion of its cargo a stock of ropes belonging to the shipowner and intended to be utilised eventually as rigging, their loss could not be claimed on the ground that they formed part of the ship's tackle ; and this even though, in the event of necessity, they might be applied to that purpose ; since they would be in reality something beyond what would be " necessary, suitable, or usual " for that particular voyage, and would relate to the cargo rather than to the ship.

The shorthand expression f.o.b. (free on board) may here be explained. The clause introduced into the policy protecting the goods during their transit to the vessel by means of lighters or barges does not operate where the goods are purchased f.o.b., that is to say, where the vendor has contracted to place them on the ship by which they are to be conveyed—the risk in transit being, in this event, devolved upon the seller and his underwriter.

The policy provides protection for the merchandise until it be

" discharged and safely landed " at the place of arrival covered by the contract. If the merchandise is not discharged from the vessel on to a quay or pier or other usual place of debarkation, but into a barge, the point of discharge, it is clear, is not identical with that of safe landing, for the latter expression implies delivery at the customary places for the landing of goods.

As a general proposition, however, it has been decided that if it be the practice of any trade to effect the landing of goods by means of lighters or barges, the clause in the contracts still covers the risk of this intermediate transport.

It has been stated that the term " voyage " implies technically the course of navigation customarily pursued between port and port ; so that goods insured from London to Sydney and loaded on board a ship sailing from London to Melbourne would not be covered. Hence with a view to providing for possible deviations from ordinary routes, the policy on goods usually contains a clause that if a change of voyage occur the deviation shall be held covered at a premium to be arranged, provided prompt intimation be given by the owner of the goods of the altered passage. A clause of analogous tenor is generally found in voyage policies on hulls of vessels. Permissible deviation, it may be mentioned, from the voyage covered by the policy, may occur. We need merely cite four forms : (1) where the alteration of route happens beyond the control of the captain and owner ; the violence of the weather, for example, already referred to, may blow a ship out of her course ; (2) where the deviation is reasonably demanded by the safety of the vessel or cargo insured : a ship damaged by tempest may seek a port of refuge for repair ; (3) where the vessel leaves the course to save human life or to help a distressed ship where life may be endangered ; and (4) where the alteration is rendered compulsory by the barratrous action of the master or crew if the peril of barratry be one of the risks included in the insurance.

We have referred to Open Policies where no value is expressed, and Valued Policies where a value is furnished ; and it is the general practice in England to adopt the latter form of insurance for goods and ships, while a large proportion of freight policies are of the open character.

The etymological sense of freight is cargo or load, but the term is now employed in English marine practice for the hire for carrying

goods. A vessel is hired from the owner by a merchant, and the contract of hiring either with persons who take a ship or with persons who desire their merchandise to be conveyed in her, constitutes a contract of "freightment," and the "freight" is the sum paid to the owner for transporting the cargo. It may further be explained that when a vessel is hired to another person for the purpose of conveying goods, the person so hiring the vessel is termed the "Charterer," and the agreement between him and the owner is described as a "charter party." If the cargo is not restricted to one class of commodity, such as a cargo of wheat, but comprises different kinds of goods, the owner of the vessel on receipt of the articles for conveyance to some specified destination gives an acknowledgment for the goods, which is termed a "bill of lading."

But the term "open" obviously requires at all events some provisional or customary definition, and the judicial decisions expressing the limits of the term direct that, in respect of merchandise, the value is formed by the prime cost of the goods added to the expense of shipping and the premium for the insurance; that as regards the vessel itself, the valuation shall consist of the value it possessed at the beginning of the voyage, comprising also the value of the stores and provisions for the crew, the outfit, and the charge for insurance; while the valuation in freight shall be the gross freight due to the vessel for the transport of the goods when she reaches her destination, together with the cost of the insurance.

It will be obvious that many anomalies are involved in these decisions, and hence open policies are seldom adopted.

A judge has expressed the advantage of the system of valued policies; which consists in providing the insured with the full value of the article insured and enabling the underwriter to secure a larger amount of profit: thus producing a fair result on both sides. The difficulty of excessive valuation of course occurs, which, however, cannot be here discussed; and an authoritative writer has justly pointed out that instances have happened where the insurance of four times the amount of the invoice cost would be warranted: such, for example, as a shipment of silver to Japan with the intention of securing gold in exchange at the Japanese ratio of 4 to 1, when the interchangeable value elsewhere at the time was about $15\frac{1}{2}$ to 1.

The form of policy in marine insurance as a descendant practically unaltered of the rudimentary document prepared in ancient and different times, has been the subject of continual judicial animadversion, on account of its laxity of construction ; and here, as we have stated, we possess an eminent example of the righteous and equitable manner in which complicated transactions can be settled upon the basis of an antiquated and indefinite document when mutual confidence and personal honour act as the agents of interpretation, regarding the spirit of the contract beyond the literal form.

It has been judicially laid down that the words of the policy must be understood in their plain obvious sense, unless they possess a current special and customary significance in the particular trade, and unless the context is of such a character that the ordinary meaning of the words will not apply. It will be remembered that, in addition to the printed body of the text of the policy, the terms may be contained also in marginal printed clauses, in printed or stamped clauses impressed upon or attached to the policy, and in clauses written upon the face of the policy. Hence a process of interpretation is rendered the more difficult. It is only necessary to observe upon this point that the provisions in the text and clauses of the policy in favour of the insured are taken to be cumulative and not exclusive or restrictive of one another ; that is, additional clauses attached to the policy for the purpose of increasing the extent of the insured's indemnity (that is, augmenting the area of insurance payable) will not deprive him of any indemnity he may possess under the original text ; and that, in accordance with the universal principles of the construction of documents, any ambiguity is interpreted in the sense least favourable to the underwriter on the equitable ground that he is the framer of the form. Where, however, a clause is framed by the insured and introduced into the policy for his own benefit, it would, of course, appear, on the same principle, that its terms would be construed against him.

Assignments

The Act of 31 and 32 Vict. c. 86, provides that whenever a policy of insurance on any ship, or on the goods in a ship or on any freight, is assigned to any person entitled to the property insured, the assignee is empowered to sue upon the policy in his own name.

The Perils of the Sea

In the form of Policy the underwriters undertake certain risks specified as "adventures and perils." A leading judgment defined the expression "perils of the sea" to the effect that it did not comprise every casualty or accident which might occur to the subject of the insurance on the sea : it must be a peril "of" the sea. And it has also been finally settled that the expression does not include every damage or loss of which the sea is the immediate cause ; for example, the incessant and natural action of the sea which produces "wear and tear" of fabric. There must occur some casualty which could not be foreseen as one of the necessary incidents of the voyage ; for the object of the insurance is to provide an indemnity against accidents which *may* happen, and not against events which *must* happen. In a certain case before the Courts it appeared that a vessel proceeding from London to Honduras was detained at Antigua in consequence of the illness of the crew. While remaining there, holes in the bottom of the ship were made by rats, which rendered the vessel unfit for continuing her voyage, and the goods were sold at a loss. It was held that this resulting loss was not one which entailed liability upon the underwriters who insured the property only against immediate consequences of the perils of the sea. Had the damage been directly caused by sea perils the fact that it was originally due to rats would not have relieved the underwriter from liability. For example, had sea-water reached the cargo through the holes which the rats had made it may be presumed that the policy would have been liable. By custom, no liability attaches for the loss of articles placed in improper or insecure positions, such as water-casks on deck or ropes lying on deck when the vessel is not entering or leaving port. Comprised within the phrase of "perils of the sea" are foundering, stranding, damage produced by collision with another vessel or in consequence of adverse weather. Loss of the vessel by fire or by lightning is protected. In a judgment delivered in 1807 it was affirmed that the claim would be recoverable in the event of fire, even if the fire were an act performed in duty to the State. Thus, a vessel which had been insured was chased by a French privateer, and when escape became impossible the captain and crew burnt the ship in order to prevent its capture. The underwriters were held to be liable. To the lay mind the reason of the decision is not very

evident. As a general statement, it seems that underwriters of policies effected upon goods are not liable for damage to those goods by a fire produced by the condition in which the goods were shipped on board.

Jettison is the act of throwing cargo overboard for the purpose of lightening the vessel and thus conducing to the chance of her escape from danger. If the danger be real, so that the sacrifice is imperative, and the goods of a merchant are cast away to prevent a threatened loss being transformed into an actual one, the merchant is not placed in any worse position than would have been the case had the loss occurred, and the same remark applies to the underwriter. On the assumption that jettison was absolutely demanded, it is to be noted that under an ordinary policy the underwriter is not liable for jettison or abandonment of cargo carried *on* the deck : the goods covered by a policy, unless an express clause to the contrary be inserted or unless it be the universal usage of the particular trade to convey cargo exclusively on deck, are goods which are stowed *under* deck. It need hardly be added that goods cast overboard by reason of intrinsic defect (for example, meat which has become putrid and dangerous to the health of the crew) cannot be claimed against the underwriter under an ordinary description of policy.

A fraudulent jettison, one which is not demanded by genuine maritime emergency, performed by the master of the vessel and the crew on their own motion and without the shipowner's approval, is termed Barratry. If the shipowner concur, the liability for the value of the perished goods is entailed upon him.

The precise definition of barratry is difficult, for the definition must exclude simple mistakes, misjudgments, and, to an extent, negligence, while it must comprise, on the other hand, acts of the captain against the owners fraudulently committed for his own advantage at the owners' expense, and acts of the crew against the captain and owners (for example, a mutiny and seizure of the vessel, and, in the opinion of some authorities, theft), and must thus embrace an element of criminality.

The scuttling of a vessel, the intentional running of a vessel ashore for the purpose of casting her away, setting the ship deliberately on fire, illegally disposing of a ship and cargo and appropriating the proceeds, and diverging from the proper course of the vessel for

the captain's private business, are included among barratrous proceedings.

In former days, the fleets of one nation warred with those of another, not only by means of regular ships of war, but also with the armed vessels of private individuals (privateers), licensed by the State to assist in damaging the enemy. These risks of capture and seizure are often exempted from the policy by the addition of the clause "free of capture and seizure" (or, symbolically, F. C. and S.).

The Loss of Vessel and Cargo

A total loss occurs when, by the perils insured against, the subject insured is destroyed, or so injured as to become valueless or of trifling value to the insured for the purposes and uses for which it was intended, or is removed from the possession and control of the insured so that he is entirely deprived of it. We may assume a special example: a properly-equipped vessel leaves a port for a voyage which ordinarily lasts for three months; at the expiration of six months no news is received of arrival at the destined port, nor has she been reported as having passed islands or stations situated along the course she was intended to pursue, nor has any "speaking" been obtained of her by other ships following or crossing the track in question or any sea-path in its neighbourhood. Inquiry continues to be instituted; and finally the vessel is "posted" at Lloyd's as missing. No actual proof of the cause of loss can obviously be secured; the presumption, however, arises that as she sailed in sound condition she must have perished by some peril of the sea expressed in the policy. A loss is thus constituted which falls upon the underwriter. Even although a vessel may not be completely lost, a total loss may yet ensue. If she run upon rocks and sustain damage to the bottom, it may be that she will never be removed from the position as a ship: she may be ground away by attrition against the rocks, and sink into the ocean as a mass merely of timber and iron; while, even if she could be floated, the expense of repair would prove so burdensome as to render the removal a loss in any commercial sense. In an instance of this kind the owner must furnish notice to the underwriter of his intention to abandon the interest insured. This notice forms a condition precedent to the right to recover a Constructive Total Loss, and must

be intimated in order that the underwriter may be informed of the insurer's election and thus possess an opportunity of protecting his own interests as affected by the decision. The abandonment is invariably refused. If the facts confirm the owner's judgment that the vessel, though existing *in specie*, cannot be preserved from actual total loss or only at a cost greater than its value would be if such expenditure were incurred, the loss is termed a Constructive Total Loss ; in other words, a loss which is not an actual one, but a loss in the view of construction by law. The underwriter would thus, on payment of the claim, possess the salvage and obtain the ownership of the vessel and any subsequent earnings. The subject is often technical in character, and its further consideration cannot be pursued in an elementary and explanatory manual. And the loss arising in respect of other interests in ships or goods could not be dealt with.

Average

In what is technically termed the memorandum added to the policy occur the words " free from average " in respect of the insurance.

The word average signifies a loss less than total, and resulting from sea-damage. For example, corn, fish, salt, fruit, flour and seed are warranted in the policy free from average, unless general or the ship be stranded : sugar, tobacco, hemp, flax, hides and skins are warranted free from average under £5 per cent. ; all other goods, together with the ship and freight, are warranted free from average under £3 per cent., unless general or the ship be stranded. The effect of this clause (taking as an example the first group of goods) is to relieve the underwriter from any extent of deterioration of the articles by sea-damage, however great, which does not amount to a total loss ; that is, he is free from claims for particular average (or partial damage), or (regarding the second and third groups) from such claims if under the specified percentages, unless the vessel be stranded ; in the latter event, he remains responsible as to all the groups for such claims whether caused by the stranding or not.

As the subject of average is perplexing an additional explanation may be attempted. In the Law of Marine Insurance, all loss which arises in consequence of extraordinary sacrifices made, or expenses incurred, for the preservation of the ship and cargo, are comprised

within General Average, and must be borne proportionably by all who are interested, or, as the Rhodian Law* expressed it, "that which has been given for all shall be replaced by the contribution of all." Losses occasioned by jettison of cargo for the common safety, or in consequence of a forced discharge with a view to floating a stranded ship, or by the cutting away of a mast to avert a common danger, and by other acts of a similar nature, would form the subject of a General Average. A particular average is the average or damage accidentally caused by some sea peril, and consists in either a deterioration, or a total loss, of part of the subject insured, and this loss is borne solely by the person upon whom it falls. If, upon a vessel conveying as part of its cargo a supply of tea, a certain number of the packets of the tea were damaged by salt water in consequence of a leak, the tea would be alone affected, and the owner of that article or the underwriter (who had insured the tea) would be the person to sustain the loss, while the holders of other interests —the ship and remaining cargo—would be relieved from loss. With regard to the clause freeing the underwriter from claims, that is, averages, not amounting to 3 per cent. on the sum assured, it may be pointed out that great inconvenience and trouble were occasioned by claims of a trivial amount, and hence the provision was adopted that in connexion with the ship and cargo generally the underwriter should be relieved from claims (that is, averages) which did not amount to 3 per cent. of the sum insured. In summary: the word average, in the marine sense, implies damage to the property insured, either actual damage or necessary expenses, and any damage which is less than total loss is "average"; part of a shipment, for example, of sugar may be spoilt by sea-water; a portion of a shipment of heavy articles may be jettisoned or thrown overboard for the purpose of lightening the ship and enabling her to proceed; silk by contact with the water may become deteriorated in value; and all these damages are "average." If cargo is cast overboard for the benefit of the safety of the vessel, the loss is incurred for the common advantage, and the average (or damage) is accordingly described as "general average,"—being made good proportionably by all persons concerned in the adventure, whereas,

* The Rhodians early acquired a great commercial reputation, and a compilation of sea laws existed (known as the Maritime Law of the Rhodians) which were generally observed in the trade of the Levant. It is inferred that these laws prevailed between 900 and 700 B.C.

as has just been stated, if the average (or damage) be accidentally occasioned by some peril insured against, and not deliberately by the act of the person for the common benefit, that loss must be sustained exclusively by the person upon whom it falls, and is termed a Particular Average; and this whether the damage occur to a whole cargo, such as grain or sugar, or to a particular portion of it, or to one or more interests in a general cargo.

Free of Particular Average or the F.P.A. Clause

Particular average, so far as it can be rendered intelligible to persons not practically trained in the business of marine insurance, has been defined, as we have pointed out, as the liability attaching to a marine policy in respect of damage or partial loss, that is, deterioration or diminution, accidentally and immediately caused by some of the perils insured against to some particular interest (such as the ship alone or the cargo alone) which has arrived at its destination. As may be supposed, such accidental damage may easily arise from any of the perils enumerated in the policy, and thus a serious liability is cast upon the underwriter. The extent of this liability has been somewhat restricted by the permanent addition to the policy, introduced in 1749, of the clause already referred to as the "Memorandum." But the list of goods specified in the first sentence of that clause is but meagre, and soon became supplemented by a special clause to be adopted as required, termed the F.P.A. clause, which, while it protected the underwriter, concurrently offered the insured cheaper terms of insurance, and thus accommodated conflicting interests. Broadly speaking, it provides that the underwriter shall not pay for any loss or damage short of a total loss unless the vessel be stranded, sunk, burnt, or in collision.

Free of all Average (F.A.A.) Policies

arise where the underwriter is to discharge no claim for deterioration or partial loss. He remains liable only for total loss, absolute or constructive, and for particular charges incurred to avert a total loss.

Wear and Tear

In judging of the nature of the liability of an underwriter under the common form of policy, an important question occurs: the element, namely, of the inherent or natural tendency in physical

objects to defect and decay which may be described as wear and tear. Ships, for example, from the intrinsic nature of their materials, progressively and continuously proceed in deterioration. It may be truly stated that a considerable proportion of the casualties and damages sustained at sea do not arise from the perils of the sea or other extraordinary events alone or from wear and tear exclusively, but from the combined action of all these causes. The determination of the limit consequently where wear and tear terminate is one of the troubled problems of marine insurance.

Stranding

The underwriter's exemption from claim for anything short of a total loss of the specified articles or of loss of a stated percentage of value in other cases (mentioned previously) is removed by the stranding of the vessel, and the act of stranding must be fortuitous or accidental. A mere striking or touching any rock or reef or other object does not amount to a stranding : the ship must remain fixed for some definite time. It was stated in a judgment that stranding must imply a settling of the ship (not a settling down), an interruption of the voyage, so that for the time being and in respect of its intended purpose the vessel might be considered to be wrecked, and from which misfortune much damage occasionally results. When the stranding is intentional, the vessel being beached to prevent sinking in deep water, the act is deemed to constitute a strand, since the ship was stranded not in the ordinary course of navigation, but in order to avoid impending danger.

It may be regarded as settled that for the constitution of a stranding within the scope of the memorandum to the policy, the interest insured must be in one common adventure with the ship at the time when she took the ground.

Burning

The case of burning may occur. If the property insured be a cargo of flour, for example, and the flour takes fire and is destroyed without damage resulting from the fire to the vessel, the underwriter is free of claim for partial loss or damage of the flour. It is the ship that must be burnt or part of it. The destruction of a cabin therefore by fire removes the exception, while a fire in the cargo itself fails to do so.

All other Perils, Losses, and Misfortunes.

In concluding the consideration of the liabilities of the policy, a brief attention must be devoted to the effect of what are termed the "general" words—"all other perils, etc." Although these words have formed the subject of important decisions, it is still difficult to frame any adequate definition of their meaning. It would appear that the cause of loss must be accidental as opposed to one that is foreseen; external to the subject insured in contra-distinction from inherent defect; and consisting of some irresistible force or agency as opposed to wear and tear. The most important decision of recent years upon the construction of these words was that relating to the s.s. *Inchmaree*. The effect of the judgment in the House of Lords (in 1887) was to exclude damage through negligence, explosions, bursting of boilers, breakage of shafts, or any latent defect. Since that decision, these liabilities have been specially added to the policy, where needed, by a clause designated the "Inchmaree" clause.

Time Insurance

In the preceding pages we have dealt with goods in transport from one port to another where the specification of any particular time during which the risk continued is needless—a voyage insurance, in brief. In time policies the settlement of the period of risk is the principal feature. The system of insuring vessels by means of policies of this nature is very ancient. It is referred to in the Ordinances of France of 1681, while it forms at the present day a most valuable section of an underwriter's business. And it is manifestly convenient. For if vessels be engaged in regular employment or upon short voyages, it would involve needless anxiety and expense to maintain the protection of insurance voyage by voyage. Freight is also commonly insured for time, but the terms of insurance are somewhat open to discussion, and their legal significance remains to be more precisely defined. The effects of captains and officers are also usually insured for time. Goods, however, are rarely so insured, for the ordinary floating policy is a more convenient contract for that purpose, subsisting, as it does, for no arbitrary period, but until the sum insured is exhausted by declarations of goods shipped. The warranty usually inserted in floating policies that the goods shall be shipped by a certain date does not constitute the insurance one for time.

(By the Stamp Act of 1891, 54 and 55 Victoria, it is enacted that no policy of sea insurance made for time shall be effected for any period exceeding twelve months.)

In connexion with the presumption of loss under a time policy, an interesting case upon the point was decided seventeen years ago. In this instance a policy for twelve months lapsed eighteen days after the starting of the ship on a voyage of twenty-five days. The policy was not renewed; no intelligence of the ship was ever again received; and no direct evidence of the date of her loss could be obtained. It was held by the judge that the insured was not bound to prove that the loss occurred during the duration of the policy, and that if the available evidence indicated the probability of loss of the vessel before the time when the policy lapsed, the underwriter was liable.

The Essential Elements of a Valid Marine Insurance

These consist of the completion of the vessel's passage in the voyage insured, or the customary route of navigation, the legality of the traffic in which the voyage is made, and the seaworthiness of the ship. Consequently, the causes which entail a vitiation of the policy are deviation, illegality of trade, and unseaworthiness. The implied warranty of seaworthiness does not apply to a "time" policy: nevertheless, in such case, if the vessel be lost through unseaworthiness without the intervention of any peril insured against, the underwriter will not be liable. Beyond the three implied warranties just mentioned, certain other essential requirements must be present in every policy of Marine Insurance. These necessary conditions are imposed by three authorities—the Common Law applicable to all contracts, Statutes other than Stamp Acts, and Stamp Acts. The requirements of these three authorities are identical in respect of certain particulars and would appear to consist of: writing, the parties, their mutual assent, the valuable consideration, the particular risk or adventure, the date, the place where the contract is made, the sum insured, execution, and the stamp.

The Adjustment of Losses

The work of distinguishing the items of account which should be respectively charged to the merchant, the shipowner and the

underwriter, demands specialized knowledge and experience. Hence a class of experts has arisen who devote their time exclusively to the adjustment of marine losses by apportioning the shares which appertain to the several persons interested as just mentioned.

The average adjuster, in respect of claims upon goods, determines the insured value of the deterioration or diminution. This latter result he arrives at by a comparison of sound and damaged values based upon the certificates of surveyors or sworn brokers, and with the aid of other documents he prepares his average (or loss) statement, which expresses the total amount due from the underwriters.

Bottomry Bond

If a ship is involved in trouble, and, for the purpose of completing the voyage or removing the difficulty, money must be raised, the captain is entitled to obtain the necessary sum by creating a charge upon the ship, and the deed of mortgage is termed a "Bottomry." The mortgagee would insure his advance, as a protection against the subsequent loss of the vessel, until repayment. With the increased facilities of communication between the various parts of the world, the ramifications of international exchanges, and the consequent ease of remittance, the practice of raising money by Bottomry is now seldom adopted.

Demurrage

If, for example, a ship be hired and she is not discharged at the time mentioned in the charter party, the person responsible for discharging is liable for the sum, termed Demurrage, entailed by the delay.

In respect of any detention of a vessel at a port of refuge, the underwriter is not responsible for any claim for loss (for example, wages and provisions) to the shipowner resulting from the delay, that is to say, employing the technical term, he is not liable for demurrage; neither does he incur liability for the loss of any interest on the value of the goods which the owner of those goods may sustain in consequence of the delay. A claim of this character would obviously be one not for loss actually incurred, but for the failure of the full anticipated profit. It is usual, however, in most policies to undertake to pay General Average supplemented

by the provisions of the York-Antwerp Rules,* in which case such expenses as the wages and provisions, above referred to, would be allowed. The underwriter may also become responsible for demurrage where the vessel he has insured is liable for damage to another vessel through collision. In 1836 the policy in the common form was held to be free from any liability for damage caused, since the obligation to pay for the damage was neither a necessary nor a proximate effect of the perils of the sea, but arose out of an arbitrary provision in the law of nations. For the purpose of overcoming the effect of this decision, the collision-clause was introduced. Although appended to the policy on the ship, it forms a totally distinct contract of insurance in istelf. By its terms the underwriter engages to repay to the insured his due proportion of all sums which the insured may defray by way of damages in consequence of his liability for any collision, or his share of ¾ths of such damages, as the clause may provide. It is customary for the underwriter to limit his liability to ¾ths only of the shipowners' risk, under the original conception that some incentive to the exercise of care should be cast upon him and his servants. But this restriction was speedily disregarded, and it is always possible for a shipowner to secure full protection either with Lloyd's and the Companies alone, or by entering his vessel, as has been described, in some Shipowners' Club or association. The general practice of granting only a three-fourths' indemnity is to-day merely a consideration of premium. An invariable item in claims for damage produced is one for demurrage. The space already occupied in this chapter forbids a consideration of the incidence of the liability for damage occasioned by collision according as one or both of the vessels may be in fault, controlled, as it is, by the statutory protection of the shipowner from any responsibility in excess of £15 a ton where the loss of life alone, or the loss of life and of ship and cargo, is involved, and £8 a ton where the loss of cargo and ship alone is caused : nor obviously can we deal with the application of such liability to the policy under the terms of the collision-clause in particular cases.

* In 1860, a Congress was formed of persons (British and foreign) interested in marine business and insurance, and a code of International General Average Rules was prepared at York, known as the York Rules. They do not appear to have passed into general practice ; but in 1877 at Antwerp, the Association for the Reform and Codification of the Law of Nations completed a set of regulations upon their basis, to which the title of the " York-Antwerp Rules " is applied. These have been subsequently revised and extended.

CHAPTER XV

Accident Insurance; and other Descriptions of Insurance against Casualties

The statistics available are not sufficiently codified, so far as the public are concerned, to permit the presentation of a formal basis for these modes of insurance in this book.

The financial experience of ordinary accident companies adequately proves that their premiums are founded upon a careful and express comparison (which all these forms of insurance must involve, more or less systematically) of the numbers of persons or articles exposed to the risk of accidents or damage in a year with the number of casualties which occur. In personal accidents the element of age is not important, as in Life Assurance, since in ordinary exposure to accidents no special or progressive liability to risk is entailed; the force of accidental injury does not advance in a corresponding manner with the rate of mortality age by age; and where the age is increased, and the chance, through vital failure, of the occurrence of accident is thereby augmented, this enhanced probability is compensated by prudential diminution of exposure to risk. On the other hand, the business of indemnity against accidents connected with the Workmen's Compensation Acts must to a considerable extent—as in the early stages of every novel description of enterprise—be experimental, and dependent rather upon sagacity than amplitude or precision of data. Experience of the results of these Acts (and, admitting their beneficial tendency, they also apparently and indirectly involve an influence upon the social future of workmen, especially at their advanced ages, which does not promise to prove altogether advantageous) is at present too restricted to enable any approximately close numerical measure to be formed of the onerous liabilities they will ultimately entail. It is accordingly proposed in this chapter to describe briefly the history of Accident Insurance in its customary forms, and to classify the various modes in which the general principles of insurance have been applied as a provision against specific casualties.

The astonishing development of railway construction sixty years ago, and the dangers attendant on the conveyance of large

numbers of passengers in a single train, prompted the establishment of a system of insurance protection in connexion with railway accidents, and in the period from 1845 to 1848, eleven companies were projected for this purpose of which only one (founded in 1848) has survived.

The serious burden of stamp duty then levied introduced grave difficulty, but ultimately the Government agreed to the issue of unstamped tickets in consideration of a composition duty assessed upon the receipts. It is curious to note that the legislature expressed the fear that, under this scheme of insurance, unscrupulous mothers might be tempted to obtain a gain of money by the death or maiming of children where the family was too numerous, and hence a provision was enacted that no child under the age of twelve should be granted the benefit of an insurance ticket. And we understand that to this day the issue of a ticket to such persons is prohibited.

Difficulties of interpretation arose with respect to the meaning of death incident to, and consequent upon, railway travelling, and finally by an Act of Parliament passed in June, 1864 (27 and 28 Vict., cap. cxxv.) a railway accident was defined as an accident to the train, and the following scale of allowances was there prescribed :—

Premium.	Fatal accidents.	Non-fatal accidents.	
		Total disablement.	Partial disablement.
d.	£	£ s. d.	£ s. d.
3	1,000	6 0 0 per week	1 10 0 per week
2	500	3 0 0 ,, ,,	15 0 ,, ,,
1	200	1 5 0 ,, ,,	6 3 ,, ,,

The payment of compensation was limited to 26 weeks, and travellers by excursion trains could not recover more than £500.

In January, 1850, a Company was formed for insuring against the risk of death resulting within three months from the occurrence of an accident of any description. This scheme was almost immediately extended to include the insurance of compensation for non-fatal accidental bodily injury, and the modern system of insurance

against general accidents dates from the June of that year. The scale of benefits was originally settled at £1,000 for death by accident and £5 per week during complete disablement for a period not exceeding 26 weeks, with a sum not exceeding £10 for medical attendance during the injury. The allowance for medical attendance was subsequently abandoned, and the payment during disablement increased to £6 per week.

The originally limited sphere of accident insurance has gradually been expanded, with the amplification of social needs and sources of danger, into numerous forms. Increased benefits for the total loss of limbs or eyesight—casualties which had been omitted from Accident Policies since 1855—were again introduced in 1880. More recently, the benefits payable under ordinary policies have been doubled in respect of railway accidents, and the system of granting a temporary allowance for disablement by disease has been adopted. The diseases included within the scope of insurance were at first confined to typhoid fever, typhus fever, scarlet fever, smallpox and measles, but the list has now been extended to nearly 40 specified complaints, while, under some descriptions of policy, sickness of every kind is covered. The scheme of insurance has also been modified by the grant of an annual addition of 5 per cent. to the sum payable at death until it reaches 150 per cent. of the original amount assured ; by a reduction of 10 per cent. on the premiums to total abstainers ; and by the removal of restrictions of locality in connexion with both accident and disease.

It is of interest to learn that, in an accident in 1868, when the train was burnt, among the dead was a person of whose identity the only available evidence consisted of his signet-ring, which was discovered among the ashes, and with the honour which distinguishes our insurance institutions this slender proof was accepted as sufficient. Among the many unexpected origins of claims by death are found the slipping on dead leaves, being struck by a flint from a neighbouring quarry, tripping over a telegraph wire, the scratching of a cat, and the bursting of a soda-water bottle.

We proceed now to describe briefly the several adaptations of insurance to the various possibilities of risk and loss.

Accidents

of every kind can be insured against, and it is estimated that one person out of every ten insured sustains an accident of some description and receives compensation, while one accident out of every hundred terminates fatally.

Boiler Insurance

Steam boilers can be officially inspected by trained men employed by companies, and insurance granted against damage by explosion, including injury sustained by surrounding property. Personal injuries to workpeople and other persons resulting from boiler accidents are also provided against.

Burglary Insurance

Protection is afforded against loss and damage by burglary and housebreaking, and also, in the case of private houses, against larceny or theft by servants. In some instances the burglary and housebreaking policy does not contain the "average clause," so that the company is liable in the event of loss to the extent of the total amount insured.

Judicial statistics show that upwards of 70,000 known "professional" thieves exist. It has recently been stated, on trustworthy authority, that, in the area controlled by the Metropolitan Police (excluding the non-residential city), property of the value of upwards of £500,000 was stolen during the last two years. This loss was the result of over 35,000 felonies among a population of 7,000,000.

The modern system of burglary insurance is not the product of the nineteenth century alone, for an institution of "insurance from housebreakers" was established during the prolific period of the "South Sea Bubble."

Calls upon Shares.

The possible losses arising from calls upon shares, not fully paid up, held in commercial and industrial companies within the United Kingdom can be secured by a Guarantee policy.

Carriages

Vans and vehicles are insurable against accidental damage, including the cost of repairs which may be necessary in respect

of accidents caused by collision, and by the falling, kicking, or bolting of horses. The premium is assessed upon the full cost price of the vehicle when new.

Credit.

Merchants can be protected by insurance against an abnormal percentage of loss arising from bad debts.

Cyclist and Motor Riders

may insure their machines against burglary, theft, fire, and accident, and provide compensation for accidents to themselves, to paid drivers, and to third parties.

Diseases

of almost every description can be pecuniarily encountered with a policy.

Drivers of Vehicles.

Employers of drivers can be indemnified against the liability they sustain to pay compensation for injuries to the person or property of third parties, caused by the drivers' negligence. The Common Law imposes upon the employer personal responsibility for injuries occasioned to any person through negligence or breach of duty not only on his own part, but also on the part of any servant if that servant is acting within the scope of his allotted employment. If a coachman has been directed to drive to the city by the Thames Embankment and, in disobedience of orders, proceeded by way of the Strand and there, in consequence of negligence, caused an accident, his employer would be liable since the coachman was acting within the general scope of his employment, which was to drive his master's carriage.

Fidelity Guarantees

protect the employers against any violation of trust (involving pecuniary loss) on the part of managers, clerks, travellers, or collectors. The Insurance Company having ascertained, by enquiry of previous employers and private referees, that the past history and character of the applicant for a guarantee are satisfactory, completes an agreement with the employer—based upon a statement of the system of supervision which he proposes to exercise over the accounts and the performance of the duties of the applicant—

to reimburse any loss from defaults which may be committed by the official during the currency of the policy. The security of the principal companies is accepted in respect of government and municipal officials, trustees in bankruptcy, and liquidators and receivers appointed by the Courts of Chancery. The insurance may assume the form of (1) a collective policy, where a particular sum (as the extent of the company's liability) is placed against the name of each member of the staff, or (2) a floating policy which secures the employer against any loss, to the limit of the insurance, in any one year, whether the defalcations be committed by *any one* of the members of the staff or by *any number* of them. The alterations required in the policy in consequence of additions to the staff or substitutions in the body of officials may be effected by means of an endorsement.

Forged Transfers

may be guaranteed against.

Hail Storms

Damage to growing crops can be protected, and the insurance is based upon the acreage covered by the crops. We learn from the experience of a certain company that their losses under this class of insurance, though the risks were carefully selected, have proved exceedingly severe, and so far therefore as this particular record indicates, the exposed position of agriculturists forcibly suggests the necessity of this special protection.

Horses and Cattle

can be insured, including "floating" policies upon the entire stock on a farm.

Keys and Season Tickets

If a key or season ticket be hopelessly lost, compensation is provided up to a certain amount.

Licensed Properties

Loss may be sustained by depreciation of property of this description in consequence of the deprivation of the license, and guarantees are available to compensate the diminution of value.

Lifts.

Owners or tenants of passenger lifts and other lifts can be insured against liability for damages for personal injuries resulting to individuals from accidents occasioned by or in connexion with the lifts, and against pecuniary loss entailed by the breakdown of the lift machinery.

Lost Documents.

The loss may occur of a life policy (which has to be delivered to the assuring company on the death of the assured); of the certificates of stocks or shares; and of bonds, deposit receipts, debentures, and the title deeds of properties. Indemnities are provided by guarantee companies for these missing instruments, and thus enable claims to be paid, transfers of securities to be effected, and properties to be sold or mortgaged.

Mortgage and Debenture Guarantees

However valid a mortgage or debenture may be in solidity at the date of investment, the possibility always exists of a depreciation of value, and a guarantee policy supplies the requisite protection, and involves merely the diminution of the rate of interest yielded to the extent of the premium paid. In guaranteeing the debentures of commercial and industrial undertakings, the guaranteeing company would first investigate the title and value of the properties comprised in the debenture issue as the security to the holders, and would maintain an expert oversight upon the progress of the undertaking during the currency of the bond. The individual investor cannot obviously institute these enquiries himself, and hence in accepting a debenture thus protected he is secured in the receipt of his interest throughout the term of the bond and in the return of his principal upon the expiry of that period, or at prior liquidation should it occur.

Plate Glass

The insurances granted cover breakage from all causes except fire.

Registered Articles,

such as bonds, notes and other valuable documents, despatched by registered post to any part of the world, can be insured.

Titles to Properties

The system of guarantee is extended to protect pecuniary loss due to defects in title, or to missing beneficiaries hereafter appearing and claiming an estate or a portion of it against the present holder under a presumptive title, or to lunatics recovering sanity and disposing elsewhere of property now in the possession of the next apparent successor, and providing also against the dispossession of property (a protection especially necessary to mortgagees and purchasers) which would be entailed by the birth of issue, the remarriage of a life-tenant, or the abandonment of a certain "name and arms," on the retention of which the ownership of an estate depends.

Trusteeships and Executorships.

A Company can be appointed the Executors or Trustees under wills and marriage settlements, so that the nomination to these offices of private persons, and the trouble and expense of the substitution of others as deaths occur, are thus avoided.

Workmen's Compensation Insurance.

The vast importance, from an economic and social point of view, of modern legislation in connexion with the risks of accident and death to workpeople in the various trades and occupations, suggests the propriety of a somewhat extended survey.

Under the Common Law—which previously regulated the rights of workmen against employers for compensation in connexion with injuries incurred in their service—the dependants of persons killed by accidents possessed no remedy ; and it was not until the passing of Lord Campbell's Act (9 and 10 Vict., cap. 93) in 1846 that damages could be recovered by them—the commencement of a legal action being permissible within 12 months from the workman's death from injury. The doctrine of "Common Employment," as expressed by the Courts, affirmed that when an employé, on entering into a contract of service, agreed, from the nature of the contract, to take upon himself known risks of his employment, he also contracted to assume upon himself, as one of the obvious hazards of the trade, the risk arising from the negligence of fellow-servants. Hence, at Common Law, an employé could only recover damages against the

employer when the actual negligence of the latter could be established. The employer was, however, liable for an injury resulting from his omission to provide the servant with proper materials and resources for his work or from his negligence in the choice of persons to whom he entrusted the supply of these materials and resources, or in the selection of proper servants.

In 1880 the Employers' Liability Act (43 and 44 Vict., cap. 42) was passed with the object of restricting the application of the doctrine of Common Employment and rendering the employer responsible for the negligence of those of his servants on whom he had conferred authority or the duty of superintendence. This Act provides that where personal injury is caused to a workman by defect in the condition of the ways, works, machinery or plant used in the business, or by the negligence of any person in the employers' service who is entrusted with superintendence or to whose orders the workman was bound to conform, and did conform, the workman or (in the event of the injury resulting in death) his legal representatives shall possess the same right of compensation against the employer as if he had not been in that employer's service. The amount of compensation recoverable is limited to a sum not exceeding 3 years' wages; notice of an accident must be furnished within 6 weeks from its occurrence; and an action to recover must be commenced within 6 months from the same date, or, if death has ensued, the action must be brought within 12 months from the death.

The Workmen's Compensation Act of 1897 (60 and 61 Vict., cap. 37) provides that if personal injury by accident arising "out of and in the course of" the employment is occasioned to a workman, and is not attributable to his own serious and wilful misconduct, the employer shall be liable to grant compensation on the following basis: (1) where death results from the injury, and the workman leaves dependants wholly supported by his earnings, the sum payable consists of 3 years' wages with the condition that the amount shall not be less than £150 or exceed £300; (2) if he leave dependants partially supported by his earnings, the amount of compensation is to be agreed upon, with the limitation that it shall not be in excess of the sum payable in the preceding event; while (3) if no dependants be left, the compensation consists of the expenses of his medical attendance and burial, not exceeding £10. (A paper

recently issued by an expert contained an estimate (based upon the number of deaths furnished in the Factory Inspectors' Returns) that the percentage of cases of male workmen over 18 years of age where dependants would be left at death, amounted to 56, so that in 44 per cent. of the total fatal accidents, no relatives supported by the workmen would exist as claimants.) Where total or partial incapacity for work is produced by the injury, a weekly payment is enforced during the period of disability (after the second week) which shall not exceed either one-half of the workman's average weekly earnings or £1 per week. Written notice of the accident must be supplied to the employer as soon as practicable after its occurrence and before the workman has voluntarily left his employment, while a claim must be submitted within 6 months from the date of the accident or, in the event of death, within 6 months from the time of death. The Act applies to employment "on or in or about" railways, factories, mines, quarries, or engineering works on buildings exceeding 30 feet in height which may be in course of construction, or of repair, by means of scaffolding, or of demolition, or on which machinery driven by steam, water, or other mechanical power is being used for any of the three purposes just mentioned. It may be stated, therefore, that practically the sole exception to liability under this Act occurs where the injury is attributable to the serious and wilful misconduct of the workman himself.

In 1900, the benefits of the Workmen's Compensation Act were extended (by 63 and 64 Vict., cap. 22) to workers in agriculture—this occupation being defined as comprising horticulture, forestry, the keeping or breeding of live stock, poultry, or bees, and the cultivation of fruit and vegetables. It may be added that recent decisions have removed the restriction involved in the qualification "on or in or about" as regards agricultural work.

The liabilities imposed upon employers by these Acts of Parliament form the subject of an increasingly important branch of insurance. The rates of premium are assessed as a percentage on the wages paid, but owing to the absence of adequate statistical data, the limited experience hitherto acquired of the operation of the later Acts, and the keenness of competition, varying rates are frequently quoted for an identical description of risk. Speaking generally, the premiums are adjusted according to the severity of the hazard involved in the specific occupation. The proposition has been

suggested by persons who are unacquainted with the practical difficulties and complexities of the business that each workman should be insured under a separate policy at a scale of premium increasing with the age. A slight consideration will suffice to show that this course would not only be accompanied by grave hardships to workmen whose policies were not effected at an early age, but would also introduce considerable complications in the instances of men leaving one trade for another; in respect further of casual labourers or men paid by the hour, and as regards, again, the collection of the premiums from the employer upon whom the burden of the liability is imposed. These difficulties are obviated by the system of basing the premiums upon the amount of the wage-list. The sum expended in wages evidently affords an accurate measure of the number of men effectively exposed to risk, rising and falling, as it does, when trade is brisk or depressed, and thus obviously rendering unnecessary all minute calculations of the period of actual occupation of each individual workman. The main objection to the plan in connexion with the Workmen's Compensation Act lies in the minimum limit of £150 for compensation in fatal accidents. It is evident that if wages fall, the insurance premium concurrently falls automatically, and since the minimum cost of fatal accidents, where dependants exist, remains constant, the claim-ratio in this respect becomes heavier. It would appear that the loading to the premium for the risk should be considerable on account of the cost of possible litigation and (at all events under existing circumstances) as a provision, in an undertaking of this novel character, for the contingency of the divergence of actual experience from the indications of the scanty data provisionally adopted.

Workmen's insurance usually presents several marked characteristics, namely, (1) a fairly constant ratio of fatal accidents to the number of persons exposed to risk; (2) a continuous increase in the number and cost of claims for minor accidents; and (3) in many schemes, an element of deferred or postponed liability.

In this country no comprehensive statistics, officially published, at present exist in a form adapted to provide a suitable basis for the computation of premiums for the insurance of workmen's compensation, and accordingly, when this description of business was originally undertaken by companies, the data issued by the German and Austrian Government Insurance Departments were necessarily

and provisionally utilised. The following Tables, extracted from those sources, illustrate to some extent the characteristics already mentioned.

LABOUR ACCIDENTS PER 100,000 MEN EMPLOYED AND EXPOSED TO RISK.

A : *Germany.*

Year.	Deaths.	Cases of permanent disablement.	Cases of temporary disablement lasting over 13 weeks.	Cases not compensated.
1891	71	374	110	2,639
1894	66	398	162	3,012
1897	70	362	259	3,485
1900	74	367	306	3,729
1902	64	385	358	3,792

B : *Austria.*

Year.	Deaths.	Cases of permanent disablement.	Cases of temporary disablement lasting over 4 weeks.	Cases not compensated.
1891	66	251	708	1,461
1894	68	374	826	2,799
1897	68	368	1,015	3,699
1900	69	408	1,100	3,938

The large proportion of cases not really warranting compensation will be observed. A similar increase in the number of accidents is also noticeable in the experience of an important company in Milan for the insurance of workpeople.

The following Tables expressing the experience (for the 5 years ending on the 30th of June, 1903), of the Iron Trades' Employers' Insurance Association are extracted from the Report of the Departmental Committee appointed, in 1903, to enquire into the law relating to compensation for injuries to workmen :—

TABLE I

Year	Number of firms	Number of men	Accidents			Claims		
			Number	Average number per firm	Number per 100 men	Number	Number per cent. of accidents	Number per 100 men
1st	604	205,000	12,247	20	5·9	3,277	26·75	1·6
2nd	629	212,000	12,322	20	5·8	4,031	32·71	1·9
3rd	621	202,000	11,236	18	5·6	4,090	36·40	2·0
4th	656	236,000	13,235	20	5·6	5,426	40·99	2·3
5th	695	248,000	13,663	20	5·5	5,747	42·06	2·3

TABLE II

Exhibiting the Increasing Average Cost per Claim.

Year	Number of claims	Amount of claims	Average amount
		£	£ s. d.
1st	2,815	19,935	7 1 8
2nd	3,914	31,353	8 0 3
3rd	3,920	32,727	8 6 11
4th	4,957	51,937	10 9 7
5th	5,569	54,947	9 17 4

The preceding figures are especially instructive in illustrating two important features : (1) that the number of claims increases rather than the number of accidents, and (2) that the average cost of claims also increases, so that (3) the burden of the loss-ratio is subject to two factors tending to render it continually heavier.

The element of deferred liability for compensation occurs in different forms. It may arise in connexion with the payment of compensation by way of an annuity, when a danger may exist of inadequate provision for the recurring instalments, or it may be involved in the provisions of the law by which an extended period is granted to the injured workman within which an action for damages may be instituted. In illustration of the latter difficulty, it may be mentioned that in some of the United States of America an action may be brought, to recover damages, 4, 6, or even 10 years after the accident has occurred. In the American practice, the difficulty naturally arises (since these possible acts of litigation cannot be accurately foreseen) of fixing the proper reserve for such suspended possibilities of liability : in the more reasonable British

legislation, this element is practically extinguished by the brevity of the period during which a claim can be legally enforced.

Two important dangers, requiring to be recognised, assessed, and guarded against in Workmen's Compensation Insurance consist of (1) claims for trivial injuries and (2) malingering (or feigned illness) on the part of the claimants. As a protection from the first danger it is usually provided, either by legislation or by the scheme of insurance, that the compensation shall not commence until a certain period has elapsed since the accident, while the second can approximately be minimised by fixing the allowance below the average earnings of the workman, so that he may be deprived of inducement to prolong the period of disablement artificially.

Regarding the indirect consequences of legislation for securing compensation to injured workmen, the Report of the Departmental Committee (already referred to) contains the statement that the evidence submitted proved that the Workmen's Compensation Acts had largely augmented the difficulties of old men in finding and retaining employment, and that, in the judgment of the Committee, these difficulties tended to increase. The opinion was entertained generally by the representatives both of the workmen and of the employers who were examined that, in some industries at all events, old men were more liable to accidents than young men, and further (and more important) that the seriousness of the consequences likely to ensue upon even a slight accident increased with the age, so that the chances against elderly men obtaining fresh employment when out of work and even of retaining their present employment must be materially augmented. The Report also expressed the conclusion that, on the whole, the extent of actual litigation produced by the two Compensation Acts of 1897 and 1900 had proved to be very small when compared with the great number of cases settled by agreement without any recourse whatever to legal aid or to proceedings in Court. It should be justly borne in mind that a considerable share of the litigation which has attracted public attention has been rendered imperative by the need of referring to the courts upon the proper interpretation of novel Acts, and, consequently, related rather to the establishment of principles than the adjustment of claims. An examination of these Acts will reveal numerous ambiguities of language which may (and, in many instances, must) require determination by the courts, for several

of the principal terms employed are incapable of precise definition.

The extent of workmen's insurance is very considerable, and its area will no doubt largely expand. In Germany, Austria, and Norway this branch of business is undertaken by the Government: in other countries, it is carried on by insurance companies which are usually subject to state supervision and compelled by law to deposit with the Government considerable sums in securities for the protection of the policy-holders.

So far as we are aware, in nearly every instance in which this class of insurance has been pursued, the volume of income anticipated, combined with the uncertainty of the risk involved (due essentially to the paucity of appropriate data) has led companies to underestimate the net cost and to compete for the business at inadequate rates, with the consequence at the outset of comparatively unprofitable results. This course is the more to be regretted on the ground that, owing to the tendency of the net cost to increase continuously, the sequent necessity arises of a frequent revision of rates to counterbalance that increase alone, and thus it is rendered extremely difficult to recoup any losses produced by over-sanguine underwriting in the early stages. This contingency, however, it may fairly be submitted, applies largely to all transactions of a novel character where the materials for computation are insufficient, the real hazards only competent of disclosure and assessment on experimental trial, and where accordingly hope tends to predominate over cautious prevision. The probabilities comprised in this description of risk are exceedingly complex, and an element of bargaining and negotiation enters into the settlement of claims. A workman is frequently willing to accept a capital sum in commutation of an annuity, and in some instances this course may prove to his advantage: on the other hand, it may reasonably be urged that the capital may be promptly or gradually expended without permanency of benefit, and the family accordingly be deprived of the continued aid to maintenance which the annuity provided and which it was a primary object presumably of the Acts to secure.

For the reasons already adduced it is difficult at present to proceed further than the tabulation of past experience in the form of a percentage of cost upon the total wages, and on this basis to assess the

rates. The failure, in this country, to establish a central organisation among the Companies for the combination and comparison of experience as a foundation for adjusted premiums is, we consider, accountable for the publication of widely divergent scales of rates, while the intense competition for business has entailed heavy losses in some quarters ; and it seems probable that many years must elapse before an accurate measure of the several hazards can be determined or a complete recovery secured from the effects of former errors of judgment. It is simply reasonable to add the comment, applicable, as we have stated, to every form of human enterprise, that fresh experiments in business, through the absence of serviceable information derived from the past as an index to the future, must always involve appreciable chances of misconception and mis-estimation of probabilities, with consequently temporary financial strain. But diligent collection and analysis of acquired experience ; sagacity in its interpretation ; and steadfast adherence to *facts* in graduating premiums to hazards, will in due time reward this scheme of insurance enterprise with steady and permanent prosperity and conduct it within the sphere of scientifically-constructed systems of work. And the guarantees of adequate success may be suggested as the combination of Offices for the registration and analysis of their united experiences as the data for the computation of premiums, and (pertinent to all forms of business) the dominance of the elements of financial strength and profit over mere expanse of transactions provided a sufficient basis of risks be obtained. We may conclude in general terms that the administration of an enterprise of this complex nature demands supremely a sound commercial training which gradually confers the power of commercial insight and prevision.

We may add the concluding remark that, like most legislation of a complicated character, whose ramifying and divergent consequences have not been exhaustively conceived, it seems to be undoubtedly the case that these Workmen's Acts are not entirely beneficial either to employers, the workmen, or the companies. Slight injuries previously deemed insignificant will tend to be made the foundation of claims, with a probable increase of premiums ; it is not unlikely that a spirit of litigation, with its evil results, will be gradually promoted ; and the grave consequence must, in all probability, occur that since the liability to accidents (in specially

hazardous trades) increases with the age, the older men (though otherwise thoroughly serviceable, and indeed more so by reason of their trained experience) may be compelled to relinquish work, and thereby become dependent in advanced age.

Legislation upon trade and commerce invariably confirms the wise remark of Bastiat that, in all complex problems, the consequences which are not obvious should be most diligently sought and scrutinised since, generally, they exceed, in vital significance, the results which are clearly evident ; and the remedy for an admitted evil may thus tend, without this vigilant investigation and prevision, simply to intensify its range and power.

READERS of this volume who desire instruction in the practical conduct of Insurance Business should procure a copy of

"INSURANCE OFFICE ORGANIZATION, MANAGEMENT, AND ACCOUNTS,"

BY THE AUTHOR OF "INSURANCE: A practical exposition for the Student and Business Man," AND RICHARD MASTERS, A.C.A.

Demy 8vo, 140 pp. Price 3/6 net, post-free 3/10.

LONDON:
SIR ISAAC PITMAN AND SONS, LTD.,
1 AMEN CORNER, E.C.

370
Y7
38C 5

Author Young, Thomas Emley
Title Insurance. Ed.2,rev.
82588
EcF
Y77i
DATE.

UNIVERSITY OF TORONTO
LIBRARY

Do not
remove
the card
from this
Pocket.

Acme Library Card Pocket
Under Pat. "Ref. Index File."
Made by LIBRARY BUREAU